



NSRIT

AUTONOMOUS

ANSWER KEY & SCHEME OF EVALUATION

SEE III
Second Year B. Tech.

**ACADEMIC
REGULATION
2020**

Academic Year
2020 - 2021



Semester End Examination, February, 2022

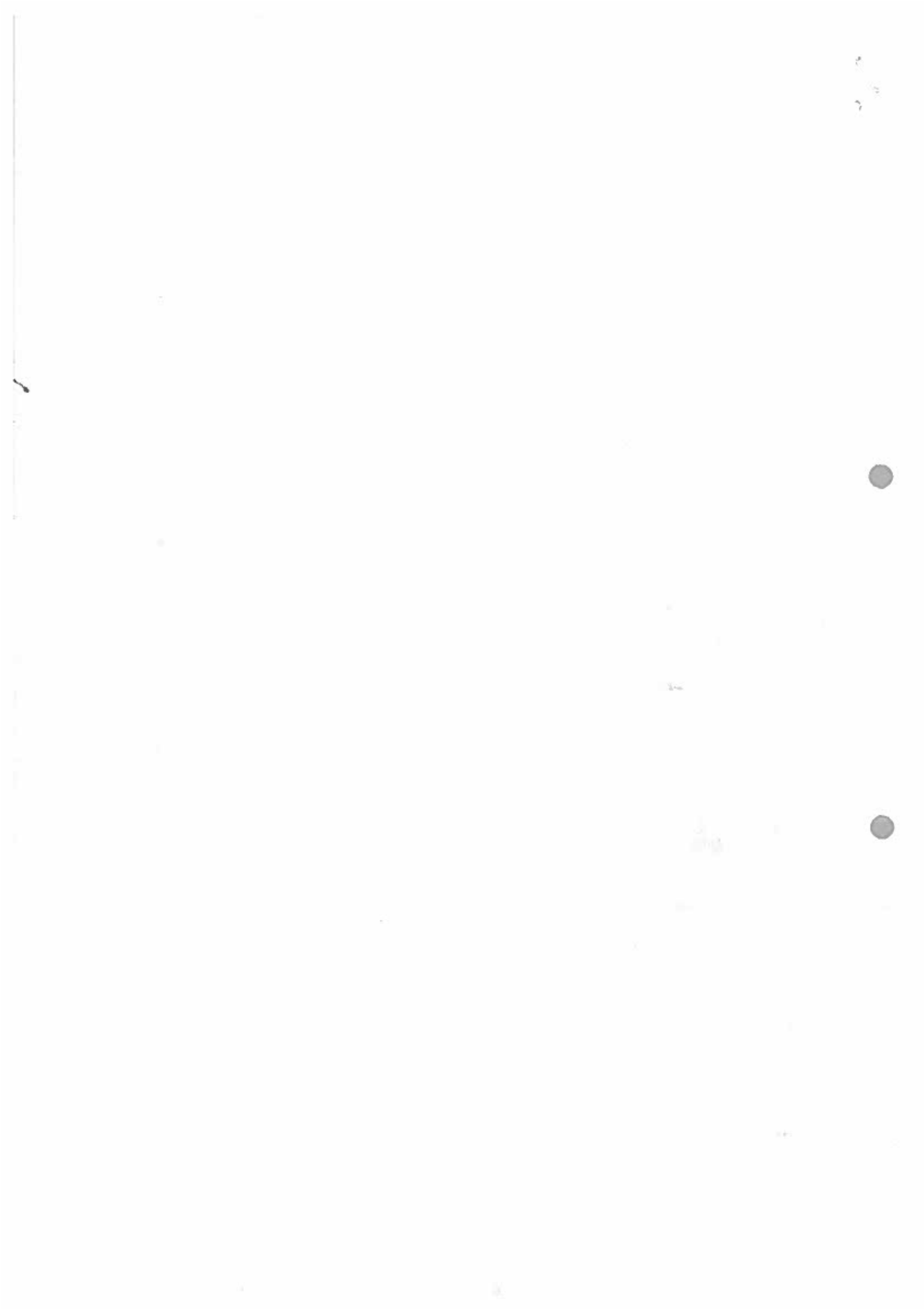
Degree	B. Tech. (U. G.)	Program	Civil Engineering	Academic Year	2021- 2022
Course Code	20CE303	Test Duration	3 Hrs. Max. Marks 70	Semester	III
Course	SURVEYING				

Part A (Short Answer Questions 5 x 2 = 10 Marks)

No.	Questions (1 through 5)	Learning Outcome (s)	DoK
1	Define principles of surveying	20CE303.1	L1
2	Distinguish between back sight and Fore sight	20CE303.2	L1
3	Define Tangent Point and Point of curve.	20CE303.3	L1
4	Define photogrammetry.	20CE303.4	L1
5	Define GIS.	20CE303.5	L1

Part B (Long Answer Questions 5 x 12 = 60 Marks)

No.	Questions (6 through 15)	Marks	Learning Outcome (s)	DoK
6	Describe the linear measurement of chaining on a slope along with their advantages and disadvantages.	12 M	20CE303.1	L1
OR				
7	What is meant by local attraction? How is it detected and corrected for local attraction.	12M	20CE303.1	L1
8	Describe the height of instrument method of reducing the levels. Compare the height of instrument method with rise and fall method.	12M	20CE303.2	L2
OR				
9	Explain the procedure to determine the tacheometric constants by a) Fixed Hair Method b) Movable Hair Method	12M	20CE303.2	L2
10	What is a transition curve? State the various types of transition curves with the help of a neat sketch. Explain briefly its necessity.	12M	20CE303.3	L2
OR				
11	Discuss about the recent advancements in total stations and their usefulness in monitoring structures.	12M	20CE303.3	L2
12	Explain the aspects of flight planning for an aerial survey and obtain an expression for the number of photographs required for given area of length and width of a survey	12M	20CE303.4	L2
OR				
13	Describe the various segments of GPS surveying. Also, give a brief account of the applicability and limitations of each technique.	12M	20CE303.4	L2
14	Discuss the possible advantages and disadvantages of using a raster GIS and vector GIS.	12M	20CE303.5	L2
OR				
15	Discuss in detail about the Supervised and Unsupervised image classification in Geographical Information System.	12M	20CE303.5	L2



ANSWER KEY AND SCHEME OF EVALUATION

Course code: 20CE303

COURSE: SURVEYING

(1) Define Principles of Surveying?

- (A)
- To work from whole to the part.
 - Triangulation divided into large triangles.
 - To control and localise minor errors.

(2) Distinguish Between Back Sight and Fore Sight?

(A) Back Sight:

Sign taken on a point of known Elevation, to obtain the height of instruments. It is taken as plus sight.

Fore Sight:

Taken on a point of unknown Elevation. It is taken as minus sight.

(3) Define Tangent point and Point of Curve?

(A) point of Curve:

It refers to the starting point of the curve where alignment changes from tangent to the curve.

Point of Tangent:

It refers to the point where

alignment changes from curve to the tangent.

(4) Define the photogrammetry?

(A) Photogrammetry:

It is the science of obtaining information about physical objects through process of recording, measuring and interpreting of photographs of the area.

(5) Define the GIS?

(A) GIS:

GIS is defined as an information system that is used to input, store, retrieve, manipulate, analyze and output geographically referenced data (or) geospatial data, in order to support decision making for planning and management of land use, natural resources, environment, transportation, urban facilities etc.

ANSWER KEY AND SCHEME OF EVALUATION

Course code: 20CE303

COURSE: SURVEYING

Q) Describe the linear measurement of chaining on a slope along with their advantages and disadvantages?

(A) Linear Measurements:

There are various methods of making linear measurements and their relative merit depends upon the degree of precision required. They can be mainly divided into three heads.

- (1) Direct measurements.
- (2) Measurement by optical means
- (3) Electro-Magnetic methods.

In case of direct measurements, distances are actually measured on the ground with help of a chain or a tape or any other instrument.

In the optical methods, observations are made through a telescope and calculations are done for the distances, such as in tachometry or triangulation.

In the electro-magnetic methods, distances are measured with instruments

that rely on propagation, reflection and subsequent reception of either radio waves, light waves or infrared waves.

Advantages of chains in Surveying:-

- chain survey is simplest and commonest method used in surveying exercises.
- The equipment used to conduct chain survey are simple to use.
- The equipment used in chain survey can easily be replaced. For example, measuring rods can be replaced with measuring tape.
- This method does not involve complicated mathematical calculation. It is a relief to those who are afraid of mathematics.
- In chain survey few people are needed to conduct the survey. Normally chain survey team has three people: Bookers, leader and follower.

Disadvantages of chains in Surveying:

- Simple chain survey cannot be conducted in built up areas and large areas.
- Simple chain survey is subject to several chances of errors of accumulation which may cause by problem of chain. The chain linkage may fail to stretch up properly and result in inaccurate data.

ANSWER KEY AND SCHEME OF EVALUATION

Course code: 20CE303

COURSE: SURVEYING

Also clogging of chain may lead to error in reading.

- It is time consuming.
- It may not be conducted in areas with steep slopes or water logged areas. chain survey is usually conducted in dry areas with gentle slopes. It becomes more complicated when survey is conducted in areas that are too wet.
- chain survey becomes more complicated method when there are raised points (obstacles) in between areas to be surveyed.

(7) what is meant by local attraction? how is it detected and corrected for local attraction?

(A) Local Attraction:-

The magnetic needle does not indicate the north direction, when it is under the influence of attractive bodies in its surroundings, such a disturbing influence is termed as "local attraction".

The disturbing magnetic substances may be

iron ore, steel structures, rails, electric cables carrying current, lamp posts, fences and many materials such as bunch of keys, knife, spectacles etc. with the surveyor,

Detection of local Attraction:-

unless the local attraction at a place is found-out and corrected, the bearings taken with a compass are not correct. To detect the presence of local Attraction, the fore and back bearings of a line should be taken. If the difference of the fore and back bearings of the line is exactly 180° , then there is no local attraction.

If the difference in F.B and B.B of a line is not equal to 180° , then needle is said to be affected by local attraction, provided there is no instrumental error.

Application of Correction for local Attraction:-

There are two methods for the application of correction, they are discussed below.

(1) Direct Method:

From the given table of observations, the unaffected line is first founded. Then, commencing from the unaffected line, the bearing of the other affected lines are

ANSWER KEY AND SCHEME OF EVALUATION

Course code: 20CE303

COURSE: SURVEYING

Correction at Each Station. This method is Simple and generally Employed.

(2) Interior Angle Method:-

In this method first, all the interior angles of a traverse are calculated from the observed bearings, then an angular check is applied. The sum of the interior angles should be equal to $(2n-4) \times 90^\circ$ where n is the number of sides of the traverse. If it is not so, the total error is equally distributed along all the angles of the traverse.

Then starting from the unaffected line, the bearings of all the lines are corrected by using the corrected interior angles. This method is very laborious and hence generally not employed.

(8) Describe the height of instrument method of reducing the levels. Compare the height of instrument method with rise and fall method.

(A) Booking and Reducing the levels:-

As soon as after observing a staff reading should be entered in a level field book, in the respective column. A level field book or level book is used for booking and reducing the levels of various points. There are two methods of booking and reducing the levels of various points. There are two methods of booking and reducing the levels of point from two observed staff readings.

(a) Height of Instrument Method [Collimation Method]:

- In this method, the Elevation of the plane of collimation (i.e. height of instrument) is found out by adding the backsight reading to the RL of the BM, on which the backsight is taken.
- The RL of intermediate points and the first change point is obtained by subtracting the respective staff readings from the Height of instrument (HI).
- Shift the instrument to the second position and again calculate the Elevation of the plane of collimation, by adding the BS reading to the RL of change point.

ANSWER KEY AND SCHEME OF EVALUATION

Course code: 20CE303

COURSE: SURVEYING

- By means of backsight and foresight taken on the change point, correlate the levels of two planes of collimation.
 - Find out the reduced level of successive points and the second change point by subtracting their staff readings from the elevation of the new plane of collimation.
- Arithmetical check:

$$\sum B.S - \sum F.S = \text{last R.L} - \text{first R.L.}$$

Height of Instrument Method:

- It is rapid or it involves few calculation.
- There is no check on the R.L's of intermediate points.
- There are two arithmetic checks
 $\sum B.S - \sum F.S = \text{R.L of last Station} - \text{R.L of first Station.}$
- Most Suitable for longitudinal or cross-sectional leveling and ~~cont~~ contouring, where there are number of intermediate sights.

- Errors, if any, in the intermediate sights are not detected.

Rise and Fall Method:

- It is laborious, involving several calculation.
- There is check on the R.L. of intermediate points.
- There are three arithmetical checks
 $\sum B.S - \sum F.S = \sum Rise - \sum fall = R.L \text{ of last Station} - R.L \text{ first Station}.$
- This method is suitable for fly levelling and for establishing new bench marks, where there are no intermediate sights.
- Errors, if any, in the intermediate intermediate sights are also detected.

(Q) Explain the procedure to determine the tachometric constants by

- (a) Fixed Hair Method.
- (b) Movable Hair Method.

(A) (a) Fixed Hair Method:-

This is the most commonly used method of tachometry. In this method the stadia hair are kept at fixed interval and the value of staff intercept (S) varies with the distance from instrument station.

ANSWER KEY AND SCHEME OF EVALUATION

Course code: 20CE303

COURSE: SURVEYING

This method is also used even when horizontal sights are not possible. For inclined sights, readings may be taken by holding the staff either vertical or normal to the line of sight. [not considering horizontal distance].

(b) Moveable Hair Method:-

In this method the staff intercept on the leveling staff is kept constant by fixing two targets or vanes at a known distance apart. During observation the distance between stadia hairs is ~~to~~ so adjusted that the upper hair coincides the upper large target and the lower hair coincides with the lower target, by adjusting the micrometer screws fitted on the drop diaphragm. The variable ~~the~~ micrometer stadia intercept is measured and the required distance is then calculated. Generally, this method is not adopted.

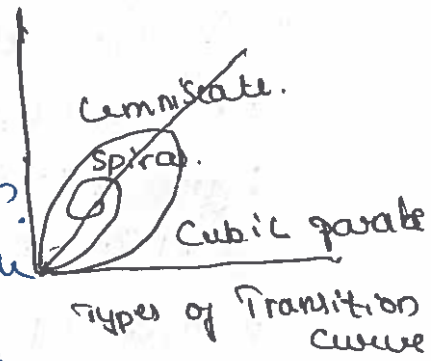
(10) what is a transition curve? state the various types of transition curves with the help of a neat sketch. explain briefly its necessity?

(A) Transition Curve:-

It is a curve usually introduced between a simple circular curve and a straight, or between two simple circular curves. It is also known as a easement curve. A transition curve has a radius, gradually changing from a finite to infinite value or vice versa. It is widely used on highways and railways, since its radius increases or decreases in a very gradual manner.

Types of Transition Curve:

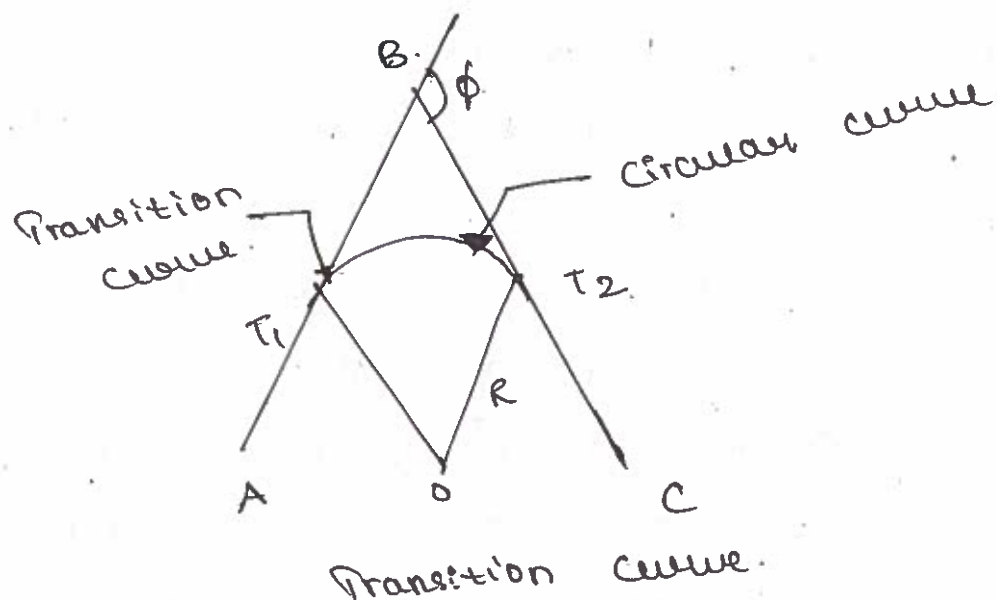
- (1) Spiral or clothoid
 - (2) Cubic parabola.
 - (3) Lemniscate.
- The length of transition curve is designed to fulfill the three conditions.
 - length of transition curve by the rate of change of radial acceleration.
 - length of Transition Curve by rate of change of super elevation.
 - Empirical formula for the length of transition curve recommended by IRC.



ANSWER KEY AND SCHEME OF EVALUATION

Course code: 20CE303

COURSE: SURVEYING



(11) Discuss about the recent advancements in total stations and their usefulness in monitoring structures?

- (A)
- In field survey, the use of electronics based instruments is now so widespread that it would be difficult to imagine any contemporary site surveying without it.
 - The recent applications of electronics in the surveying instruments has enabled the surveyors to collect and process the field data much more easily and to a higher precision than is possible using routine instruments.

- Electronic theodolite is combined and interfaced with EDMs and Electronic data collectors, they become total Stations or Electronic tacheometers (ET).
- These total Stations can read and record horizontal and vertical angles, together with slope distances.
- These are operated using a multi-function keyboard which is connected to a microprocessor built into the instrument.
- Thus the microprocessor in the total Station cannot only perform a variety of mathematical operations- for example averaging multiple angles, measurements, averaging multiple distance measurements, calculation of rectangular Co-ordinates, calculation of slope corrections, distances between remote points, remote object elevations, atmospheric and instrumental corrections but in some cases, can also store observations directly using an internal memory.
- Though number of companies are manufacturing total Stations, like Leica TCA 1800 and Nikon C-100 total Stations.
- However, the features common to the majority of total Stations.

ANSWER KEY AND SCHEME OF EVALUATION

Course code: 20CE303

COURSE: SURVEYING

(12) Explain the aspects of flight planning for an aerial survey and obtain an expression for the number of photographs required for given area of length and width of a survey.

(A) Number of photographs required to cover the area :

A = Total area to be photographed.

L = length of photo in direction of flight.

w = width of photo normal to direction.

S = Scale of photograph f/H .

L = net ground distance corresponding to l .

w = net ground width corresponding to w .

a = net ground area covered by each

photo.

P_l = % overlap in direction of flight (length).

P_w = % overlap in side (width).

$$\text{Number of photographs} = \frac{\text{Total area to be photographed}}{\text{net ground area covered by each photo}}$$

$$= \frac{A}{a}$$

The size of each photograph = $(1 - P_l) \times (1 - P_w) w$.

Ground area covered by each photograph

$$a = S(1-P_1)L \times S(1-P_w)W$$

• If actual geometry L and B known no. of photographs calculated as under.

= Total number of flights $\times$ number of photo in each flights.

$$\text{Total number of flight lines} = \frac{\text{width of Area}}{(1-P_w)Sw} + 1$$

$$\text{Total number of photographs in each flight line} = \frac{\text{length of flight strip}}{(1-P_1)Sl} + 1$$

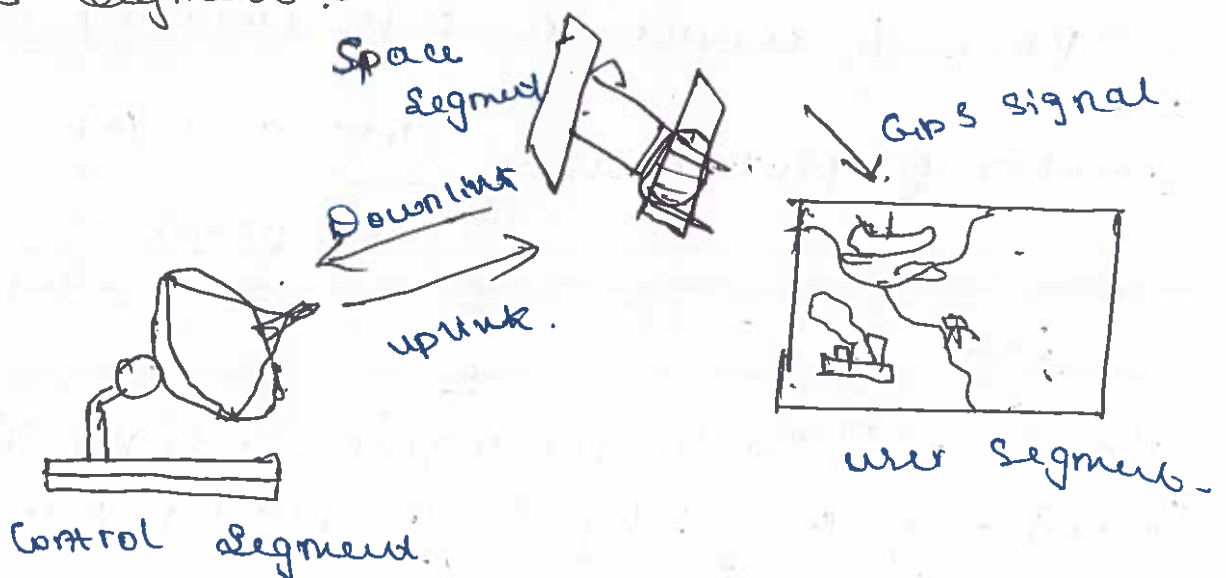
Time lag between exposure $T = \frac{3600 \times L}{S}$ sec.

L = Ground length of each photo $(1-P_1)Sl$ in km.

S = speed of aeroplane in km/hour.

(13) Describe the various segments of GPS Surveying. Also, give a brief account of the applicability and limitations of each technique?

(A). GPS Segments :-



ANSWER KEY AND SCHEME OF EVALUATION

Course code: 20CE303

COURSE: SURVEYING

The GPS Consists of three major segments.

1. Space Segment.
2. Control Segment.
3. User Segment.

1) Space Segment:-

Each satellite broadcasts a navigation message with exact position, clock, status and orbit details of itself [called Ephemeris] and general position and health information of all other satellites [called Almanac]. The nominal space segment consists of 24 satellites.

- Six fixed orbital planes.
- Altitude 20,200 km (10,981 NM).
- Orbit period is almost 12 hours.
- Orbital planes are equally spaced 60 degrees apart.
- Inclined 55 degrees with respect to the equatorial plane.
- 4 satellites in each orbit.
- 3 satellites are active spares which can replace failed satellites.
- Satellites repeat the same track / configuration over any point on Earth every 24 hrs.

- At any time there are 4 to 8 satellites "visible" above 15° elevation.
- Above 100 there may be 10 visible.

(2) Ground Segment:

- GPS satellites are under surveillance of the Control Segment.
- Five monitoring ground stations constantly track the satellite transmissions, clocks, status and orbits.
- The ground stations are located at Hawaii, Colorado Springs, Ascension Island, Diego Garcia and Kwajalein.

(3) User Segment:-

- The user segment includes all GPS users.
- User equipment consists of a GPS - antenna and GPS - receiver with built-in microcomputer or GPS - navigator.
- GPS receiver designs differ depending on user requirements and cost.

(14) Discuss the possible advantages and disadvantages of using a raster GIS and vector GIS?

(A) Raster GIS:

Advantages:

- The data structure is much simpler.
- It is easy to overlay and combine layers.

ANSWER KEY AND SCHEME OF EVALUATION

Course code: 20CE303

COURSE: SURVEYING

Disadvantages:

- Because a value must be stored for each and every cell in a grid, there is a great deal of redundancy.

Vector GIS:-

Advantages:-

- Accurately representing true shape and size.
- Conserving disk space.
- It is efficient in terms of data storage.
- useful for network analysis and modeling flows of linear feature.

Disadvantages:

- The data structure is more complex. Especially when you have fully encoded topology [eg, using the arc-node models.]
- It is more difficult to write computer programs to manipulate data.
- Spatial analysis (over laying images) operations can be more difficult.

(15) Discuss in detail about the supervised and unsupervised image classification in Geographical Information System?

(A) Supervised classification:-

- Supervised classification is an idea based which enables a user to choose pixel samples from images which represent specific classes.
- The software for processing images will direct training sites also referred in the form of references to group all pixels on the image.
- The training sites also referred to as input classes or testing sets, are selected depending on how the user understands. You can also set up bounds to determine the relationship of other pixels to put them together.
- The bounds are frequently selected according to spectral characteristics of testing sets.
- Also, the maximum and minimum of strength in particular spectral bands.
- The user can design the total classes which specific image has been classifying and most analytics use both unsupervised and supervised classification for coming up with final grouped maps and output analysis.

unsupervised classification:-

- unsupervised classification involves grouping of images with similar characteristics and is

ANSWER KEY AND SCHEME OF EVALUATION

Course code: 20CE303

COURSE: SURVEYING

Software - based.

- For that reason, the user is not required to produce sample classes.
- The Software the techniques for determining images that are related and put them in classes.
- You will need to specify the type of algorithm to be used and a number of classes.
- However, a user should know more about the place being classified.
- This is necessary during the classification of images with similar characteristics that are provided by the computer.
- The pixels are compared to real features on Earth, such as forests, wetlands and urban areas.

1. The first of these is the fact that the
the world is a very different place
than it was a few years ago. The
economy is in a state of flux, and
the social and political landscape is
changing rapidly. This is due to a
number of factors, including the
global financial crisis, the rise of
new technologies, and the increasing
awareness of environmental issues.
2. The second factor is the increasing
diversity of the world's population.
As more people from different
cultures and backgrounds migrate to
other parts of the world, the
cultural and ethnic mix is becoming
more complex. This has led to a
greater understanding of different
perspectives, but it has also created
challenges in terms of social cohesion
and integration.
3. The third factor is the rapid
pace of technological change. The
internet, mobile phones, and social
media have transformed the way we
communicate and interact. While this
has brought many benefits, it has
also led to a loss of privacy and
an increase in cyber threats.
4. Finally, the world is facing
significant environmental challenges.
Climate change, deforestation, and
pollution are all threats to the
planet's health and the well-being of
future generations. These issues
require global cooperation and action
to address.

Semester End Examination, January/February, 2022

Degree	B. Tech. (U.G.)	Program	Civil Engineering	Academic Year	2021 - 2022
Course Code	20CE304	Test Duration	3 Hrs. Max. Marks 70	Semester	III
Course	Strength of Materials				

Part A (Short Answer Questions 5 x 2 = 10 Marks)

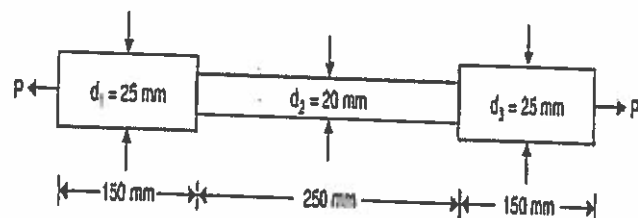
No.	Questions (1 through 5)	Learning Outcome (s)	DoK
1	What is the use of Mohr's circle?	20CE304.1	L1
2	Write the equation for the simple bending theory	20CE304.2	L1
3	Calculate the maximum deflection of a simply supported beam carrying a point load of 85 kN at mid span. Span = 5 m, $E = 21000 \text{ kN/m}^2$	20CE304.3	L1
4	Define (i) Slenderness ratio (ii) Radius of Gyration	20CE304.4	L1
5	Distinguish between Circumferential stress (or hoop stress) and Longitudinal stress	20CE304.5	L1

Part B (Long Answer Questions 5 x 12 = 60 Marks)

No.	Questions (6 through 15)	Marks	Learning Outcome (s)	DoK

The bar shown in Fig. 1 is tested in universal testing machine. It is observed that at a load of 40 kN the total extension of the bar is 0.280 mm. Determine the Young's modulus of the material.

6



12M

20CE304.1

L1

Figure.1

OR

A specimen of steel 20 mm diameter with a gauge length of 200 mm is tested to destruction. It has an extension of 0.25 mm under a load of 80 kN and the load at elastic limit is 102 kN. The maximum load is 130 kN. The total extension at fracture is 56 mm and diameter at neck is 15 mm. Find

7

- The stress at elastic limit
- Young's modulus
- Percentage elongation
- Percentage reduction in area
- Ultimate tensile stress

12M

L2

20CE304.1

The unsymmetric I-section as shown in Figure. 2 is the cross-section of a beam, which is subjected to a shear force of 60 kN. Draw the shear stress variation diagram across the depth

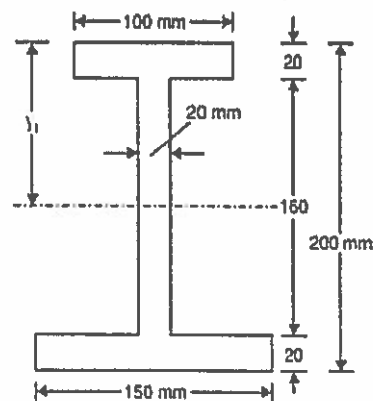


Figure 2

OR

A circular steel pipe of external diameter 60 mm and thickness 8 mm is used as a simply supported beam over an effective span of 2 m. If permissible stress in steel is 150 N/mm^2 , determine the maximum concentrated load that can be carried by it at mid span as shown in Figure. 3

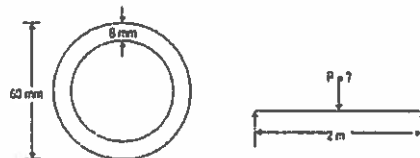


Figure 3

- 8
- 10 A simply supported beam span 14m, carrying concentrated loads of 12kN and 8kN at two points 3mts and 4.5m from the two ends respectively. Moment of Inertia I for the beam is $160 \times 10^3 \text{ mm}^4$ and $E = 210 \text{ kN/mm}^2$. Calculate deflection of the beam at points under the two loads by moment area method

12M

20CE304.2

L3

OR

- 11 A beam AB of 8 m span is simply supported at the ends. It carries a point load of 10 kN at a distance of 1 m from the end A and a concentrated load of 15 kN at a distance of 2 m from the end B. If $I = 10 \times 10^6 \text{ m}^4$, Using Macaulay's Method, Determine:
(a) Deflection at the mid-span,
(b) Slope at the end A.

12M

20CE304.3

L2

- 12 A simply supported beam of length 6m is subjected to uniformly distributed load of 80 kN/m over the whole span and deflects 25 mm at the centre. Determine the crippling loads when the beam is used as column with the following conditions.

12M

20CE304.4

L3

- I. Both the ends fixed
- II. Both the ends pin jointed

OR

- 13 A hollow cylindrical cast iron column is 3 m long with both ends fixed. Determine the minimum diameter of the column if it has to carry a safe load of 150 kN with a factor of safety. Take the internal diameter as 0.6 times the external diameter

12M

20CE304.4

L3

Take $\sigma_c = 450 \text{ N/mm}^2$ and $\alpha = \frac{1}{1700}$ in Rankine's formula

14	A hollow shaft is to transmit 200 kW at 80 r.p.m. If the shear stress is not to exceed 60 MPa and internal diameter is 0.6 of the external diameter, find the diameters of the shaft.	12M	20CE304.5	L2

OR

15	A thin cylindrical shell is 3m long and 1m in internal diameter. It is subjected to internal pressure of 1.2 MPa. If the thickness of the sheet is 12mm, find the circumferential stress, longitudinal stress, changes in diameter, length and volume. Take $E = 200 \text{ GPa}$ and $\mu = 0.3$	12M	20CE304.5	L2

ANSWER KEY AND SCHEME OF EVALUATION

Subject: - strength of Materials

course code: 20CE304.

PART-A (5x2 = 10M)

1. What is the use of Moho's circle?

Ans: - Moho's circle is a graphical method of finding normal, tangential and resultant stresses on an oblique plane.

→ Moho's circle is a two dimensional representation of transformation equation for plane stress.

→ It can also be used to calculate the principal stresses, maximum shear stress and stress on inclined planes.

→ Moho's circle often used in calculations relating to Mechanical engineering for materials strength, geotechnical engineering for strength of soils and structural engineering of built structures.

2. write the equation for the simple bending theory?

Ans:-

simple bending equation,

$$\boxed{\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}}$$

where M is expressed in N-mm.

Moment of Inertia in mm^4 .

σ is expressed in N/mm^2 .

Young's Modulus (E) = N/mm^2 .

R = Radius in mm.

3. Calculate the maximum deflection of a simply supported beam carrying a point load of 85 kN at mid span = 5m.
 $E = 21000 \text{ kN/m}^2$.

Ans:- Given data:-

length of span = 5m.

Point load or Concentrated load (w) = 85 kN.

Young's Modulus (E) = 21000 kN/m^2 .

ANSWER KEY AND SCHEME OF EVALUATION

Assume Moment of Inertia (I) = $2 \times 10^5 \text{ mm}^4$.

Assume cross-section of beam = $300 \times 450 \text{ mm}$.

$$I = \frac{bd^3}{12} = \frac{300 \times 450^3}{12} = 2278 \times 10^9 \text{ mm}^4.$$

deflection (or) δ = ?



→ simply supported beam subjected to point load at the centre.

$$\text{deflection (or) } \delta = \frac{wl^3}{48EI}.$$

$$\delta = \frac{wl^3}{48EI} = \frac{85 \times 10^3 \times 5000^3}{48 \times 21000 \times 10^3 \times 2.278 \times 10^9}$$

$$\delta =$$

4. define (i) slenderness ratio (ii) Radius of Gyration.

Ans:- slenderness ratio :- The ratio of effective length to least radius of gyration is called slenderness ratio.

$$\text{slenderness ratio} = \frac{l}{k_{\min}} \text{ or } \frac{l}{r_{\min}}$$

$r_{\min} = k_{\min} = \text{least radius of gyration.}$

→ The load carrying capacity of long column depends upon the slenderness ratio.

→ For good design the slenderness ratio as small as possible

Radius of Gyration :- The ratio square root of moment of inertia (I) to the cross sectional area (A) is called.

Radius of Gyration.

$$k = \sqrt{\frac{I}{A}}$$

where $I = \text{Moment of Inertia,}$

$A = \text{Area of cross-section in mm}^2,$

$k = \text{Radius of Gyration in mm}$

ANSWER KEY AND SCHEME OF EVALUATION

5. distinguish between circumferential stress or (hoop stress) and longitudinal stress?

Ans:- Circumferential stress :- (or) hoop stress :-

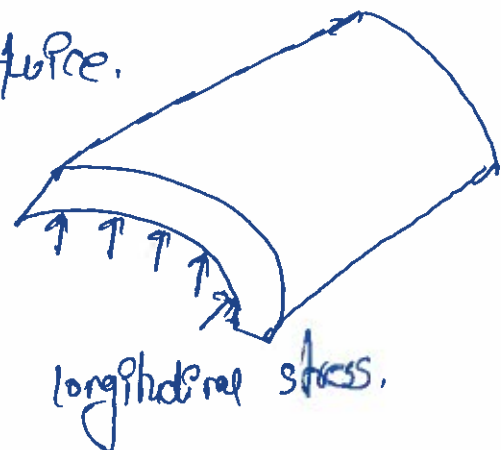
→ The hoop stress in thick cylinders will not be uniform across the thickness. 

→ The hoop stress will vary from a maximum value at the inner circumference to a minimum value at the outer circumference.

→ The hoop stress in case of thin cylinders are reduced by wire winding on the cylinders.

longitudinal stress :- longitudinal stress twice circumferential stress.

=



the 11th of the month of September 1891

at the city of New York

(Seal) I hereby certify that

the above is a true and correct copy

of the original as it appears in the

records of the City of New York

in the office of the City Clerk

and is a true and correct copy

of the original as it appears in the

records of the City of New York

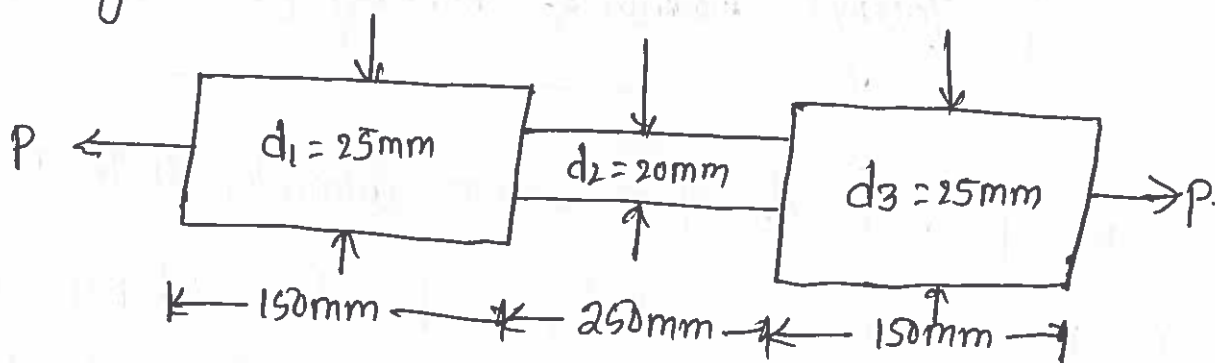
in the office of the City Clerk

and is a true and correct copy

ANSWER KEY AND SCHEME OF EVALUATION

PART-B

6. The bar shown in fig, tested in UTM, it is observed that at load of 40 kN the total extension of the bar is 0.280 mm, determine the young's modulus of the material.



Ans:-

Given data : -

Tensile load = 40 kN = $40 \times 10^3 \text{ N}$.

Extension of the bar = 0.280 mm.

Young's modulus of the material (E) = ?

Formula:-

Total change in the length of the bar.

$$\Delta L = \Delta L_1 + \Delta L_2 + \Delta L_3 = \frac{PL_1}{A_1 E} + \frac{PL_2}{A_2 E} + \frac{PL_3}{A_3 E}$$

$$\Delta L = \frac{P}{E} \left[\frac{L_1}{A_1} + \frac{L_2}{A_2} + \frac{L_3}{A_3} \right]$$

$$E = \frac{P}{\Delta L} \left[\frac{L_1}{A_1} + \frac{L_2}{A_2} + \frac{L_3}{A_3} \right]$$

$$E = \frac{40 \times 1000}{0.280} \left[\frac{150}{\frac{\pi}{4} \times 25^2} + \frac{250}{\frac{\pi}{4} \times 20^2} + \frac{150}{\frac{\pi}{4} \times 25^2} \right]$$

$$E = 200989.95 \text{ kN/m}^2 \text{ or } 200989.95 \times 10^3$$

$$E = 200.98 \text{ N/mm}^2 \text{ or } 200.989 \times 10^3 \text{ N/mm}^2$$

Young's modulus of material (E) = 200.98 N/mm²

7) A specimen of steel 20mm diameter with a gauge length 200mm is tested to destruction. It has an extension of 0.25mm under a load of 80kN and the load at elastic limit is 102kN. The maximum load is 130kN, the total extension at fracture is 56mm and diameter at neck is 15mm. Find.

(i) the stress at elastic limit.

(ii) young's modulus (E)

(iii) percentage elongation (%)

(iv) percentage reduction (%)

ANSWER KEY AND SCHEME OF EVALUATION

Q.4

percentage reduction! - ultimate Tensile stress.

Ans:-

Given data:-

diameter of the steel rod = 20mm,

Gauge length = 200mm.

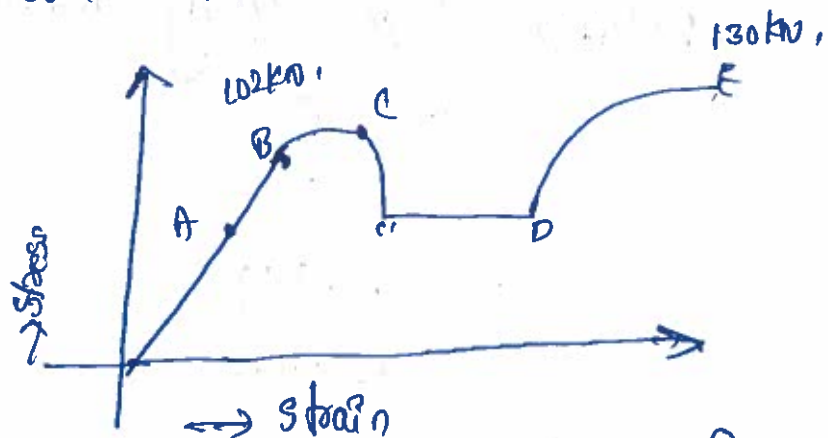
extension under load of 80kN = 0.25mm.

Elastic limit = 102kN = 102×10^3 N.

maximum load = 130 kN.

extension at fracture = 56mm.

diameter at neck = 15mm.



stress - strain diagram for mild steel.

Ans ① stress at elastic limit $= \frac{P}{A}$ at elastic limit.

$$\text{Area of the steel (A)} = \frac{\pi}{4} \times 20^2 \text{ mm}^2.$$

$$\begin{aligned} \text{stress at elastic limit} &= \frac{102 \times 10^3}{\frac{\pi}{4} \times 10^2} \\ &= 324,674 \text{ N/mm}^2 \end{aligned}$$

④ Young's modulus (E) = $\frac{\text{stress}}{\text{strain}}$

$$\sigma = 80 \times 10^3 \text{ N.}$$

$$\text{strain} = \frac{\text{change in length}}{\text{original length}} = \frac{\delta L}{L}$$

$$E = \frac{0.25}{200} = 1.25 \times 10^{-3}$$

Relation b/w Young's modulus, stress & strain.

$$E = \frac{\sigma}{\epsilon}$$

$$E = \frac{80 \times 10^3}{1.25 \times 10^{-3}}$$

$$\boxed{E = 203720 \text{ N/mm}^2}$$

ANSWER KEY AND SCHEME OF EVALUATION

6a) Ans:- $\text{Percentage elongation (\%)} = \frac{\text{Total change in length}}{\text{original length.}} \times$

$$\boxed{\% \text{ elongation} = \frac{\Delta l}{l} \times 100.}$$

$$\% \text{ elongation} = \frac{56}{200} \times 100 = 28\%$$

(ii) Percentage reduction in Area.

$$\% \text{ reduction in Area} = \frac{\frac{\pi}{4} D_1^2 - \frac{\pi}{4} D_2^2}{\frac{\pi}{4} D_1^2}$$

$$= \frac{D_1^2 - D_2^2}{D_1^2}$$

$$\% \text{ reduction in Area} = \left(\frac{20^2 - 15^2}{20^2} \right) \times 100$$

$$\% \text{ reduction in Area} = \underline{\underline{43.75\%}}$$

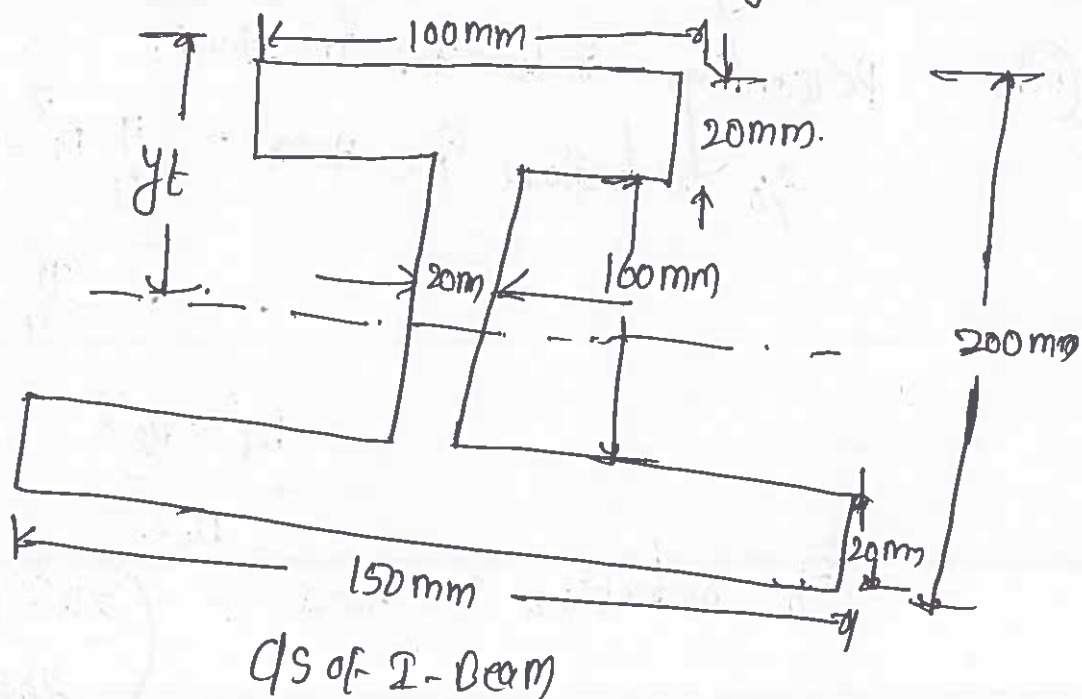
(4)

$$\text{ultimate tensile stress} = \frac{\text{ultimate load or maximum load}}{\text{original CS area.}}$$

$$\text{ultimate tensile stress} = \frac{130 \times 10^3}{\frac{\pi}{4} \times 20^2} = 413.80 \text{ N/mm}^2$$

unit-II

- 8) The unsymmetric I-section as shown in fig, the cross section of a beam, which is subjected to a shear force of 60 kN. draw the shear stress variation diagram across the depth.



ANSWER KEY AND SCHEME OF EVALUATION

SAns:

Given data: →

Shear force (F) = 60 kN = $60 \times 10^3 \text{ N}$.

→ The given I-section is unsymmetrical about x-x axis, but symmetrical about y-y axis.

→ Let us first calculate the position of N.A. This is obtained. we know the position of the C.G. of the given section.

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3}$$

$$\bar{y} = \frac{[(100 \times 20) \times 190] + [(160 \times 20) \times 100] + [(150 \times 20) \times 10]}{(100 \times 20) + (160 \times 20) + (150 \times 20)}$$

$\bar{y} = 89.02 \text{ mm from bottom,}$

$\bar{y} = 110.98 \text{ mm} \approx 111 \text{ mm from top.}$

calculating moment of inertia of section about N.A.

$$I = \left[\frac{b_1 d_1^3}{12} + A_1 h_1^2 \right] + \left[\frac{b_2 d_2^3}{12} + A_2 h_2^2 \right] + \left[\frac{b_3 d_3^3}{12} + A_3 h_3^2 \right]$$

$$= \left[\frac{100 \times 20^3}{12} + (100 \times 20) \times (110 - 10)^2 \right] + \left[\frac{20 \times 160^3}{12} + (20 \times 160) \times (111 - 10)^2 \right] + \left[\frac{150 \times 20^3}{12} + (150 \times 20) \times (89 - 10)^2 \right]$$

$$\boxed{I = 46.506 \times 10^6 \text{ mm}^4}$$

→ Shear stress at a section $q = \frac{F A \bar{y}}{I b}$

where q = Shear stress at any section,

F = Shear force.

b = Width of the section at that level.

$A \bar{y}$ = moment of the area of the portion above that section about N.A. of the section.

I = moment of inertia of section about N.A.

① At section (i) top of the flange, $q = 0$.

② At section (ii). (Bottom of the top flange)

$$q = \frac{60 \times 10^3}{46.506 \times 10^6 \times 100} \times [100 \times 20 \times (111 - 10)]$$

$q = 2.66 \text{ N/mm}^2$ ~~$q = 13.302 \text{ N/mm}^2$~~

ANSWER KEY AND SCHEME OF EVALUATION

At section (in web at junction) with flange where $b=20\text{mm}$.

$$q = \frac{60 \times 10^3}{46.506 \times 10^6 \times 20} \left[100 \times 20 \times (110 - 10) \right]$$

$$q = 13.303 \text{ N/mm}^2.$$

(10) At section 3. at n.a level; $b=20\text{mm}$.

$$q = \frac{60 \times 10^3}{46.506 \times 10^6 \times 20} \left[100 \times 20 \times (111 - 10) \right]$$

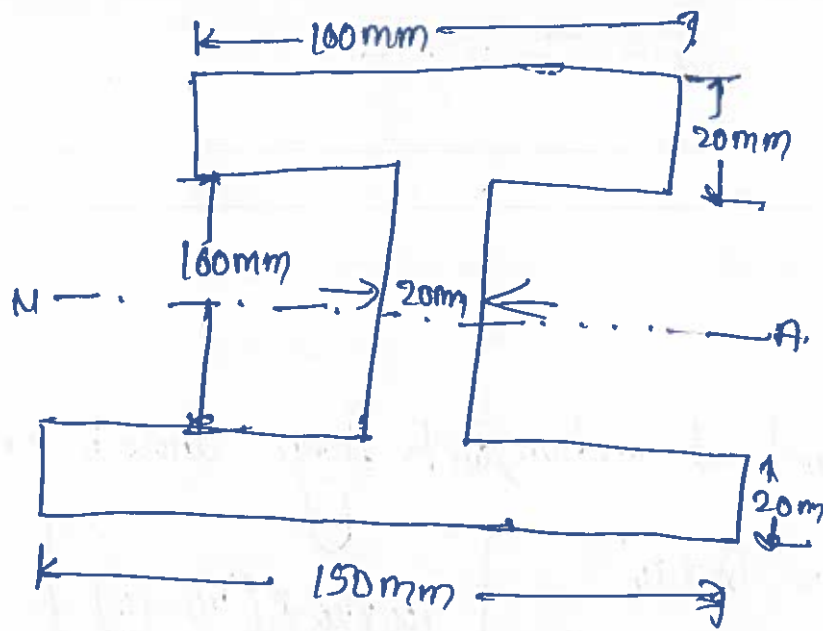
$$+ (111 - 120) \times 20 \times \left(\frac{111 - 20}{2} \right)$$

$$q = 18.37 \text{ N/mm}^2$$

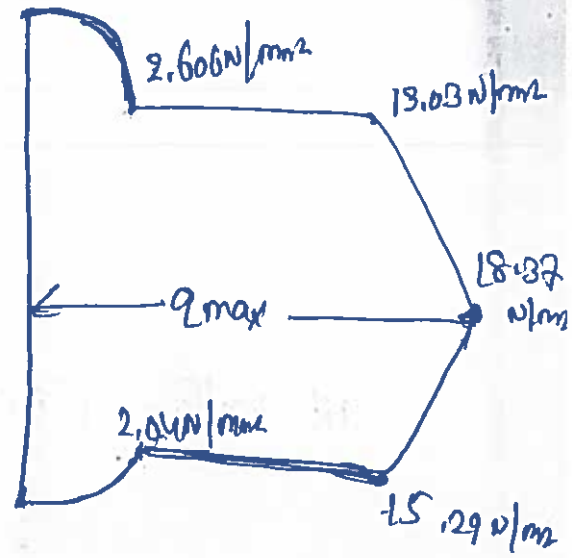
(11) At section A, Cut lower fibre at lower flange

$$q = \frac{60 \times 10^3}{46.506 \times 10^6 \times 170} \times (150 \times 20) \quad (\text{fig. 10})$$

$$q = 2.04 \text{ N/mm}^2$$



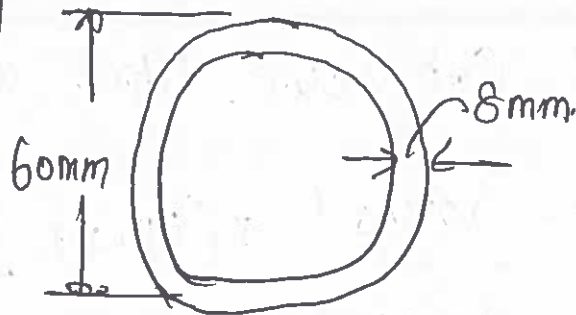
c/s of I-section



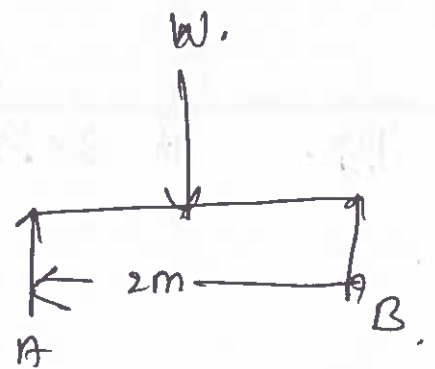
shear stress variation across depth of I-section

- 9). A circular steel pipe of external diameter 60 mm and thickness 8 mm is used as simply supported beam over an effective span of 2 m. If permissible stress in steel is 150 N/mm^2 , determine the maximum concentrated load that can be carried by it at mid span as shown.

In fig.



c/s of beam steel pipe



Simply supported beam

ANSWER KEY AND SCHEME OF EVALUATION

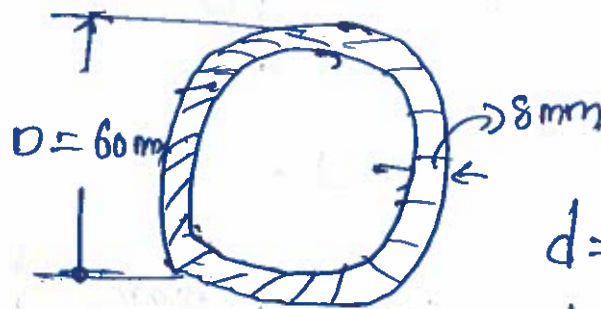
9 Ans).

Given, data :-

External diameter (D) = 60mm,

thickness (t) = 8mm

From fig,



$$d = D - t - t$$

$$d = D - 2t$$

Internal diameter of Pipe = 44mm $d = 60 - 2 \times 8$

permissible stress (σ_{pu}) = 150 N/mm². $d = 60 - 16 = 44\text{mm}$

By Bending equation,

$$\boxed{\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}}$$

where, M = Bending moment kN-m,

I = Moment of Inertia mm⁴,

σ = Bending stress N/mm²

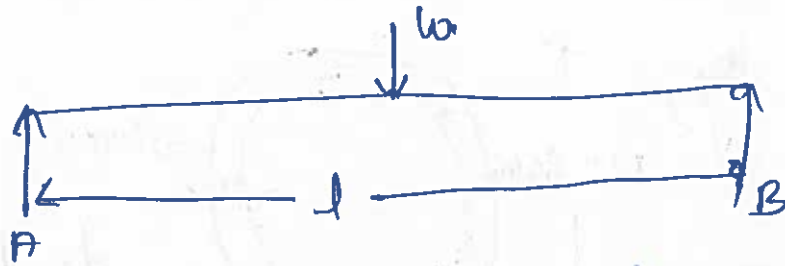
y = depth of section about N.A.
Radius mm

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

$$M = \frac{\sigma}{y} \times I.$$

$$\text{Bending Moment} = \frac{150 \times \frac{\pi}{64} [60^4 - 44^4]}{30.}$$

$$\text{Bending moment} = 2.26 \times 10^6 \text{ N-mm.}$$



For simply supported beam, it subjected to concentrated load @ the centre. Maximum Bending moment

$$M = \frac{Wl}{4}$$

$$\Rightarrow W = \frac{M \times 4}{l}$$

$$W = \frac{4 \times 2.26 \times 10^6}{2000}$$

$$W = \frac{4 \times 2.26 \times 10^6}{2 \times 10^3}$$

$$W = 4.52 \text{ kN}$$

∴ total point load = 4.52 kN.

ANSWER KEY AND SCHEME OF EVALUATION

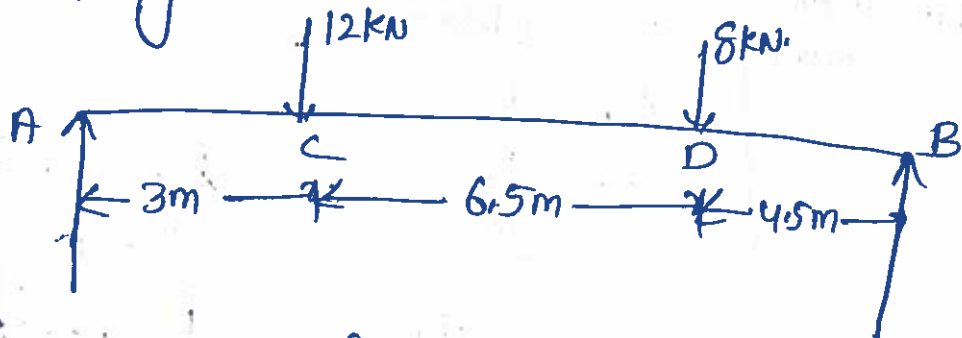
- 10) A simply supported beam span 14m, carrying concentrated loads of 12kN and 8kN at two points 3m and 4.5m from the two ends respectively. moment of inertia for the beam is $160 \times 10^3 \text{ mm}^4$ and $E = 210 \text{ kN/mm}^2$, calculate deflection of the beam at points under the two loads by moment area method?

Ans:-

type of beam = simply supported beam.

Moment of Inertia (I) = $160 \times 10^3 \text{ mm}^4$.

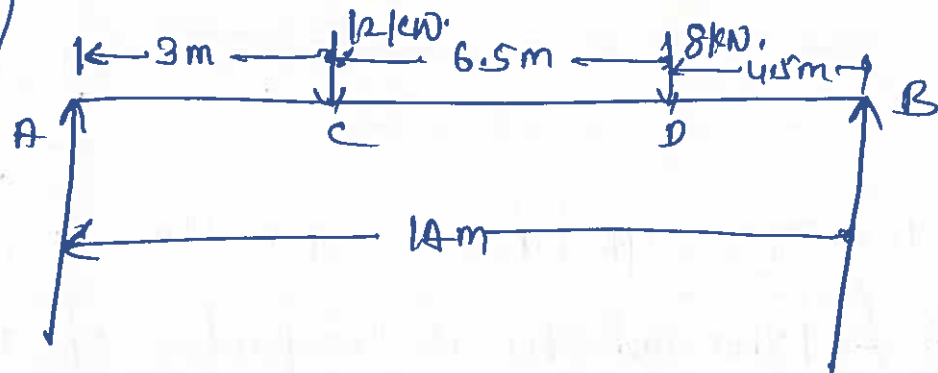
Young's modulus (E) = 210 kN/mm^2 .



Deflection of the beam under loads = ?

step-1: using equilibrium condition, calculate the support Reactions

Taking moment about A, moment @ A.



moment @ A,

$$R_B \times 14 - 12 \times 3 - 8 \times 9.5 = 0$$

$\text{Moment} = \text{Force} \times \text{distance}$

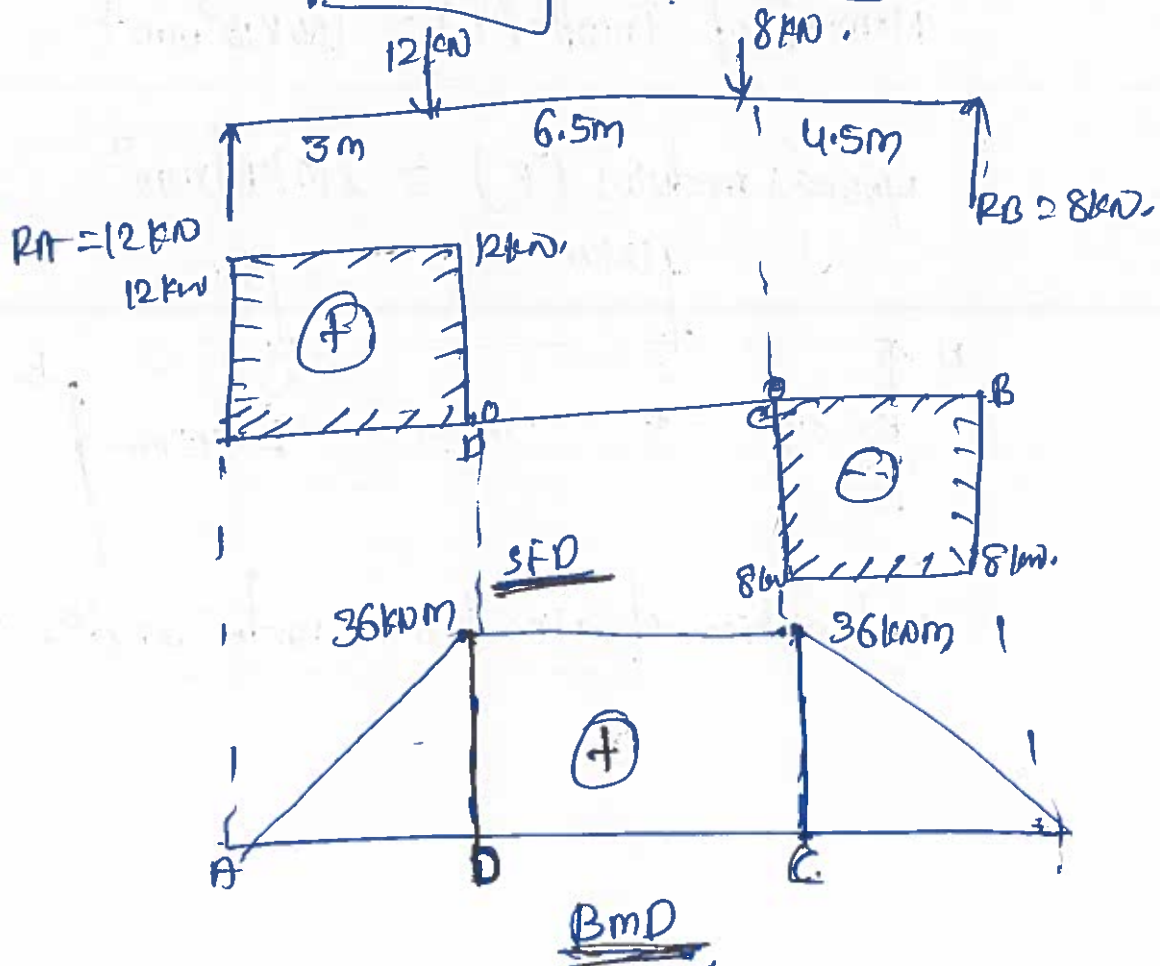
$$R_B \times 14 - 12 \times 3 - 8 \times 9.5 = 0$$

$$R_B \times 14 = (12 \times 3) + 64.5 \quad (8 \times 9.5)$$

$$R_B = 8 \text{ kN} \quad ; \quad R_A = 12 \text{ kN}$$

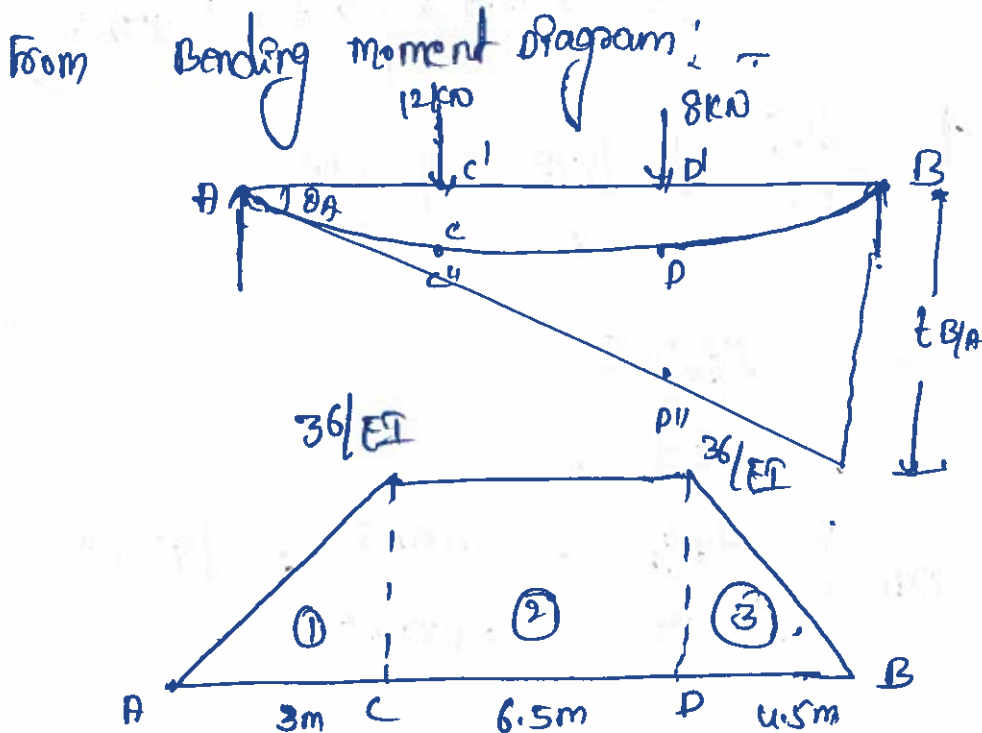
$R_B = 8 \text{ kN}$

$R_A = 12 \text{ kN}$



ANSWER KEY AND SCHEME OF EVALUATION

10 Ans)



From Mohr's theorem:-

$$\text{Deflection } (\delta) = \frac{A \bar{x}}{EI}; \quad \text{slope } (\theta) = \frac{A_{area}}{EI} = \frac{\Delta}{EI}$$

Where A = Area of Bending moment diagram

$\bar{x}$ = Centre of gravity from the left
@ 1 right support of the beam

E = Young's modulus N/mm²

I = moment of Inertia of the Beam mm⁴.

From fig, Moment Area theorem-II,

$$t_{B/A} = \frac{1}{2} \times 3 \times \frac{36}{EI} \left(\frac{3}{3} + 11 \right) + 6.5 \times \frac{36}{EI} \times \left(\frac{6.5}{2} + 4.5 \right) + \frac{1}{2} \times 4.5 \times \frac{36}{EI} \times \left(\frac{2}{3} \times 4.5 \right)$$

$$= \frac{648}{EI} + \frac{1818.5}{EI} + \frac{243}{EI}$$

$$t_{B/A} = \frac{2704.5}{EI}$$

$$\theta_A = \frac{t_{B/A}}{14} = \frac{2704.5}{EI \times 14} = \frac{193.18}{EI}$$

$$y_c = \text{deflection @ } c = c'c'' - t_{C/A}$$

$$= 3 \times \theta_A - \frac{1}{2} \times 3 \times \frac{36}{EI} \times \left(\frac{3}{3} \right)$$

$$= 3 \times \frac{193.18}{EI} - \frac{54}{EI} = \frac{525.54}{EI}$$

$$= \frac{525.54 \times 10^{12}}{210 \times 10^3 \times 10 \times 10^9} = 15641.07 \text{ mm}$$

Y at C.

$$\boxed{\text{Approximate value} = 156.41 \text{ mm}}$$

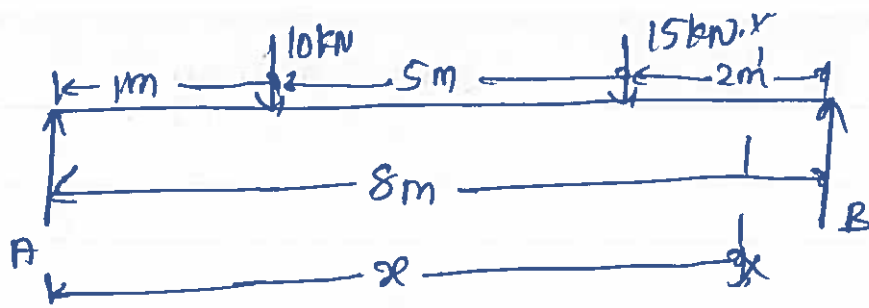
ANSWER KEY AND SCHEME OF EVALUATION

$$\begin{aligned}
 y_{at\ 0} &= \delta'_{\delta''} - t_{\theta A, \delta\delta''} \\
 &= 9.58A - \left[\frac{1}{2} \times 3 \times \frac{36}{EI} \times (1+6.5) + 6.5 \times \frac{36 \times 6.5}{EI} \right] \\
 &= \frac{1835.2}{EI} - \left[\frac{405}{EI} + \frac{760.5}{EI} \right] \\
 &= \frac{669.7}{EI} = \frac{669.71 \times 10^{12}}{210 \times 10^3 \times 160 \times 10^3} = \underline{\underline{19931.8\text{ mm}}}
 \end{aligned}$$

$y_{at\ 0} :-$ Approximately $y_{at\ 0} = 19931\text{ mm}$, Redesign
the section

- 11) A beam AB of 8m span is simply supported at the ends. It carries a point load of 10kN at a distance of 1m from the end A and a concentrated load of 15kN at a distance of 2m from the end B, if $I = 10 \times 10^6 \text{ m}^4$, using Macaulay's method, determine,
- deflection at mid span.
 - slope at the end A.

11 Marks)



Given data: Moment of inertia (I) = $10 \times 10^6 \text{ m}^4$

using equilibrium conditions:- $\sum V = 0$

$$\sum M = 0$$

$$\sum H = 0$$

Step-1 To find out the support reactions.

$$\boxed{\sum M = 0}$$

$$R_B \times 8 - 15 \times 6 - 10 \times 1 = 0$$

$$R_B \times 8 = (15 \times 6) + (10 \times 1)$$

$$\boxed{\uparrow = \downarrow}$$

$$R_B = 12.5 \text{ kN}$$

$$\boxed{\uparrow \downarrow = \sum V = 0}$$

$$R_A = 12.5 \text{ kN}$$

$\Rightarrow$ Moment equation $EI \frac{d^2 y}{dx^2} = m$

$$EI \frac{d^2 y}{dx^2} = 12.5x - 10(x-1) - 15x(x-6)$$

$\rightarrow (1)$

$$EI \frac{dy}{dx} = 12.5 \frac{x^2}{2} + C_1 - 10 \frac{(x-1)^2}{2} - \frac{15x^3}{2}$$

$\Rightarrow$ slope equation

ANSWER KEY AND SCHEME OF EVALUATION

→ Integrating the above equation again we get.

$$EI \frac{d^2 y}{dx^2} = 12.5 \times \frac{x^2}{2} + C_1 x + C_2 \Big| - \frac{10(x-1)^3}{6} - \frac{20(x-6)^3}{6} \rightarrow \text{deflection equation}$$

→ Find the values of C_1 and C_2 , use two boundary conditions, the boundary conditions are.

① at $x=0$; $y=0$ and ② at $x=6\text{m}$; $y=0$.

→ substituting the first boundary condition i.e. at $x=0$, $y=0$ in equation ① and considering the equation upto first dotted line (as $x=0$ lies in the first part of the beam) we get

$$0 = 0 + 0 + C_2 ; C_2 = 0$$

$$\boxed{C_2 = 0}$$

At $x=8m$ $y=0$.

$$0 = \frac{12.5 \times 8^3}{6} + 8C_1 - \frac{10 \times 7^3}{6} - \frac{15 \times 2^3}{6}$$

$$\boxed{C_1 = -59.375} \text{ from point}$$

$$EI \frac{dy}{dx} = 12.5 \frac{x^2}{2} - 59.375 - \frac{10(x-1)^2}{2} - \frac{15(x-6)^2}{2}$$

→ Final slope equation.

$$EI y = \frac{12.5 x^3}{6} - 59.375x - \frac{10(x-1)^3}{6} - \frac{15(x-6)^3}{6}$$

⇒ Final deflection equation

a) Deflection at mid span i.e. $x=4$.

$$EI y = \frac{12.5 \times 4^3}{6} - 59.375 \times 4 - \frac{10(4-1)^3}{6}$$

$$y = \frac{-149.17}{EI} = \frac{-149.17}{10 \times 10^6 \times E} \text{ Assumed } E = 2 \times 10^8 \text{ N/m}^2$$

$y_{atc} = 116 \text{ mm}$

b) slope at the end A. i.e. $x=0$.

$$EI \frac{dy}{dx} = 0 - 59.375 \Rightarrow$$

$$\frac{dy}{dx} = \frac{-59.375}{EI} = \frac{-59.375}{10 \times 10^6 \times E} \text{ Assumed } E = 2 \times 10^8 \text{ N/m}^2$$

ANSWER KEY AND SCHEME OF EVALUATION

- 12) A simply supported beam of length 6m is subjected to uniform distributed load of 80 kN/m over the whole span and deflects 25mm at the centre. Determine the crippling loads when the beam is used as column with the following conditions
- ① Both the ends are fixed.
 - ② Both the ends pin jointed.

ANS:-

Given data :-

Type of beam = simply supported beam.

Type of load = UDL (Uniformly distributed load)

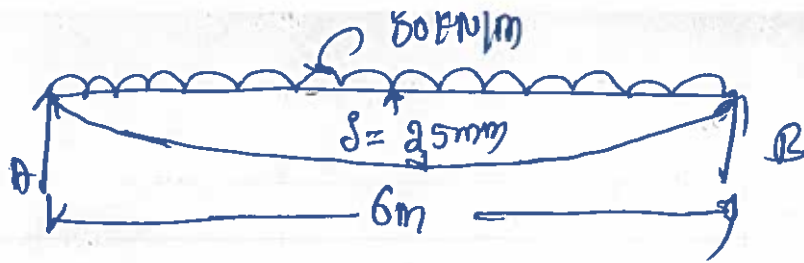
UDL = 80 kN/m over the whole span.

Deflection = 25mm at centre.

Span = 6m
Crippling load = ?

- ① Crippling load at both the ends are fixed
- ② Crippling load at ends pin jointed.

12A121



For simply supported beam with UDL,

$$\text{deflection } (\delta) = \frac{5}{384} \frac{wL^4}{EI}$$

$$\left(\frac{25}{1000} \right) = \frac{5}{384} \times \frac{80 \times 6^4}{EI}$$

$\frac{N}{mm^2} \times mm^4$

$$EI = 54000 \text{ kN} \cdot \text{m}^2$$

i) crippling load when both ends are fixed
effective length $= \frac{l}{2} = \frac{6}{2} = 3 \text{ m}$,



$$P_{cr} = \frac{\pi^2 EI}{l^2} = \frac{\pi^2 \times 54000}{3^2}$$

$$P_{cr} = 59217.6 \text{ kN}$$

ii) P_{cr} when both ends pin jointed,

$$l_e = l = 6 \text{ m}$$

$$P_{cr} = \frac{\pi^2 \times 54000}{6^2}$$

$$P_{cr} = 14804.4 \text{ kN}$$

ANSWER KEY AND SCHEME OF EVALUATION

13. A cylindrical cast iron column is 3m long with both ends fixed. Determine the minimum diameter of the column if it has to carry a safe load of 150 kN with a factor of safety. Take the external diameter as 0.6 times the internal diameter. Take $\sigma_c = 450 \text{ N/mm}^2$, and $\alpha = \frac{1}{1700}$ in Rankine's formula.

Ans:-

Given data: — length of column = 3m

type of material used as column = cast iron

End conditions = both ends fixed.

safe load = 150 kN.

Assumed factor of safety = 5.

Internal diameter = $0.6 D$.

$\sigma_c = 450 \text{ N/mm}^2$

$\alpha = \frac{1}{1700}$ in Rankine's formula.

According to Rankine's: —

$$P = \frac{\sigma_c \times A}{1 + \alpha \left[\frac{L_e}{r} \right]^2}$$

Effective length of column for both ends fixed.

$$L_e = \frac{L}{2} = \frac{3000}{2} = 1500 \text{ mm.}$$

$$\text{Factor of safety} = \frac{\text{cripping load}}{\text{safe load.}}$$

$$\text{cripping load} = \text{safe load} \times \text{factor of safety.}$$

$$= 150 \times 5$$

$$P = 750 \text{ kN} = 750 \times 10^3 \text{ N.}$$

$$\begin{aligned} \text{Area of column (A)} &= \frac{\pi}{4} (D^2 - d^2) \\ &= \frac{\pi}{4} (D^2 - (0.6D)^2) \end{aligned}$$

$$A = \pi \times 0.16 D^2$$

$$\text{Moment of Inertia (I)} = \frac{\pi}{64} [D^4 - (0.6D)^4]$$

$$I = \pi \times 0.0136 D^4.$$

$$I = A \times k^2 \text{ where } k \text{ is radius of gyration}$$

$$k = \sqrt{\frac{I}{A}}$$

$$k = \sqrt{\frac{0.0136 \times \pi \times D^4}{0.16 \times \pi \times D^2}}$$

$$\boxed{k = 0.29 D}$$

ANSWER KEY AND SCHEME OF EVALUATION

13 Ans: -

using equation
$$P = \frac{\sigma_c \times A}{1 + \alpha \left[\frac{L_e}{r_c} \right]^2}$$

$$750 \times 10^3 = \frac{450 \times \pi \times 0.16^2}{1 + \frac{1}{1700} \times \left[\frac{1500}{0.29D} \right]^2}$$

$$\frac{750 \times 10^3}{450 \times \pi \times 0.16} = \frac{D^2}{1 + \frac{15737.56}{D^2}}$$

$$3315.72 = \frac{D^4}{D^2 + 15737.56}$$

$$D^4 - 3315.72 D^2 - [3315.72 \times 15737.56] = 0$$

$$D^2 = 9069.33$$

$$D = 95.23 \text{ mm}$$

⇒ External diameter of the column (D) = 95.23 mm ≈ 95.5 mm

Internal diameter of column (d) = 0.6 × D
= 0.6 × 95.5

$$d = 57.3 \text{ mm}$$

1A) A hollow shaft is to transmit 200kW at 800.r.p.m. If the shear stress is not to exceed 60MPa and internal diameter is 0.6 of the external diameter, find the dimensions of the shaft.

Ans:-

Given data:-

Power Transmitted (P) = 200kW.

r.p.m (N) = 800.r.p.m

shear stress τ_s or $f_s = 60\text{MPa} = 60\text{N/mm}^2$.

Internal diameter (d) = 0.6D.

$$P = \frac{2\pi NT}{60} = \frac{2 \times \pi \times 80 \times T}{60}$$

$$T = \frac{200 \times 10^3 \times 60}{2 \times \pi \times 80}$$

$$T = 23873.24 \text{ Nm}$$

By using Torsion equation:-

$$\frac{T}{J} = \frac{f_s}{r}$$

$$\frac{23873.24}{\frac{\pi}{32} [D^4 - d^4]} = \frac{60}{\left[\frac{D}{2}\right]}$$

ANSWER KEY AND SCHEME OF EVALUATION

$$\frac{23873.24 \times 1000}{60} = \frac{\frac{\pi}{16} [0.4 \cdot d^4]}{d/2}$$

$$= \frac{\pi}{16} [0.8704 d^3]$$

$$D = \sqrt[3]{2328.152.055} = \underline{\underline{132.5 \text{ mm}}}$$

$$\text{Internal dia} = 0.6 D.$$

$$= 0.6 \times 132.5$$

$$\boxed{\underline{\underline{d = 79.5 \text{ mm}}}}$$

- 15) A thin cylindrical shell is 3m long and 1m in internal diameter. It is subjected to internal pressure of 1.2 MPa. If the thickness of the shell is 12mm, find the circumferential stress, longitudinal stress, changes in diameter, length and volume. Take $E = 200 \text{ GPa}$ and $\mu = 0.3$.

ISANS:-

Given data:-

Internal diameter of shell (d) = 1000 mm

Length of shell (L) = 3000 mm.

Thickness of shell (t) = 12 mm.

Internal pressure (P) = 1.2 N/mm²

Young's modulus (E) = 2.0×10^5 N/mm².

Poisson's ratio (μ) = 0.25.

Circumferential stress (σ_1) =

$$\sigma_1 = \frac{Pd}{2t}$$

$$= \frac{1.2 \times 1.0}{2 \times 0.012}$$

$$= 50 \text{ N/mm}^2$$

$$\boxed{\sigma_1 = 50 \text{ N/mm}^2}$$

Longitudinal stress (σ_2) = $\sigma_2 = \frac{Pd}{4t}$

$$= \frac{1.2 \times 1.0}{4 \times 0.012}$$

$$= 25 \text{ N/mm}^2$$

$$\boxed{\sigma_2 = 25 \text{ N/mm}^2}$$

= .

ANSWER KEY AND SCHEME OF EVALUATION

change in diameter :- (δd) :-

$$\delta d = \frac{Pd^2}{2tE} \left[1 - \frac{1}{2}\mu \right]$$

$$= \frac{1.2 \times 1000^2}{2 \times 12 \times 2 \times 10^5} \left[1 - \frac{1}{2} \times 0.25 \right]$$

$$\boxed{\delta d = 0.21875 \text{ mm}}$$

change in length :- (δL) :-

$$\delta L = \frac{PdL}{2tE} \left[\frac{1}{2} - \mu \right]$$

$$= \frac{1.2 \times 1000 \times 3000}{2 \times 12 \times 2 \times 10^5} \left[\frac{1}{2} - 0.25 \right]$$

$$\boxed{\delta L = 0.1875 \text{ mm}}$$

Change in Volume: (δV):-

$$\frac{\delta V}{V} = 2 \frac{\delta d}{d} + \frac{\delta l}{l}$$

$$= \left[2 \times \frac{0.21875}{1000} \right] + \left[\frac{0.1875}{3000} \right]$$

$$= 4.375 \times 10^{-4} + 6.25 \times 10^{-5}$$

$$u = \frac{\pi}{4} \times d^2 \times l$$

$$= \frac{\pi}{4} \times 1000^2 \times 3000$$

$$= 2.356 \times 10^9$$

$$= 4.375 \times 10^{-4} + 6.25 \times 10^{-5}$$

$$\delta V = 5 \times 10^{-4} \text{ m}^3$$

$$\delta V = 1.178 \times 10^6 \text{ mm}^3$$

Semester End Examination, February, 2022

Degree	B. Tech. (U. G.)	Program	Civil Engineering	Academic Year	2021 - 2022
Course Code	20CE305	Test Duration	3 Hrs. Max. Marks 70	Semester	III
Course	Fluid Mechanics				

Part A (Short Answer Questions 5 x 2 = 10 Marks)

No.	Questions (1 through 5)	Learning Outcome (s)	DoK
1	Define mass density and weight density.	20CE305.1	L1
2	Classify the velocity potential function and stream function.	20CE305.2	L2
3	What are Newtonian and Non-Newtonian fluids?	20CE305.3	L1
4	Define draft tube.	20CE305.4	L1
5	Write four methods to control boundary layer.	20CE305.5	L1

Part B (Long Answer Questions 5 x 12 = 60 Marks)

No.	Questions (6 through 15)	Marks	Learning Outcome (s)	DoK
6 (a)	A Rectangular plane surface 1 m wide and 3 m deep lies in water in such a way that its plane surface makes an angle of 300 with the free surface of water. Determine the total pressure and the depth of centre of pressure when upper edge of the plate is 2 m below the free surface.	6M	20CE305.1	L3
6 (b)	Explain Simple Manometers.	6M	20CE305.1	L2
OR				
7 (a)	The left leg of a U-tube mercury manometer is connected to pipe-line conveying water, the level of mercury in the leg being 0.8 m below the center of pipe line, and the right leg is open to atmosphere. The level of mercury in the right leg is 0.65 m above that in the left leg and the space above mercury in the right leg contains Benzene (Specific gravity 0.87) to a height of 0.4 m. Find the pressure in the pipe?	6M	20CE305.1	L3
7 (b)	Develop an equation for centre of pressure for an inclined plane surface submerged in a liquid.	6M	20CE305.1	L2
8 (a)	The velocity components in a two Dimensional flow field for an incompressible fluid are as follows, $u = y^3/3 + 2x - x^2y$; $v = xy^2 - 2y - x^3/3$. Obtain Stream function and velocity potential function.	6M	20CE305.2	L3
8 (b)	What is the Principle involved in the functioning of Orifice meter?	6M	20CE305.2	L2
OR				
9 (a)	Define the equation of Continuity. And obtain an expression for continuity equation for 3-D flow.	6M	20CE305.2	L3
9 (b)	Explain various types of fluid flows.	6M	20CE305.2	L2
10 (a)	Explain about pipes in parallel connection.	6M	20CE305.3	L2
10 (b)	Derive Darcy-Weisbach equation.	6M	20CE305.3	L2
OR				
11 (a)	Explain briefly about different types of losses in pipe flow.	6M	20CE305.3	L2
11 (b)	Derive the expression for the velocity distribution for the flow of fluid between the parallel plates.	6M	20CE305.3	L2
12 (a)	What are the performance characteristics of a turbine?	6M	20CE305.4	L2
12 (b)	Explain geometric similarity.	6M	20CE305.4	L2

OR				
13	Explain in detail about Francis turbine with a neat diagram.	12M	20CE305.4	L2
14	Explain the characteristics of Laminar and Turbulent boundary layer over a thin flat plate with a neat figure.	12M	20CE305.5	L2
OR				
15 (a)	Discuss characteristics of laminar and turbulent boundary layer.	6M	20CE305.5	L2
15 (b)	Write about the development of lift in immersed bodies.	6M	20CE305.5	L2



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ANSWER KEY AND SCHEME OF EVALUATION

Part - A

1. Define Mass density and Weight density.

A. Density (or) Mass density :-

It is defined as the ratio of the mass of fluid to its volume. Thus mass per unit volume of a fluid is called density. It is denoted by the symbol ρ (rho). Its units are kg/m^3 .

$$\rho = \frac{\text{Mass of Fluid}}{\text{Volume of Fluid}}$$

density of water is 1 gm/cm^3 (or) 1000 kg/m^3 .

Specific Weight (or) Weight density :-

It is defined as the ratio between weight of fluid to its Volume. Thus weight per unit volume of a fluid is called weight density. It is denoted by (W) .

$$W = \frac{\text{Weight of fluid}}{\text{Volume of fluid}} = \frac{W}{V}$$

$$W = \rho g.$$

2. Classify the velocity potential Function and Stream function.

A Velocity Potential Function :-

It is defined as a scalar function of Space and time such that its negative derivative with respect to any direction gives the fluid velocity in that direction.

It is defined by ϕ .

$$\phi = f(x, y, z) \text{ for Steady flow.}$$

$$u = -\frac{\partial \phi}{\partial x}$$

$$v = -\frac{\partial \phi}{\partial y}$$

$$w = -\frac{\partial \phi}{\partial z}$$

Where u , v and w are components of Velocity in x , y & z directions respectively.

Stream Function :-

It is defined as the scalar function of Space and time such that its partial derivative with respect to any direction gives the velocity component at right angles to that directions.

It is denoted by ψ .

$$\psi = f(x, y)$$

$$\frac{\partial \psi}{\partial x} = v$$

$$\frac{\partial \psi}{\partial y} = -u$$

ANSWER KEY AND SCHEME OF EVALUATION

3. What are Newtonian and Non-Newtonian fluids?

A Newton's Law of Viscosity :-

It states that the shear stress (τ) on a fluid element layer is directly proportional to the rate of shear strain. The constant of proportionality is called the co-efficient of viscosity.

$$\tau = \mu \frac{du}{dy}$$

Fluids which obey the above relation are known as Newtonian fluids and the fluids which do not obey the above relation are called Non-Newtonian fluids.

4. Define draft tube.

A The draft-tube is a pipe of gradually increasing area which connects the outlet of the runner to the tail race. It is used for discharging water from the exit of the turbine to the tail race. This pipe of gradually increasing area is called a draft-tube. One end of the draft-tube. One end of the draft-tube is connected to the outlet of the runner while the other end is submerged below the level of water in the tail race.

5. Write four methods to control boundary layer.

A. The following are the methods for preventing the separation of boundary layer.

- Suction of the slow moving fluid by a suction slot.
- Supplying additional energy from a blower.
- Providing a bypass in the slotted wing.
- Rotating boundary in the direction of flow.
- Providing small divergence in a diffuser.
- Providing guide - blades in a bend.
- Providing a trip - wire ring in the laminar region for the flow over a sphere.

ANSWER KEY AND SCHEME OF EVALUATION

Part - B

6.(a) A rectangular plane surface 1m wide and 3m deep lies in water in such a way that its plane surface makes an angle of 30° with the free surface of water. Determine the total pressure and the depth of centre of pressure when upper edge of the plate is 2m below the free surface.

Sol. Given data :-

Width of plane surface ; $b = 1\text{m}$.

Depth $d = 3\text{m}$.

Angle $\theta = 30^\circ$.

Distance of upper edge from free water surface = 2m.

(i) Total pressure force ; $F = \rho g A \bar{h}$ [where $\rho = 1000 \text{ kg/m}^3$]

$$A = b \times d = 3 \times 2 = 6 \text{ m}^2.$$

$$\bar{h} = \text{depth of C.G.} = 2 + 1.5 \sin(30^\circ)$$

$$\therefore F = 1000 \times 9.81 \times 6 \times 2.75 = \underline{161865 \text{ N}}.$$

(ii) Centre of Pressure

$$h^* = \frac{I_G \sin^2 \theta}{A \bar{h}} + \bar{h} \quad \left[I_G = \frac{(bd^3)}{12} = 2.25 \text{ m}^4 \right]$$

$$h^* = \frac{2.25 \times 0.25}{6 \times 2.75} + 2.75 = \underline{2.784 \text{ m}}.$$

6(b). Explain Simple Manometers.

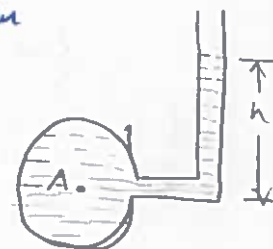
A. Simple Manometers :-

It consists of a glass tube having one of its ends connected to a point where pressure is to be measured and other end remains open to atmosphere. Common types of simple manometers are.

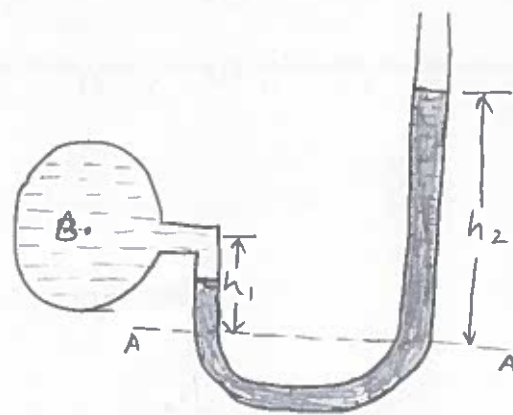
- Piezometer.
- U-tube manometer and.
- Single column Manometer.

Piezometer :- It is the simplest form of manometer used for measuring gauge pressures. One end of this manometer is connected to point where pressure is to be measured and other end is open to atmosphere.

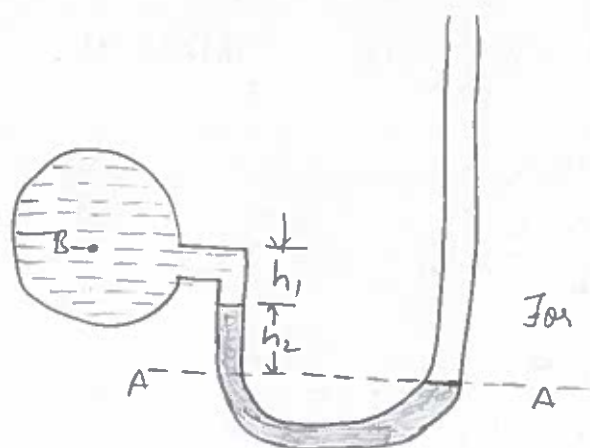
Pressure at A $P_A = \rho \times g \times h \text{ N/m}^2$.



U-tube Manometer :- It consists of glass tube bent in U-shape, one end of which is connected to a point at which pressure is to be measured and other end remains open to the atmosphere. The tube contains mercury.



For gauge pressure



For vacuum pressure.

ANSWER KEY AND SCHEME OF EVALUATION

7(a). The left leg of a U-tube mercury manometer is connected to pipe-line conveying water, the level of mercury in the leg being 0.8 m below the center of pipe line, and the right leg is open to atmosphere. The level of mercury in the right leg is 0.65 m above that in the left leg and the space above mercury in the right leg contains Benzene to a height of 0.4 m. Find the pressure in the pipe?

A.

$$\frac{P_A}{\rho} + 0.8 = (0.65 \times 13.6) + (0.87 \times 0.4)$$

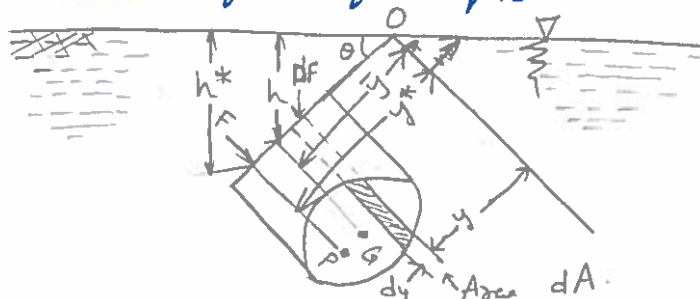
$$\rho = 1$$

$$P_A = 8.388 \text{ m of water.}$$

7(b) Develop an equilibrium equation for centre of pressure for an Inclined plane surface submerged in a liquid.

A. Inclined Plane Surface Submerged in liquid :-

Consider a plane surface of arbitrary shape immersed in a liquid in such a way that the plane of the surface makes an angle θ with the free surface of liquid.



Pressure intensity on the Elementary strip $P = \rho gh$.

$$\therefore \text{Pressure intensity force, } dF, \text{ on strip, } dF = P \times \text{Area of strip} \\ = \rho gh \times dA.$$

$$\text{Total pressure force, } F = \int dF = \int \rho gh \cdot dA.$$

$$\frac{h}{y} = \frac{\bar{h}}{\bar{y}} = \frac{h^*}{y^*} = \sin \theta.$$

$$h = y \sin \theta.$$

$$F = \int \rho \rho g \times y \times \sin \theta \times dA \\ = \rho g \sin \theta \int y \cdot dA.$$

$$\text{But } \int y dA = A \bar{y}.$$

$$\therefore F = \rho \rho g \cdot \sin \theta \bar{y} \times A. \\ = \rho g A \bar{h}.$$

Centrat pressure Centre of pressure (h^*):-

$$\text{Pressure force on strip, } dF = \rho gh dA = \rho g y \sin \theta dA.$$

$$\text{Moment of force about axis O-O} = \rho g \sin \theta y^2 dA.$$

$$\text{Sum of moments about axis O-O} = \rho \rho g \sin \theta \int y^2 dA.$$

$$F \times y^* = \rho g \sin \theta I_0.$$

$$y^* = \frac{\rho g \sin \theta I_0}{F}.$$

$$\Rightarrow \frac{h^*}{\sin \theta} = \frac{\rho g \sin \theta}{\rho g A \bar{h}} \left[I_G + A \bar{y}^2 \right].$$

$$\therefore h^* = \frac{\sin^2 \theta}{A \bar{h}} \left[I_G + A \times \frac{\bar{h}^2}{\sin^2 \theta} \right].$$

ANSWER KEY AND SCHEME OF EVALUATION

8(a). The velocity components in a two dimensional flow for an incompressible fluid are as follows, $u = \frac{y^3}{3} + 2x - x^2y$; $v = xy^2 - 2y - \frac{x^3}{3}$. Obtain Stream function and velocity potential function.

Sol. Given :-

$$u = \frac{y^3}{3} + 2x - x^2y.$$

$$v = xy^2 - 2y - \frac{x^3}{3}.$$

Velocity Components

$$\frac{\partial \psi}{\partial x} = v = xy^2 - 2y - \frac{x^3}{3} \quad \text{--- (i)}$$

$$\frac{\partial \psi}{\partial y} = -u = -\frac{y^3}{3} - 2x + x^2y \quad \text{--- (ii)}$$

Integrating (i) w.r.t. x , we get $\psi = \int (xy^2 - 2y - \frac{x^3}{3}) dx$.

$$\psi = \frac{x^2y^2}{2} - 2xy - \frac{x^4}{4 \times 3} + k \quad \text{--- (iii)}$$

Differentiate (iii) w.r.t. y we get

$$\frac{\partial \psi}{\partial y} = \frac{2x^2y}{2} - 2x + \frac{\partial k}{\partial y} = x^2y - 2x + \frac{\partial k}{\partial y}.$$

$$\therefore \frac{\partial k}{\partial y} = -y^3/3$$

we get by Simplifying above equation

Integrating, we get $k = \int (-\frac{y^3}{3}) dy = -\frac{y^4}{4 \times 3} = -\frac{y^4}{12}.$

Relation between stream function and Velocity potential function.

$$u = -\frac{\partial \phi}{\partial x} \quad \text{and} \quad v = -\frac{\partial \phi}{\partial y}$$

$$\text{and} \quad u = -\frac{\partial \psi}{\partial y} \quad \text{and} \quad v = \frac{\partial \psi}{\partial x}$$

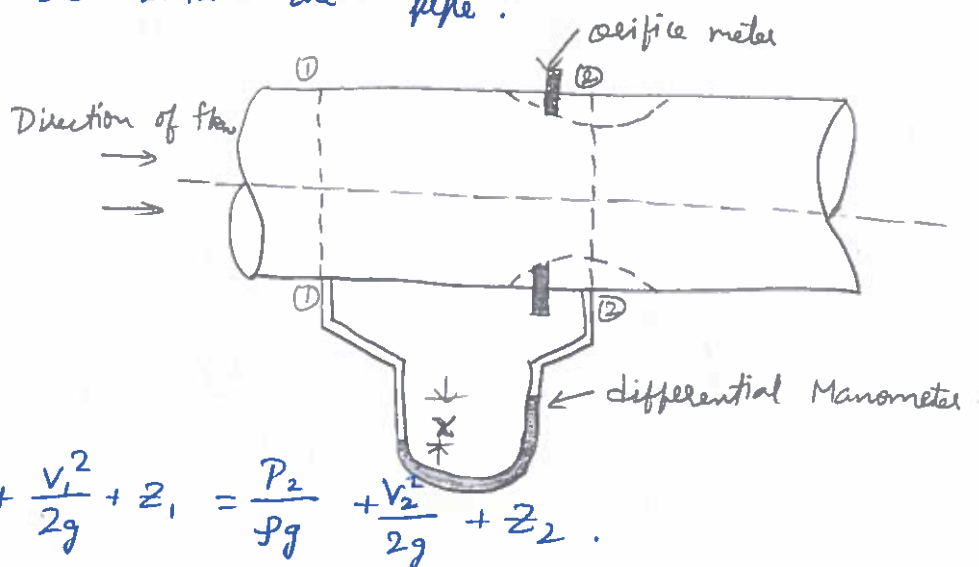
$$\text{Thus} \quad u = -\frac{\partial \phi}{\partial x} = -\frac{\partial \psi}{\partial y} \quad \text{and} \quad v = -\frac{\partial \phi}{\partial y} = \frac{\partial \psi}{\partial x}$$

$$\text{hence} \quad \frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y}$$
$$\frac{\partial \phi}{\partial y} = -\frac{\partial \psi}{\partial x}$$

From this we can find the velocity potential function using the above-computed stream function.

8(b) What is the principle involved in the functioning of Orifice meter?

A. Orifice meter :- It is a device used for measuring the rate of flow of a fluid through a pipe. It is a cheaper device as compared to venturimeter. It also works on the same principle as that of venturimeter. It consists of a flat circular plate which has a circular sharp edged hole called ~~or~~ orifice, which is concentric with the pipe.



$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

ANSWER KEY AND SCHEME OF EVALUATION

9(a) Define the equation of Continuity and obtain an expression for continuity equation for 3-D flow.

Ans. Continuity Equation in Three-dimensions :-

Consider a fluid element of lengths dx , dy & dz in the direction of x , y & z . Let u , v , and w are the inlet velocity components in x , y & z directions respectively.

Mass of fluid entering the face ABCD per second.

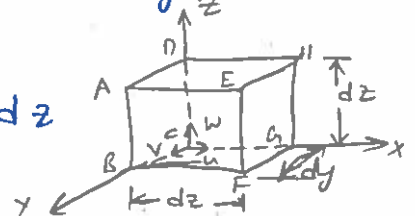
$$= \rho \times \text{velocity in } x\text{-direction} \times \text{Area of ABCD}$$

$$= \rho \times u \times (dy \times dz)$$

$$\therefore \text{Gain of Mass in } x\text{-direction} = -\frac{\partial}{\partial x} (\rho u) dx dy dz.$$

$$\text{Similarly the net gain of mass in } y\text{-direction} = -\frac{\partial}{\partial y} (\rho v) dx dy dz.$$

$$\text{and in } z\text{-direction} = -\frac{\partial}{\partial z} (\rho w) dx dy dz$$



$$\therefore \text{Net gain of Masses} = -\left[\frac{\partial}{\partial x} (\rho u) + \frac{\partial}{\partial y} (\rho v) + \frac{\partial}{\partial z} (\rho w)\right] dx dy dz.$$

$$\therefore \frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x} (\rho u) + \frac{\partial}{\partial y} (\rho v) + \frac{\partial}{\partial z} (\rho w) = 0$$

This equation is applicable for (i) Steady and unsteady flow, (ii) Uniform and non uniform flow & (iii) Compressible and Incompressible flow.

Q(6) Explain Various types of fluid flows :-

A. Types of Fluid Flow :-

- (i) Steady and unsteady flow.
- (ii) Uniform and non-uniform flow.
- (iii) Laminar and Turbulent flow.
- (iv) Compressible and in-compressible flow.
- (v) Rotational and irrotational flow.
- (vi) One, two and three dimensional flow.

→ Steady and Unsteady flow :-

Steady ^{flow} is defined as that type of flow in which the fluid characteristics like velocity, pressure, density, etc at a point do not change with time.

$$\left[\frac{\partial v}{\partial t} \right]_{x_0, y_0, z_0} = 0 ; \left(\frac{\partial P}{\partial t} \right) = 0 ; \left(\frac{\partial \rho}{\partial t} \right) = 0.$$

Unsteady flow is that type of flow, in which the velocity, pressure or density at a point changes with time.

$$\frac{\partial v}{\partial t} \neq 0 ; \frac{\partial P}{\partial t} \neq 0 \quad \frac{\partial \rho}{\partial t} \neq 0.$$

→ Uniform and non-uniform flows :-

Uniform flow is that type of flow in which the velocity at any given time doesn't change with respect to ^{Space} ~~time~~.

$$\frac{\partial v}{\partial s} = 0.$$

Non uniform flow is that type of flow in which the velocity at any given time changes with respect to space.

$$\frac{\partial v}{\partial s} \neq 0.$$



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ANSWER KEY AND SCHEME OF EVALUATION

→ Laminar and Turbulent flow :-

Laminar flow is that type of flow in which the fluid particles move along well defined paths or ~~layers~~ ^{gliding} stream line and all the stream lines are straight and parallel.

Turbulent flow is that type of flow in which the fluid particles move in a zig-zag way.

→ Compressible and Incompressible flows :-

Compressible flow is that type of flow in which the density of the fluid changes from point to point or in other words the density is not constant for fluid. " $\rho \neq \text{constant}$ ".

Incompressible flow is that type of flow in which the density is constant for the fluid flow. " $\rho = \text{constant}$ ".

→ Rotational and Irrational flow :-

Rotational flow is that type of flow in which the fluid particles while flowing along stream lines, also rotates about their own axis. And if the fluid particles while flowing along stream lines, do not rotate about their own axis then that type of flow is called irrational flow.

10(a) Explain about pipes in parallel connection.

A. Flow through parallel pipes :-

Consider a main pipe which divides into two or more branches. ~~as~~ and again joins together down stream to form a single pipe, then the branch pipes are said to be connected in parallel. ~~The~~ The discharge through the main is ~~increasing~~ increased by connecting pipes in parallel.

The rate of flow in the main pipe is equal to rate of flow through branch pipes. Hence from fig.

we get

$$Q = Q_1 + Q_2.$$

$$\Rightarrow \frac{4 f_1 L_1 V_1^2}{d_1 \times 2g} = \frac{4 f_2 L_2 V_2^2}{d_2 \times 2g}.$$

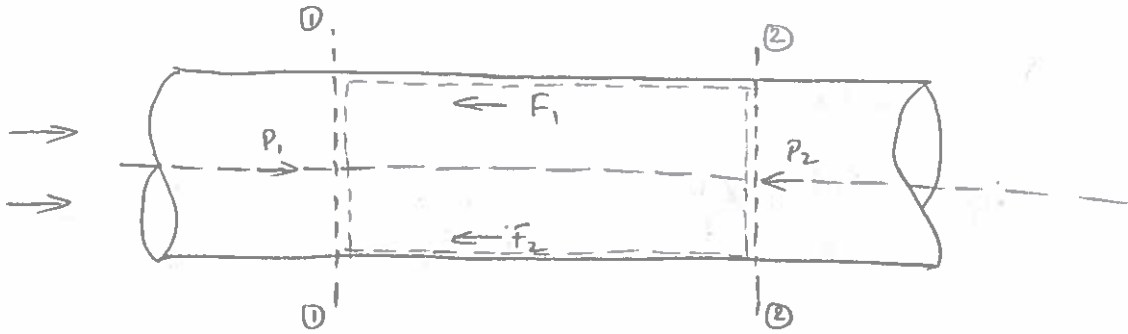
If f_1 & f_2 are equal then $\frac{L_1 V_1^2}{d_1 \times 2g} = \frac{L_2 V_2^2}{d_2 \times 2g}.$

10(b) Derive Darcy - Weisbach equation.

A. Expression for loss of head due to Friction in pipes.

Consider a uniform horizontal pipe, having steady flow

ANSWER KEY AND SCHEME OF EVALUATION



Applying bernoulli's equations at sections 1-1 and 2-2.

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_f$$

$$\therefore \frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_f \quad (\text{or}) \quad h_f = \frac{P_1}{\rho g} - \frac{P_2}{\rho g} \quad \because z_1 = z_2 \text{ \& } V_1 = V_2$$

frictional resistance $F_f = f' \times P \times L \times V^2$.

Pressure force at 1-1 = $P_1 \times A$.

Pressure force at 2-2 = $P_2 \times A$.

$$P_1 A - P_2 A - F_f = 0.$$

$$(P_1 - P_2) A = F_f$$

$$\Rightarrow f' \times P \times V^2 \times L = F_f.$$

$$(\text{or}) (P_1 - P_2) = \frac{f' \times P \times L \times V^2}{A}.$$

$$h_f = \frac{f'}{2g}$$

$$h_f = \frac{f'}{pg} \times \frac{p}{A} \times L \times v^2.$$

$$\therefore h_f = \frac{f'}{pg} \times \frac{4Lv^2}{d}$$

The final equation is

$$h_f = \frac{4 \cdot f}{2g} \cdot \frac{Lv^2}{d} = \frac{4f \cdot L \cdot v^2}{d \times 2g}.$$

[f is known as friction factor.]

11)a) Explain briefly about different types of losses in pipe flow

A) Loss of Energy in Pipes

When a fluid is flowing through a pipe, the fluid experiences some resistance due to which some of the energy of fluid is lost. This loss of energy is classified as two types

1. Major energy losses

This is due to friction and it's calculated by the following formulae

a) Darcy-Weisbach Formula

b) Chezy's Formula

2) Minor energy losses

This is due to

a) Sudden Expansion of pipe

b) Sudden contraction of pipe

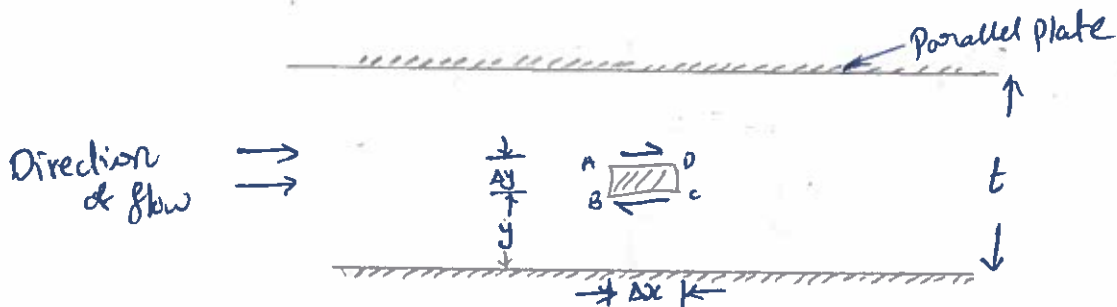
c) Bend in pipe

d) pipe fittings

e) An obstruction in pipe etc.

ANSWER KEY AND SCHEME OF EVALUATION

11 b) Derive the expression for the velocity distribution for the flow of fluid between parallel plates



Viscous flow between two parallel plates

Consider two parallel plates kept at a distance t as shown in figure. A viscous fluid is flowing between these two plates from left to right. Consider a fluid element of length Δx and thickness Δy at a distance y from the lower fixed plate

Intensity of pressure on face AB = P

" " " " " CD = $P + \frac{\partial P}{\partial x} \Delta x$

Let, shear stress acting on face ABC = τ

" " " " " AD = $\tau + \frac{\partial \tau}{\partial y} \Delta y$

If width of element in the direction $\perp$ to the paper is unity then forces acting on the fluid element are

1. Pressure force on face AB is $P \times \Delta y \times 1$

2) " " " " " CD is $(P + \frac{\partial P}{\partial x} \Delta x) \Delta y \times 1$

3) The shear force on face BC is $\tau \times \Delta x \times 1$

4) " " " " " AD is $(\tau + \frac{\partial \tau}{\partial y} \Delta y) \Delta x \times 1$

For steady and uniform flow, there is no acceleration and hence the resultant force in the direction of flow is zero

$$P(\Delta y \times 1) - (P + \frac{\partial P}{\partial x} \Delta x) \Delta y \times 1 - \tau(\Delta x \times 1) + (\tau + \frac{\partial \tau}{\partial y} \Delta y) \Delta x \times 1 = 0$$

$$-\frac{\partial P}{\partial x} \Delta x \Delta y + \frac{\partial \tau}{\partial y} \Delta y \Delta x = 0$$

$$\frac{\partial P}{\partial x} = \frac{\partial \tau}{\partial y} \rightarrow (1)$$

Velocity distribution:

To obtain a velocity distribution across a section the value of shear stress $\tau = \mu \frac{du}{dy}$ from Newton's law of viscosity

$$\frac{\partial P}{\partial x} = \frac{\partial}{\partial y} \left(\mu \frac{du}{dy} \right) = \mu \frac{\partial^2 u}{\partial y^2}$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{1}{\mu} \frac{\partial P}{\partial x}$$

Integrating the above equation w.r.t 'y' we get

$$\frac{\partial u}{\partial y} = \frac{1}{\mu} \frac{\partial P}{\partial x} y + C_1$$

Integrating again

$$u = \frac{1}{2\mu} \frac{\partial P}{\partial x} y^2 + C_1 y + C_2 \rightarrow (2)$$

Using Boundary conditions i.e @ $y=0, u=0$

@ $y=t, u=0$

Substituting boundary conditions in equation ② we get

$$C_2 = 0 \quad \& \quad C_1 = -\frac{1}{2\mu} \frac{\partial p}{\partial x} t$$

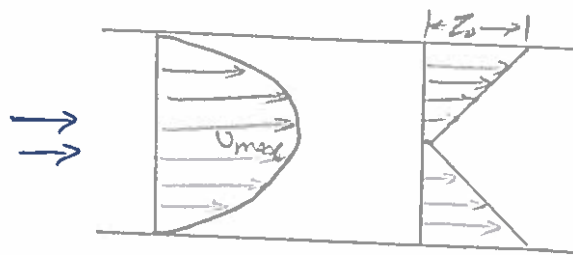
Substituting C_1 & C_2 in equation ②

$$\textcircled{2} \Rightarrow u = \frac{1}{2\mu} \frac{\partial p}{\partial x} y^2 + y \left(-\frac{1}{2\mu} \frac{\partial p}{\partial x} t \right)$$

$$u = -\frac{1}{2\mu} \frac{\partial p}{\partial x} [ty - y^2] \rightarrow \textcircled{3}$$

In the above equation, $\frac{\partial p}{\partial x}$ & t are constant.

It means u varies with the square of y .



Velocity distribution & Shear stress distribution across a section of parallel plates

12a) What are the performance characteristics of a turbine?

Ans) Efficiencies of a turbine: The following are the important efficiencies of a turbine.

- Hydraulic efficiency (η_h)
- Mechanical efficiency (η_m)
- Volumetric efficiency (η_v)
- Overall efficiency (η_o)



N S RAJU INSTITUTE OF TECHNOLOGY

(AUTONOMOUS)

SONTYAM , ANANDAPURAM, VISAKHAPATNAM – 531 173

ANSWER KEY AND SCHEME OF EVALUATION



ANSWER KEY AND SCHEME OF EVALUATION**a) Hydraulic efficiency (η_h)**

It is defined as the ratio of power given by water to the runner of a turbine to the power supplied by the water to the inlet of turbine.

$$\eta_h = \frac{\text{Power delivered to runner}}{\text{Power supplied at inlet}} = \frac{R.P}{W.P}$$

b) Mechanical efficiency (η_m):

The power delivered by water to the runner of a turbine is transmitted to the shaft of the turbine.

$$\eta_m = \frac{\text{Power at the shaft of turbine}}{\text{Power delivered by water to runner}} = \frac{S.P}{R.P}$$

c) Volumetric efficiency (η_v):

The ratio of the volume of water actually striking the runner to the volume of water supplied to the turbine is defined as volumetric efficiency.

$$\eta_v = \frac{\text{Volume of water actually striking the runner}}{\text{Volume of water supplied to the turbine.}}$$

d) Overall efficiency (η_o):

It is defined as the power available at the shaft of the turbine to power supplied by the water at the inlet of the turbine.

$$\eta_o = \frac{\text{Shaft Power}}{\text{Water Power}}$$

12b) Explain Geometric Similarity

Ans: Geometric similarity:

It is said to exist b/w the model and the prototype.

The ratio of all corresponding linear dimension in the model and prototype are equal

Let L_m = length of model

b_m = width of model

A_m = Area of model

D_m = Diameter of model

V_m = Volume of model

L_p, b_p, A_p, D_p and V_p are corresponding values of the prototype.

For geometric similarity b/w model and prototype, we have relation

$$\frac{L_p}{L_m} = \frac{b_p}{b_m} = \frac{D_p}{D_m} = L_r \text{ (Scale ratio)}$$

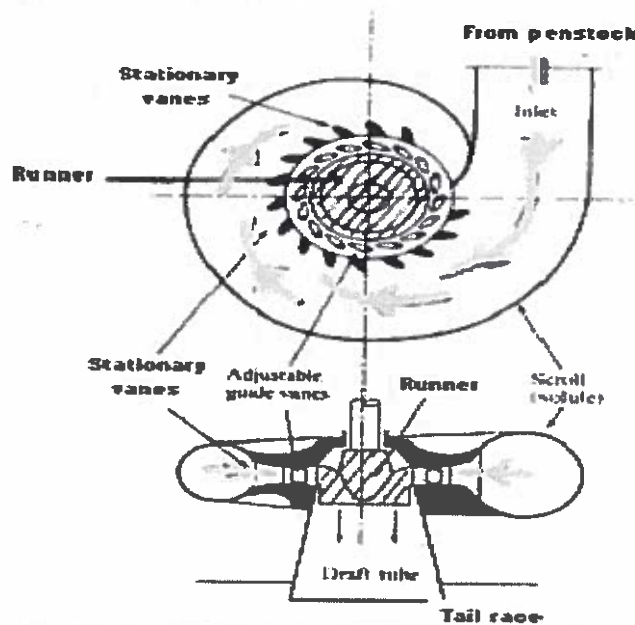
$$\text{Similarly, Areas ratio} = \frac{A_p}{A_m} = \frac{L_p \times b_p}{L_m \times b_m} = L_r \times L_r = L_r^2$$

$$\text{Volume ratio} = \frac{V_p}{V_m} = \left(\frac{L_p}{L_m}\right)^3 = \left(\frac{b_p}{b_m}\right)^3 = \left(\frac{D_p}{D_m}\right)^3$$

ANSWER KEY AND SCHEME OF EVALUATION

13) Explain in detail about Francis turbine with a neat diagram.

Ans) The inward flow reaction turbine having radial discharge at outlet is known as Francis turbine after the name of J.B. Francis. In the modern Francis turbine, the water enters the runner of the turbine in the radial direction at outlet and leaves in the axial direction at the inlet of the turbine.



Francis Turbine

The velocity triangle at inlet and out of the Francis turbine are drawn in the same way as in case of inward flow reaction turbine. As in case of Francis turbine, the discharge is radial at outlet, the velocity of whirl at outlet (ie V_{w2}) will be zero. Hence the work done by water on the runner per second will be $= PQ[V_w u_1]$

And work done per second per unit weight of water striking $= \frac{1}{g} [V_w u_1]$

Hydraulic efficiency will be given by $\eta_h = \frac{V_w u_1}{gH}$

Important relations for Francis turbine

1. The ratio of width of the wheel to its diameter is given as

$n = \frac{B_1}{D_1}$. The values of n varies from 0.1 to 0.40

2. The flow ratio is given as

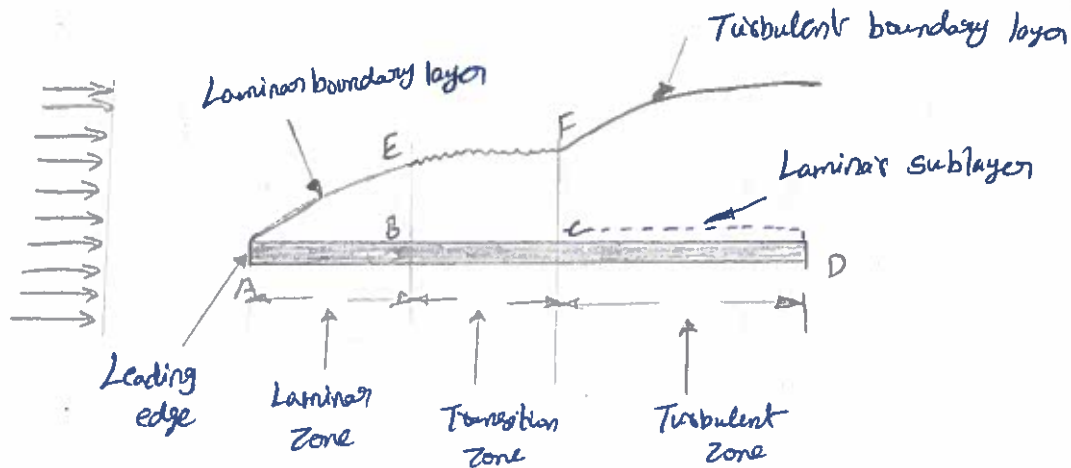
$= \frac{V_{f1}}{\sqrt{2gH}}$ and varies from 0.15 to 0.30

3. The speed ratio $= \frac{u_1}{\sqrt{2gH}}$ varies from 0.6 to 0.9

ANSWER KEY AND SCHEME OF EVALUATION

14) Explain the characteristics of laminar and Turbulent layer over a thin flat plate with a neat figure.

Ans:



Flow over a plate

For defining the boundary layer (i.e. laminar boundary layer or turbulent boundary layer) consider the flow of a fluid having ^{flow} stream velocity (U) over a smooth plate which is flat and placed parallel to the direction for free stream of fluid as show in figure above.

Characteristics of laminar and turbulent boundary layer :

Laminar boundary layers

- The laminar boundary is a very smooth flow, while the turbulent boundary contains swirls or eddies
- The laminar flow creates less skin friction drag than the turbulent flow but is less stable.

- Boundary layer flow over a wing surface begins a smooth laminar flow.
- As the flow continues back from the leading edge, the laminar boundary layer increases in thickness.

Turbulent Boundary layer:

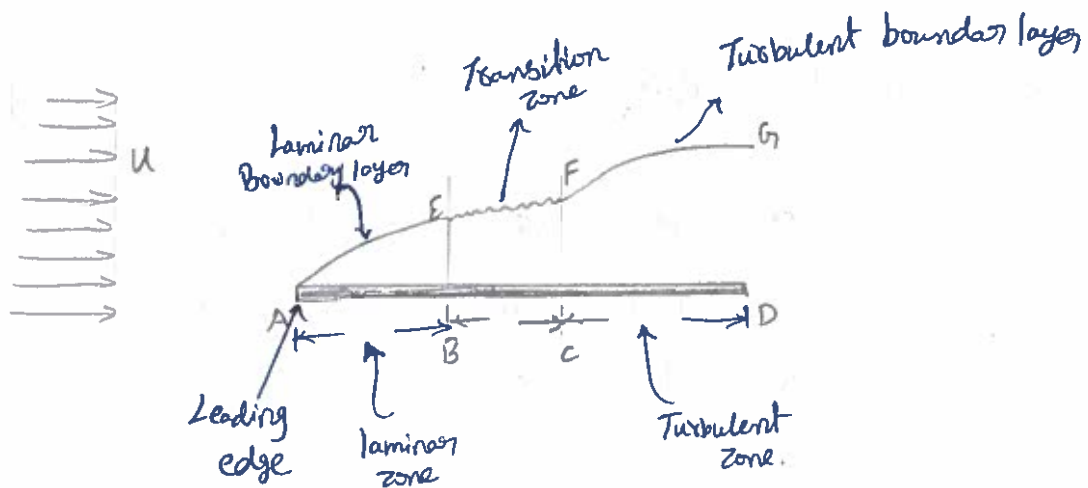
- At some distance back from the leading edge, the smooth laminar flow breaks down and transitions to a turbulent flow.
- For a drag standpoint, it is advisable to have the transition from laminar to turbulent flow as far on the wing as possible, or have a large amount of the wing surface within the laminar portion of the boundary layer.
- The low energy laminar flow, however, tends to break down more suddenly than turbulent layer.

ANSWER KEY AND SCHEME OF EVALUATION

15a) Discuss characteristics of laminar and turbulent boundary layer

Ans) Laminar Boundary layer:

For defining the boundary layer (i.e. laminar boundary layer or turbulent boundary layer) consider the flow of a fluid, having free stream velocity (U), over a smooth thin plate which is flat and placed parallel to the direction of free stream of fluid as shown in below figure. Let us consider the flow with zero pressure gradient on one side of plate, which is stationary.



Flow over a plate

The velocity of fluid on the surface of the plate should be equal to the velocity of plate. But plate is stationary and hence velocity of fluid on the surface of the plate is zero. But at a distance away from the plate, the fluid is having certain velocity.

The velocity gradient develops shear resistance, which retards the fluid. The fluid with uniform free stream velocity (U) is retarded in the vicinity of solid surface of the plate and the boundary layer region begins at the sharp leading edge.

Near the leading edge of the surface of the plate, where the thickness is small, the flow in the boundary layer is laminar though the main flow is turbulent. This layer of fluid is said to be laminar boundary layer.

The length of the plate from the leading edge, upto which laminar boundary layer exists is called laminar zone.

This shown by distance AB. The distance x from the leading edge is obtained from Reynold number equal to 5×10^5 for a plate.

$$\text{Reynolds number } (Re)_x = \frac{U \times x}{\nu}$$

x = distance from leading edge

U = Free Stream Velocity of fluid

ν = Kinematic viscosity of fluid

If the values of U & ν are known, ' x ' or the distance from the leading edge upto which laminar boundary layer exists can be calculated.

ANSWER KEY AND SCHEME OF EVALUATION

Turbulent boundary layers :

If the length of the plate is more than the distance x , calculated from the above equation, the thickness of boundary layer will go on increasing in the downstream direction.

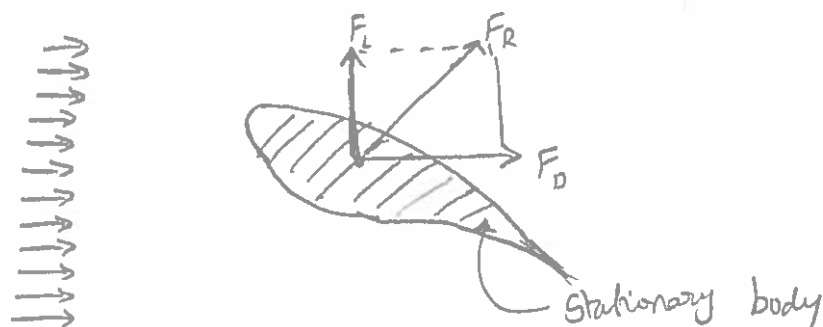
Then the laminar boundary layer becomes unstable and motion of fluid within it is disturbed and irregular which leads to a transition from laminar to turbulent boundary layer.

This short length over which the boundary layer flow changes from laminar to turbulent is called transition zone. This is shown by distance BC in figure drawn.

Further downstream, the transition zone, the boundary layer is turbulent and continues to grow in thickness. This layer of boundary is called turbulent boundary layer, which is shown by portion FG in the figure.

15b) Write about the development of lift in immersed bodies.

Ans: Consider a body held stationary in a real fluid, which is flowing at a uniform velocity U as shown in the below figure



Force exerted by a flowing fluid on a stationary body

The fluid will exert a force on the stationary body. The total force (F_R) exerted by fluid on the body is perpendicular to the surface of the body. Thus the total force is inclined to the direction of motion. The total force can be resolved in two components, one in the direction of motion and other perpendicular to the direction of motion.

Lift:

The component of the total force (F_R) in the direction perpendicular to the direction of motion is known as lift. This is denoted by F_L . Thus lift is the force exerted by the fluid in the direction perpendicular to the direction of motion. Lift force occurs only when the axis of the body is inclined to the direction of fluid flow.

If the axis of the body is parallel to the direction of fluid flow, lift force is zero. In that case only drag force acts.

Expression for lift:

$$F_L = C_L A \frac{\rho U^2}{2}$$

C_L = Coefficient of lift

A = Area of the body

U = velocity

ρ = density of fluid

Semester End Examination, February, 2022

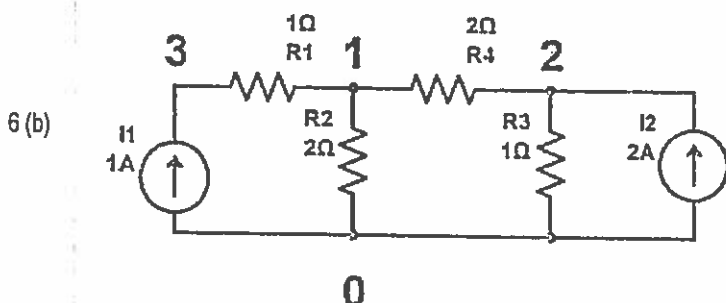
Degree	B. Tech. (U. G.)	Program	EEE	Academic Year	2021 - 2022
Course Code	20EE303	Test Duration	3 Hrs.	Max. Marks	70
Course	ELECTRICAL CIRCUIT ANALYSIS	Semester	III		

Part A (Short Answer Questions 5 x 2 = 10 Marks)

No.	Questions (1 through 5)	Learning Outcome (s)	DoK
1	An Electric iron is rated 1000W, 240V. Find the current drawn & resistance of the heating element.	20EE303.1	L3
2	What is the impedance of $(2+j5)\Omega$.	20EE303.2	L1
3	State Reciprocity Theorem	20EE303.3	L1
4	What is the relationship between Z-parameters and Y-parameters?	20EE303.4	L2
5	Write the sinusoidal response of (a) RC circuit (b) RL circuit.	20EE303.5	L1

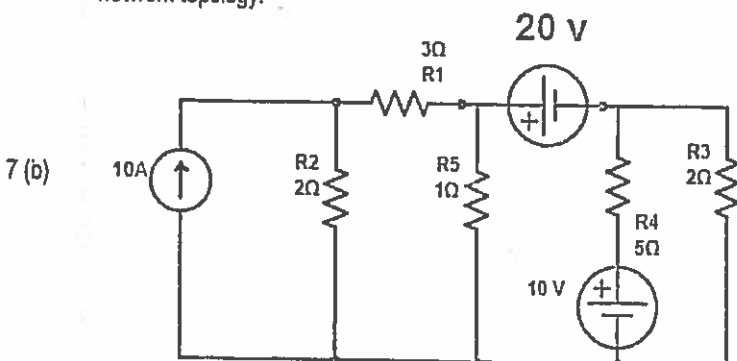
Part B (Long Answer Questions 5 x 12 = 60 Marks)

No.	Questions (6 through 15)	Marks	Learning Outcome (s)	DoK
6 (a)	Discuss the Kirchhoff's Laws with an example Determine the voltages 1 and 2 of the network in Fig. by nodal analysis.	6M	20EE303.1	L2



OR

7 (a)	Explain the Resistance parameter series and parallel combination technique with an example Determine current in 5Ω resistor for the circuit shown in figure with network topology.	6M	20EE303.1	L2



8 (a)	A series RLC circuit is supplied with a voltage source of 230 V, 50Hz. Determine circuit current and power factor if $R=40\Omega$, $L=0.2H$ and $C=50\mu F$.	6M	20EE303.2	L3

8 (b)	Draw the phasor relation for series RLC circuit.	6M	20EE303.2	L2

OR

9 (a)	A resistor of 150Ω , inductance of $200mH$ and a capacitance of $10\mu F$ are connected in series across 500V, 150Hz supply. Determine the following (i) Impedance (ii) current flowing through the circuit (iii) power factor (iv) voltage across R, L & C (v) power in watts	6M	20EE303.2	L3

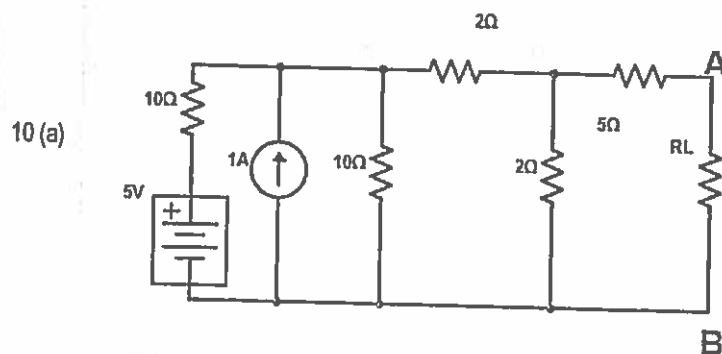
- 9 (b) A 120V AC circuit contain $10\ \Omega$ resistance and $30\ \Omega$ inductive reactance in series. What is average power of this circuit?

6M

20EE303.2

L3

In the circuit shown in Fig., obtain the condition from maximum power transfer to the load R_L . Hence determine the maximum power transferred.



7M

20EE303.3

L3

- 10 (b) State and Explain the superposition theorem.

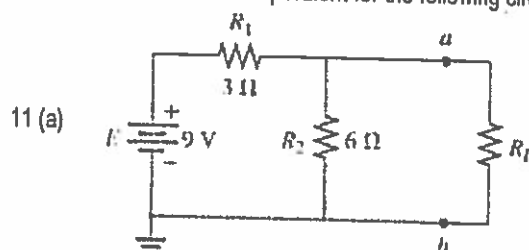
5M

20EE303.3

L2

OR

Find Thevenin's equivalent for the following circuit.



7M

20EE303.3

L3

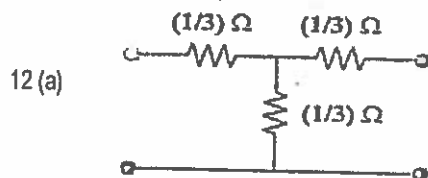
- 11 (b) State and explain millman's theorem

5M

20EE303.3

L3

Determine Y parameters of the following network



7M

20EE303.4

L3

- 12 (b) Explain about Y-parameters of a two-port network

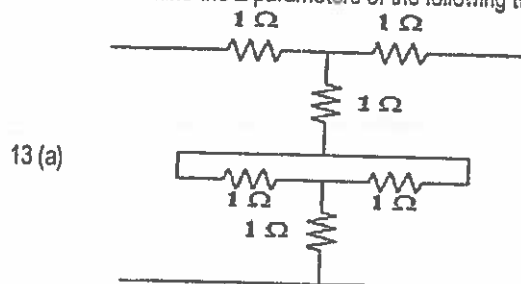
5M

20EE303.4

L2

OR

Determine the Z parameters of the following two port network.:



6M

20EE303.4

L3

- 13 (b) Derive Z parameters in terms of Y and h parameters

6M

20EE303.4

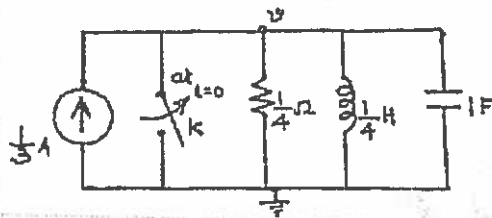
L2

- 14 (a) Figure shows RLC parallel circuit excited by a dc current source. At $t=0$ the switch is opened .find $v(t)$

7M

20EE303.5

L3



14 (b)

Determine the DC response of RL circuit and sketch the voltage transients

5M

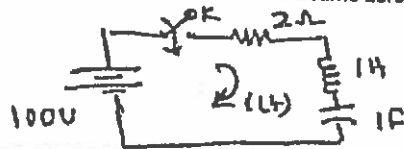
20EE303.5

L2

OR

Using laplace transform determine the current in circuit shown in figure when the switch is closed at $t = 0$. Assume zero initial condition.

15 (a)



6M

20EE303.5

L3

15 (b)

Derive the transient response of an RC circuit with DC excitation

6M

20EE303.5

L3

ANSWER KEY AND SCHEME OF EVALUATION

Electrical Circuit Analysis: 20EE303

Scheme of Evaluation:

Part-A:

1. Problem Formulae (2M)
2. Problem with formulae (2M)
3. Definition (2M)
4. Parameter Formulae (2M)
5. Response equations (2M)

6(a) Definitions:

KVL — 1M + Formulae — (2M)

KCL — 1M + Formulae — (2M)

6(b) Split of circuits node wise — 2M + 2M
Solution — 2M.

7(a) Definition & Explanation — 2M.

Series combination — 2M

Parallel combination — 2M.

7(b) Cases(i) & (ii) — One source at a time — 2M + 2M
Final solution — 2M.

8(a) Circuit — 2M

Current solution — 2M

Power factor solution — 2M

8(b) Circuit — 2M

R-L — 2M

R-C — 2M.

9(a) Each solution for (i), (ii), (iii), (iv) & (v) — 5M

Answer — 1M.

9(b) Step by step solution — 6M

10(a) Circuit — 1M

Source transformation — 1M

$R_{eq} = 2M$ $R_{th} = 1M$.

$V_{AB} = 2M$

10(b) Definition — 2M ; Example — 3M.

11(a) $R_{eq} = 2M$, Circuit — 1M, $V_{AB} = 2M$, $R_{th} = 2M$

11(b) Definition — 2M ; Example — 3M.

12(a) Equations — 4M ; Solution — 3M.

12(b) Definition — 2M ; Cases — 3M.

13(a) Cases (i) & (ii) — 3M + 3M.

13(b) Derivation — 3M + Equations — 3M.

14(a) Circuit — 2M + Calculation — 5M.

14(b) Derivation — 5M

15(a) Condition + Derivation — 3M + 3M

15(b) Derivation — 6M.

Electrical Circuit Analysis

①

End Examination

Key & Scheme

1. Rating of the Electric Iron = 1000W.

Voltage applied = $V = 240V$.

We know that

$$W = V \times I$$

$$\Rightarrow I = \frac{W}{V}$$

$$\Rightarrow I = \frac{1000}{240} = 4.167 A.$$

Therefore Current drawn = $I = 4.167 A$. (1 M)

Also, we know that

$$V = IR$$

$$\Rightarrow R = \frac{V}{I}$$

$$\Rightarrow R = \frac{240}{4.167}$$

$$\Rightarrow R = 57.59 \Omega.$$

Therefore, Resistance of heating element = R
 $= 57.59 \Omega$
(1 M).

2. Given,

$$(2+j5) \Omega$$

$$\text{Therefore } Z = 2+j5 \Omega \quad (2M)$$

$$|Z| = 5.38 \Omega$$

$$\angle \theta = 68.19^\circ$$

3. In any linear bilateral network, if a single voltage source V_a in branch 'a' produces a current I_b in branch 'b', then if the voltage source V_a is removed and inserted in branch 'b' will produce a current I_b in branch 'a'. The ratio of response to excitation is same for the two conditions mentioned above. This is called the reciprocity theorem. (2M)

$$4. \quad Z_{11} = \frac{Y_{22}}{\Delta Y} ;$$

$$Z_{12} = \frac{-Y_{12}}{\Delta Y} ;$$

(2M)

$$Z_{21} = \frac{-Y_{21}}{\Delta Y} ;$$

$$Z_{22} = \frac{Y_{11}}{\Delta Y} ;$$

$$\text{where } \Delta Y = \begin{vmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{vmatrix}$$

5. Sinusoidal response of

(a) RL circuit:

$$i = e^{-(R/L)t} \left[\frac{-V}{\sqrt{R^2 + (\omega L)^2}} \cos \left(\theta - \tan^{-1} \frac{\omega L}{R} \right) \right] + \frac{V}{\sqrt{R^2 + (\omega L)^2}} \cos \left(\omega t + \theta - \tan^{-1} \frac{\omega L}{R} \right) \quad (1M)$$

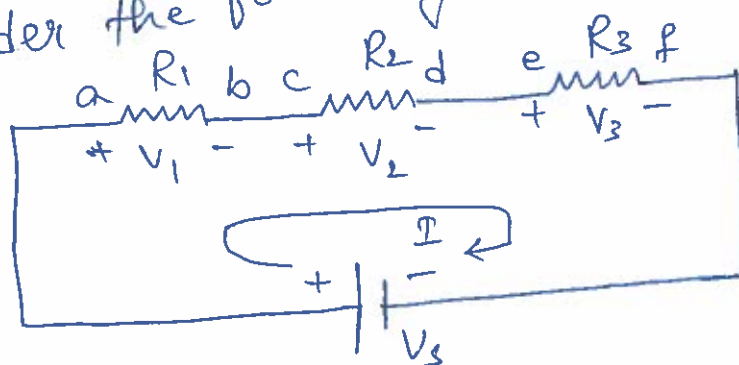
(b) RC circuit:

$$i = e^{-(t/RC)} \left[\frac{V}{R} \cos \theta - \frac{V}{\sqrt{R^2 + \left(\frac{1}{\omega C} \right)^2}} \cos \left(\theta + \tan^{-1} \frac{1}{\omega CR} \right) \right] + \frac{V}{\sqrt{R^2 + \left(\frac{1}{\omega C} \right)^2}} \cos \left(\omega t + \theta + \tan^{-1} \frac{1}{\omega CR} \right) \quad (1M)$$

6(a) Kirchoff's Voltage Law:

Kirchoff's voltage law states that the algebraic sum of all branch voltages around any closed path in a circuit is always zero at all instants of time. (1M)

Consider the following circuit,



As per the definition,

$$V_S = V_1 + V_2 + V_3 \quad (1M)$$

where $V_1 = IR_1$; $V_2 = IR_2$ & $V_3 = IR_3$

$$V_S = V.$$

$$\therefore V = IR_1 + IR_2 + IR_3$$

$$V = I(R_1 + R_2 + R_3)$$

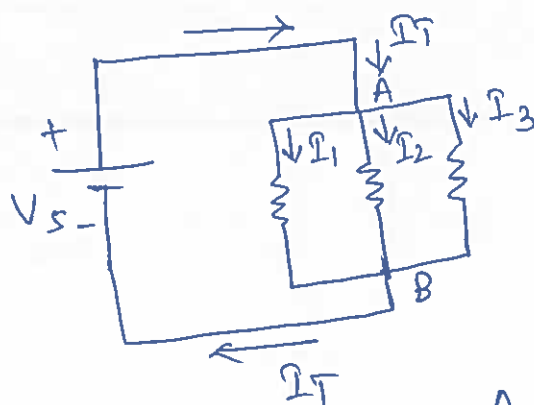
$$I = \frac{V}{R_1 + R_2 + R_3}$$

(1M)

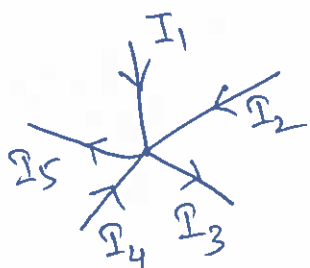
Kirchoff's Current Law:

Kirchoff's current law states that the sum of the currents entering into any node is equal to the sum of the currents leaving that node. (1M)

Consider the circuit,

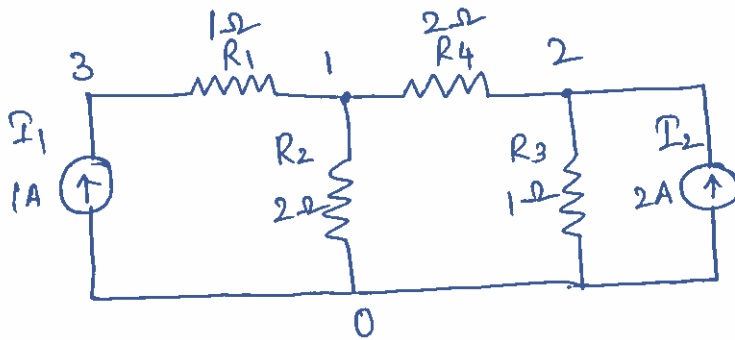
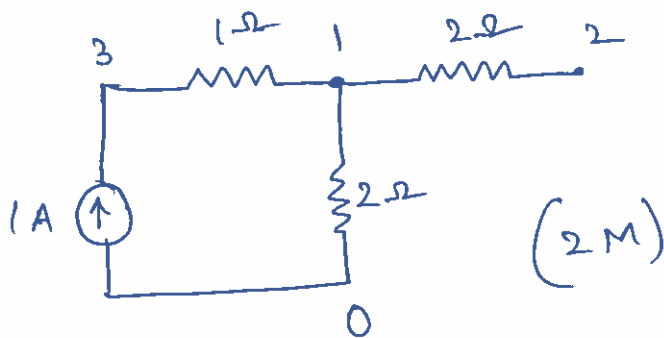


$$I_T = I_1 + I_2 + I_3 \quad (1M)$$



As per the definition,

$$I_1 + I_2 + I_4 = I_5 + I_3 \quad (1M)$$

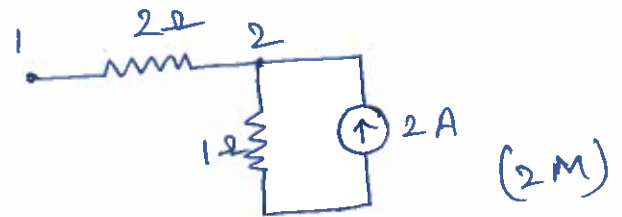
6(b) Nodal Analysis:At node 1:

Apply KCL:

$$1 = \frac{V_1 - 0}{2} + \frac{V_1 - V_2}{2}$$

$$1 = \frac{V_1}{2} + \frac{V_1}{2} - \frac{V_2}{2}$$

$$1 = V_1 - \frac{V_2}{2} \rightarrow \textcircled{1}$$

At node 2:

Apply KCL:

$$2 = \frac{V_2}{1} + \frac{V_2 - V_1}{2}$$

$$2 = V_2 + \frac{V_2}{2} - \frac{V_1}{2}$$

$$2 = \frac{3V_2}{2} - \frac{V_1}{2}$$

$$3V_2 - V_1 = 4 \rightarrow \textcircled{2}$$

On solving $\textcircled{1}$ & $\textcircled{2}$, we have,

$$2 = 2V_1 - V_2 \quad \times 1$$

$$4 = -V_1 + 3V_2 \quad \times 2$$

$$2 = 2V_1 - V_2$$

$$8 = -2V_1 + 6V_2$$

$$10 = 5V_2$$

$$\Rightarrow \underline{V_2 = 2V}$$

Substitute $V_2 = 2V$ in $\textcircled{1}$

$$1 = V_1 - \frac{2}{2}$$

$$1 = V_1 - 1$$

$$\underline{V_1 = 2V}$$

(2M)

7(a) Resistance Parameter:

When a current flows in a material, the free electrons move through the material and collide with other atoms. These collisions cause the electrons to lose some of their energy. This loss of energy per unit charge is the drop in potential across the material. The amount of energy lost by the electrons is related to the physical property of the material. These collisions restrict the movement of electrons. The property of a material to restrict the flow of electrons is called resistance. It is denoted by R . The symbol of ~~electrons~~ resistor is . The unit of the resistance is ohm (Ω).

Therefore, Ohm is defined as the resistance offered by the material when a current of one ampere flows between two terminals with one volt applied across it.

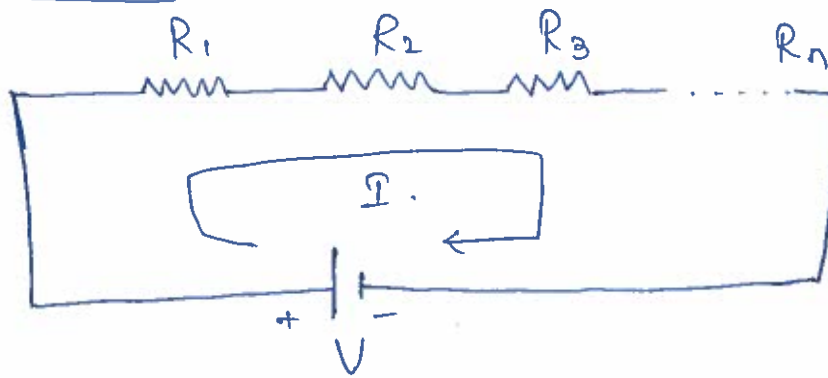
According to Ohm's law,

Current is directly proportional to the voltage and inversely proportional to the total resistance of the circuit.

$$I = \frac{V}{R}$$

(2M)

Resistance in series:



When the resistances are connected in series,

Accordingly to KVL,

$$V = IR_1 + IR_2 + IR_3 + \dots + IR_n$$

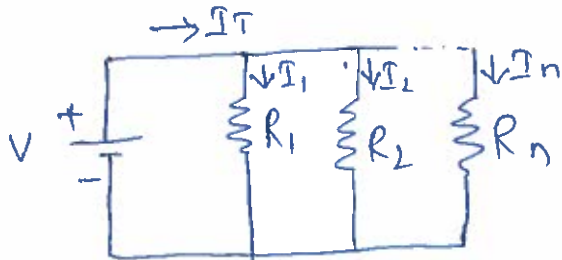
$$V = I(R_1 + R_2 + R_3 + \dots + R_n)$$

$$\Rightarrow \frac{V}{I} = R_1 + R_2 + R_3 + \dots + R_n$$

Therefore, Total resistance or Equivalent Resistance when the resistances are connected in series is given by the sum of all the series connected resistances:

$$R_{eq} = R_1 + R_2 + R_3 + \dots + R_n \quad (2M)$$

Resistances in Parallel:



When the circuit is connected in parallel, the total resistance of the circuit decreases as the number of resistors increases.

According to KCL:

$$I_T = I_1 + I_2 + \dots + I_n$$

According to Ohm's law,

$$I = \frac{V}{R}$$

$$\therefore I_T = \frac{V_s}{R_T} ; I_1 = \frac{V_s}{R_1} ; I_2 = \frac{V_s}{R_2} ; I_n = \frac{V_s}{R_n}$$

$$I_T = \frac{V_s}{R_1} + \frac{V_s}{R_2} + \dots + \frac{V_s}{R_n}$$

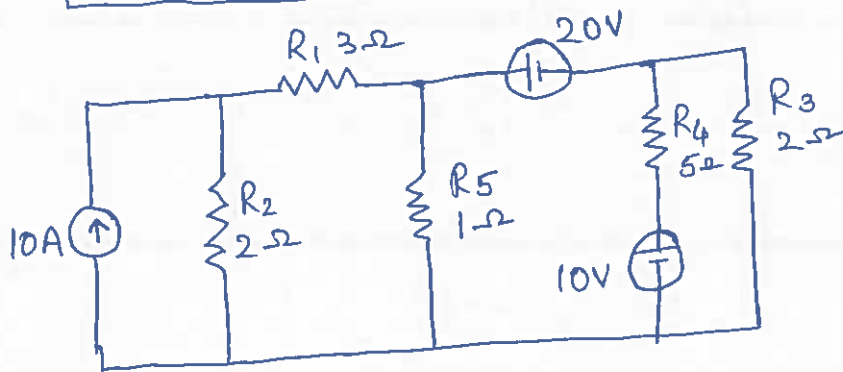
$$\Rightarrow \frac{V_s}{R_T} = \frac{V_s}{R_1} + \frac{V_s}{R_2} + \dots + \frac{V_s}{R_n}$$

$$\Rightarrow \frac{V_s}{R_T} = V_s \left[\frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n} \right]$$

(2M)

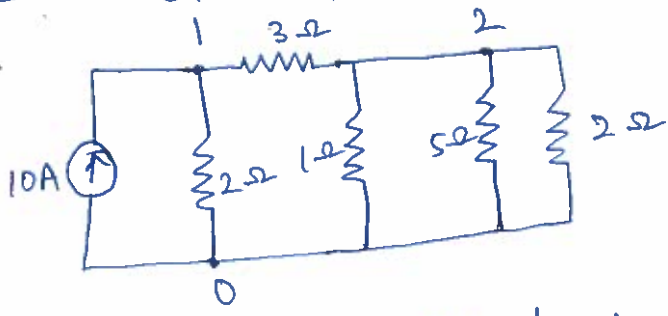
$$\therefore \boxed{\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n}}$$

7(b).



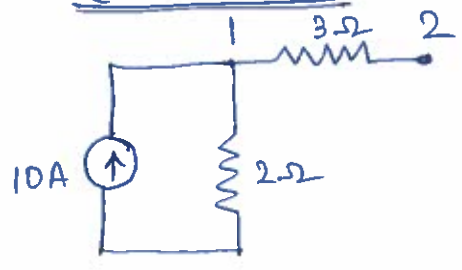
Current in 5Ω resistor shall be the sum of the currents in 5Ω resistors while considering one source at a time.

Consider 10A source:



Apply nodal analysis

@ Node 1: KCL:



$$10 = \frac{V_1}{2} + \frac{V_1 - V_2}{3}$$

$$10 = \frac{V_1}{2} + \frac{V_1}{3} - \frac{V_2}{3} \rightarrow \textcircled{1}$$

On solving $\textcircled{1}$ & $\textcircled{2}$, we have,

$$60 = 3V_1 + 2V_1 - 2V_2$$

$$\Rightarrow 60 = 5V_1 - 2V_2 \quad \times 2$$

$$0 = -10V_1 + 46V_2 \quad \times 1$$

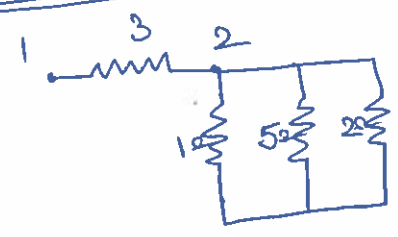
$$120 = 10V_1 - 4V_2$$

$$0 = -10V_1 + 46V_2$$

$$120 = 42V_2$$

$$\Rightarrow V_2 = 2.85V$$

@ Node 2: KCL



$$0 = \frac{V_2 - V_1}{3} + \frac{V_2}{1} + \frac{V_2}{5} + \frac{V_2}{2}$$

$$\Rightarrow 0 = \frac{V_2}{3} - \frac{V_1}{3} + \frac{V_2}{1} + \frac{V_2}{5} + \frac{V_2}{2}$$

$$\Rightarrow 0 = 10V_2 + 30V_2 + 6V_2 + 15V_2 - 10V_1$$

$$\Rightarrow 0 = 46V_2 - 10V_1 \rightarrow \textcircled{2}$$

$$\therefore 60 = 5V_1 - 2(2.85)$$

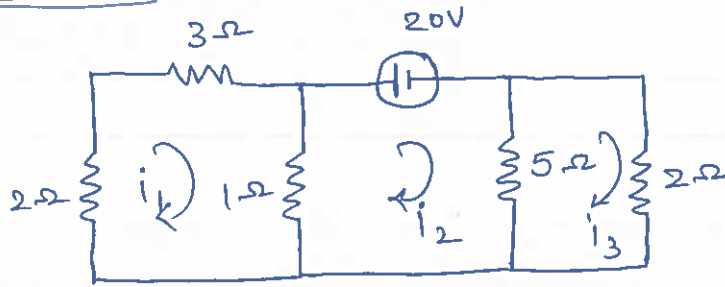
$$V_1 = 13.14V$$

Therefore, $I_{5\Omega} = \frac{V_2}{5}$

$$\Rightarrow I_{5\Omega} = I_1 = \frac{2.85}{5}$$

(2M)

$$\underline{\underline{I_1 = 0.57A}}$$

Consider 20V source:

Applying mesh analysis.

Loop 1: KVL

$$0 = 2i_1 + 3i_1 + i_1 - i_2$$

$$0 = 6i_1 - i_2 \rightarrow \textcircled{1}$$

Loop 2:

$$-20 = i_2 - i_1 + 5i_2 - 5i_3$$

$$20 = i_1 - 6i_2 + 5i_3 \rightarrow \textcircled{2}$$

Loop 3:

$$0 = 5i_3 - 5i_2 + 2i_3$$

$$\Rightarrow 0 = 7i_3 - 5i_2 \rightarrow \textcircled{3}$$

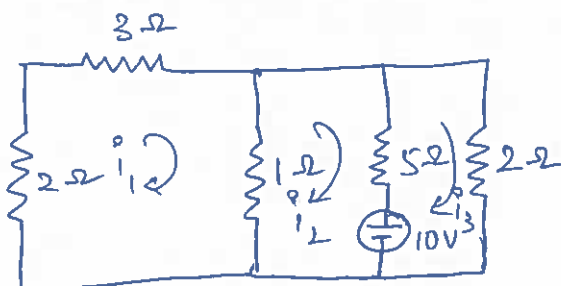
(2M)

On solving $\textcircled{1}$, $\textcircled{2}$ & $\textcircled{3}$, we have.

$$i_1 = -1.47 \text{ A} ; i_2 = -8.82 \text{ A} ; i_3 = -44.21 \text{ A}$$

$$I_{5\Omega} = I_2 = i_3 - i_2$$

$$= -44.21 + 8.82 = -35.39 \text{ A}$$

Consider 10V source:Apply loop analysis:

$$0 = 2i_1 + 3i_1 + i_1 - i_2$$

$$0 = 6i_1 - i_2 \rightarrow \textcircled{1}$$

$$-10 = i_2 - i_1 + 5i_2 - 5i_3$$

$$-10 = 6i_2 - i_1 - 5i_3$$

$$10 = i_1 - 6i_2 + 5i_3 \rightarrow \textcircled{2}$$

$$10 = 5i_3 - 5i_2 + 2i_3$$

$$10 = 7i_3 - 5i_2 \rightarrow (3)$$

On solving (1), (2) & (3), we have

$$i_1 = 0.25A; i_2 = 1.5A; i_3 = 2.5A.$$

$$I_{5\Omega} = I_3 = 2.5 - 1.5 = 1A.$$

(2M)

Therefore Total current through 5Ω resistor

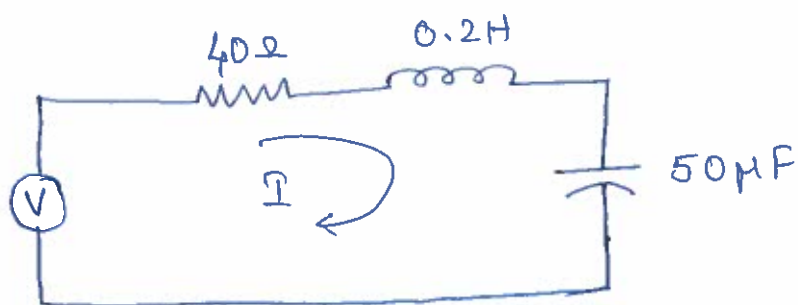
$$\begin{aligned} I_{5\Omega} &= I_1 + I_2 + I_3 \\ &= 0.57 - 35.39 + 1 \\ &= \underline{\underline{-33.82A}} \end{aligned}$$

8(a). Given,

$$\text{Voltage} = 230V$$

$$\text{Frequency} = 50\text{Hz}.$$

$$R = 40\Omega; L = 0.2H; C = 50\mu F$$



We know that,

(2M)

$$V = IZ$$

$$Z = R + j(X_L - X_C)$$

$$\begin{aligned} X_L &= 2\pi fL = 2 \times 3.14 \times 50 \times 0.2 \\ &= 62.8\Omega \end{aligned}$$

$$X_c = \frac{1}{2\pi fC} = \frac{1}{2 \times 3.14 \times 50 \times 50 \times 10^{-6}}$$

$$= 63.69 \Omega.$$

Therefore, $Z = R + j(X_L - X_c) \Omega$

Impedance = $40 + j(-0.89) \Omega$

$$Z = (40 - j0.89) \Omega$$

$$|Z| = 40.01 \Omega$$

$$\angle Z = -1.275^\circ$$

(2M)

Current in the circuit = $I = \frac{V}{Z}$

$$I = \frac{230}{|Z| \angle \angle Z} = \frac{230}{40.01 \angle -1.275^\circ}$$

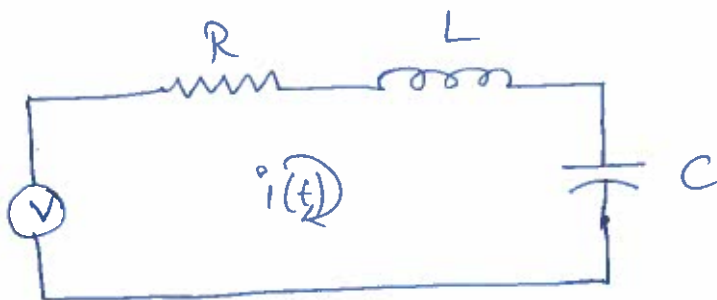
$$= 5.75 \angle 1.275^\circ$$

$$\therefore \underline{I_{ckt} = 5.75 A}$$

Power Factor = $\cos \theta = \cos(-1.275^\circ)$

$$\underline{\cos \theta = 0.999} \quad (2M)$$

8(b) Series RLC circuit:

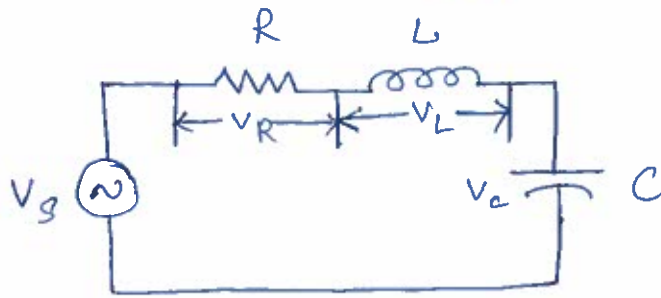


Consider the above series R-L-C circuit.

8(6) Phasor relation for series RLC circuit.

(13)

Consider the circuit as shown below.



If a sinusoidal input is applied to the circuit, the current in the circuit and all voltages across the elements are sinusoidal.

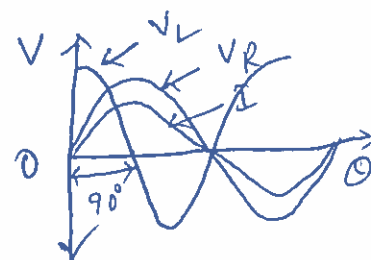
From the above circuit, we can find resistor voltage V_R , current I , Inductor Voltage V_L , Capacitance Voltage V_C for given voltage V_s .

Therefore,

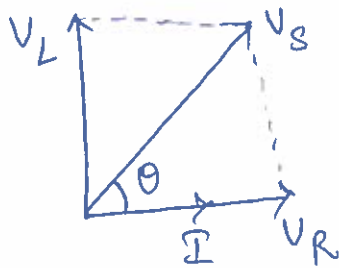
The resistor voltage V_R and current I are in phase with each other.

In R-L series ckt, the current I lags behind the source voltage V_s . But, the inductor voltage V_L leads the source voltage V_s . The phase angle between current and voltage in a pure inductor is always 90° .

Therefore, the phase relation between current and voltage in a series RL circuit is



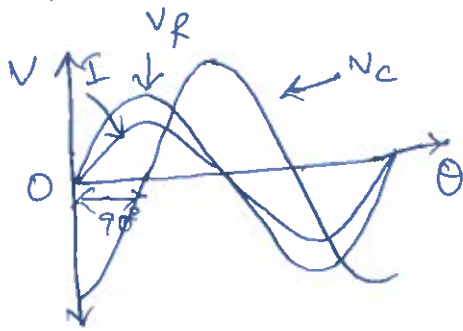
Phase relation
R-L ckt



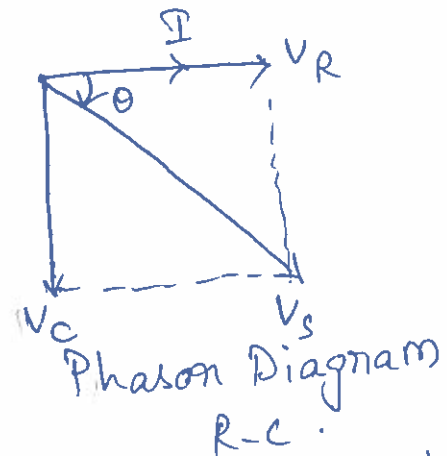
(2M)

Phasor Diagram
R-L ckt.

In R-C series ckt, the current I is the same current through all the elements. Thus the resistor voltage is in phase with the current, and the capacitor voltage lags behind the current by 90° as shown below.



Phase relationship
R-C



Phasor Diagram
R-C.

(2M)

A series RLC ckt is the combination of both R-L & R-C circuits.

Therefore, if we observe the impedance on phasor diagrams of RL & RC series circuits, the inductive reactance, X_L is displayed on the +ve j axis and the capacitive reactance X_C is displayed on the -ve j axis. These reactances are 180° apart and tend to cancel each other.

$$|Z| = 296.4 \, \Omega ; \angle \theta = 59.59^\circ \quad (1M)$$

(16)

(ii) Current flowing through the circuit

$$I = \frac{V}{Z} = \frac{500}{296.4} = \underline{\underline{1.68 \, A}} \quad (1M)$$

$$(iii) \text{ Power factor} = \cos \theta = \cos 59.59^\circ = 0.506 (1M)$$

(iv) Voltage across R, L & C

$$V_R = IR = 1.68 \times 150 = \underline{\underline{252 \, V}}$$

$$V_L = I X_L = 1.68 \times 62.8 = \underline{\underline{105.5 \, V}}$$

$$V_C = I X_C = 1.68 \times 318.47 \\ = \underline{\underline{535.03 \, V}} \quad (2M)$$

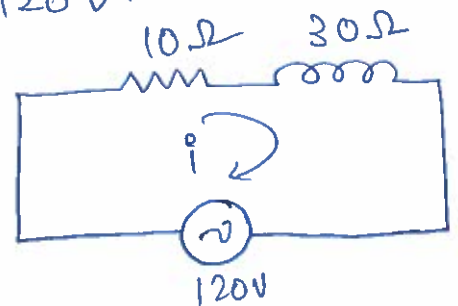
$$(v) \text{ Power in Watts} = P = VI \cos \theta \\ = 500 \times 1.68 \times 0.5 \\ = \underline{\underline{420 \, W}} \quad (1M)$$

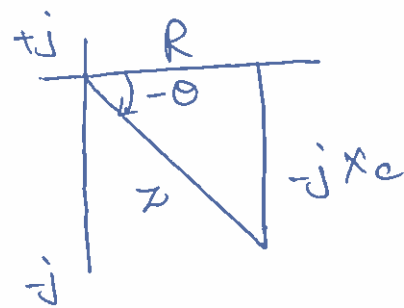
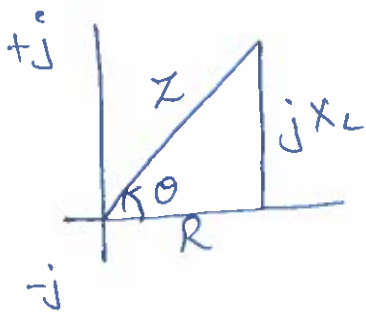
9(b) Given,

Voltage of the circuit = 120V.

Resistance = 10 Ω

Reactance = 30 Ω





(15)

Therefore, the magnitude and type of reactance in a series RLC circuit is the difference of the two reactances. The impedance for an R-L-C series ckt is given by $Z = \sqrt{R^2 + (X_L - X_C)^2}$ (2M)

$$\text{and } \theta = \tan^{-1} \left(\frac{X_L - X_C}{R} \right)$$

9(a). Given,

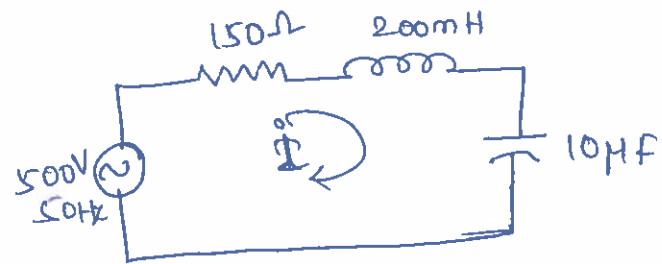
Resistance = 150Ω

Inductance = 200 mH

Capacitance = $10 \mu\text{F}$

Voltage = 500 V

Frequency = 50 Hz



(i) Impedance = $Z = R + j(X_L - X_C)$

$$X_L = 2\pi fL = 2 \times 3.14 \times 50 \times 200 \times 10^{-3} = 62.8 \Omega$$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2 \times 3.14 \times 50 \times 10 \times 10^{-6}} = 318.47 \Omega$$

$$\text{Impedance} = Z = 150 + j(62.8 - 318.47)$$

$$Z = 150 - j 255.67$$

We know that,

The average power of the R-L ckt is

$$P_{av} = \frac{1}{T} \int_0^T p(t) dt$$

Therefore,

$$P_{av} = \frac{V_m I_m}{2} \cos \theta. \quad (2M)$$

$$\Rightarrow P_{av} = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} \cos \theta.$$

$$\text{where } \frac{V_m}{\sqrt{2}} = 120V.$$

$$\frac{I_m}{\sqrt{2}} = I_{eff} = \frac{V}{Z}$$

$$Z = R + jX_L = 10 + j30$$

$$|Z| = 31.623$$

$$\angle \theta = 71.56^\circ \quad (2M)$$

$$\therefore I_{eff} = \frac{120}{31.62} = 3.79A.$$

Therefore,

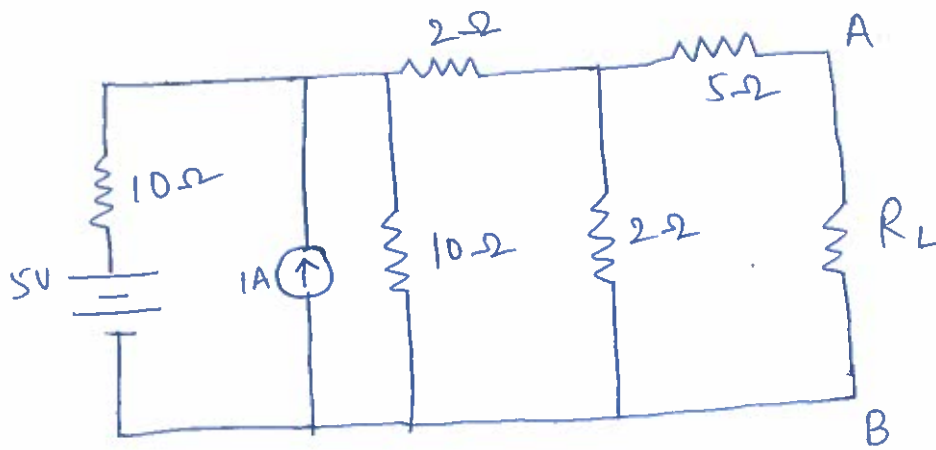
$$P_{av} = V_{eff} \cdot I_{eff} \cos \theta$$

$$= 120 \times 3.79 \times \cos(71.56^\circ)$$

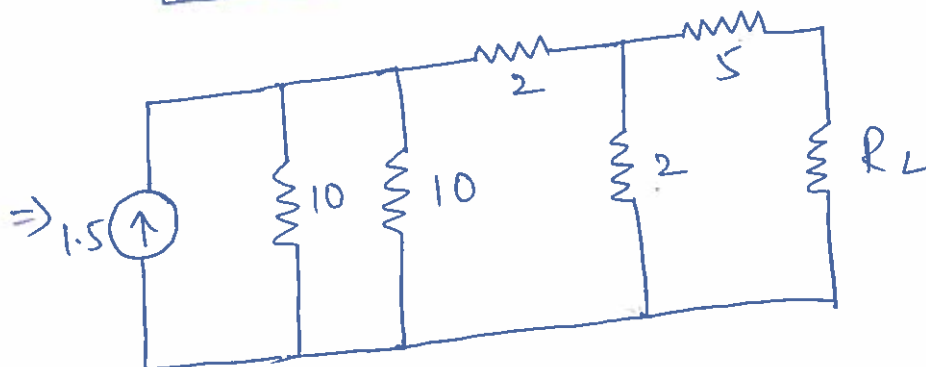
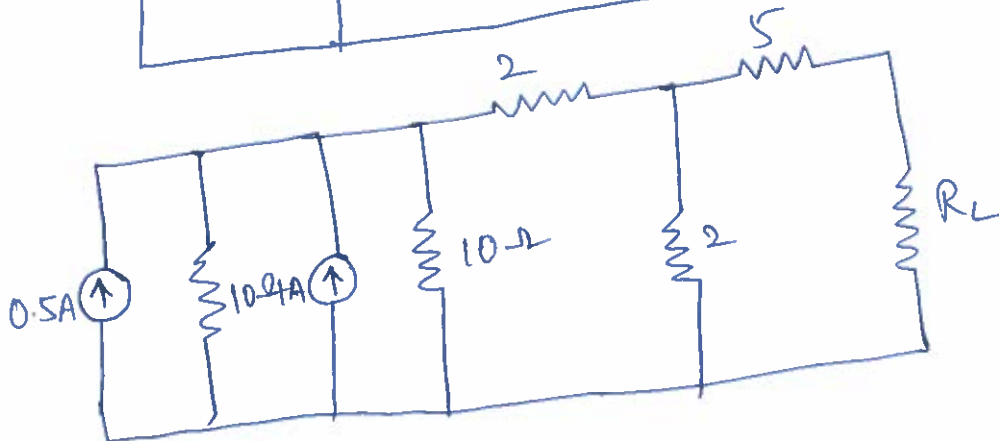
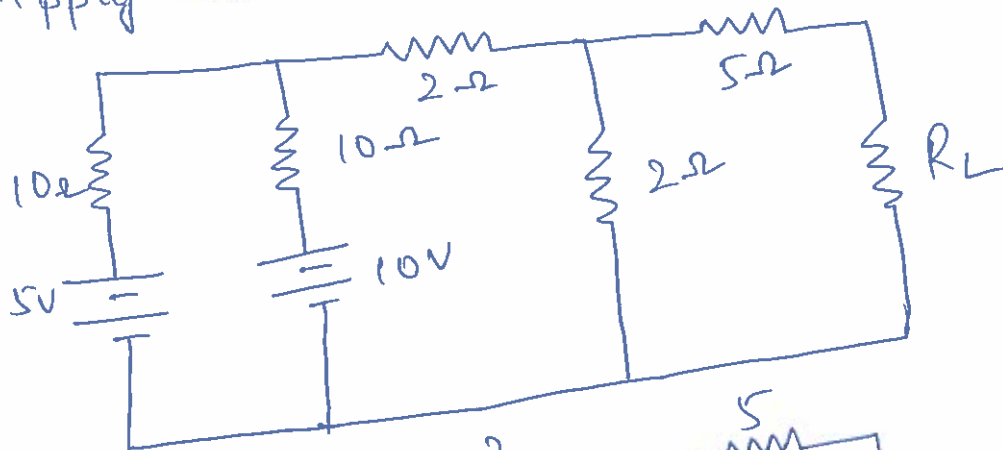
$$= \underline{\underline{143.85W.}} \quad (2M)$$

10(a)

18



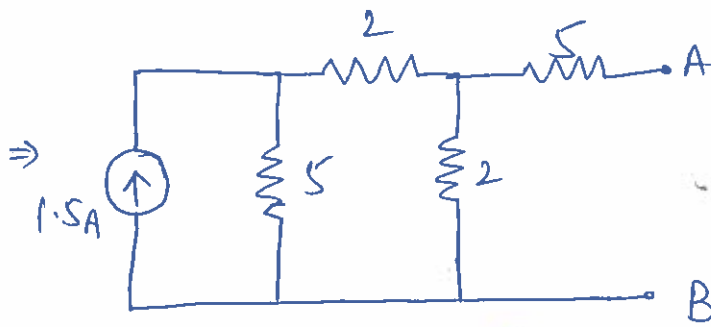
Apply the source transformation



$$R_T = \left[\left((5 \parallel 2) + 2 \right) \parallel 10 \right] \parallel 10$$

$$= \left[\left[\frac{10}{7} + 2 \right] \parallel 10 \right] \parallel 10$$

$$= 2.03 \Omega$$



$$\text{Therefore, } R_{th} = [(5+2) // 2] + 5$$

$$= 6.55 \Omega$$

Now the ckt can be drawn.



$$I = 1.5 \times \frac{5}{5+3.705}$$

$$= \frac{1.5 \times 5}{8.705}$$

$$= 0.861 \text{ A.}$$

$$I_1 = 0.861 \times \frac{2}{2+5} = 0.246 \text{ A.}$$

So the maximum power that can be delivered to R is

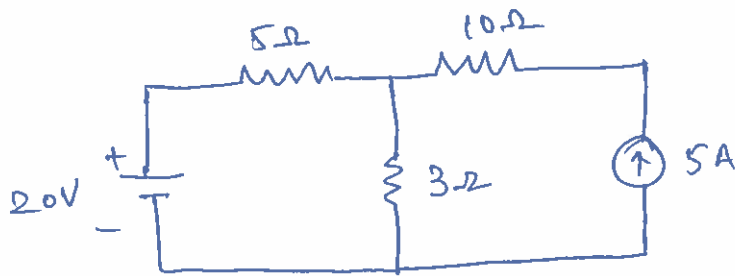
$$I_1^2 R = (0.246)^2 \times 6.55$$

$$= 0.3963 \text{ watts.}$$

10(b) Superposition theorem states that in any linear network containing two or more sources, the response in any element is equal to the algebraic sum of the responses caused by individual sources acting alone, while the other sources are non-operative; that is, while considering the effect of individual sources, other ideal voltage sources

and ideal current sources in the network are (20)
replaced by short circuit and open circuit across
their terminals. This theorem is valid only for
linear systems.

Consider the ckt



Now let us find the current passing through 3Ω
Acc superposition theorem,

The current I_2 due to the 20V VS with 5A source
open = $20 / (5 + 3) = 2.5 \text{ A}$.

The current I_5 due to 5A source with 20V source
short ckt is

$$I_5 = 5 \times \frac{5}{3 + 5} = 3.125 \text{ A}.$$

The total current passing through 3Ω resistor is
 $2.5 + 3.125 = 5.625 \text{ A}$.

Let us verify the above result by applying nodal
analysis

The current passing in the 3Ω resistor due to both
sources should be 5.625 A.

Applying nodal analysis, we have

(21)

$$\frac{V-20}{5} + \frac{V}{3} = 5.$$

$$V \left[\frac{1}{5} + \frac{1}{3} \right] = 5 + 4$$

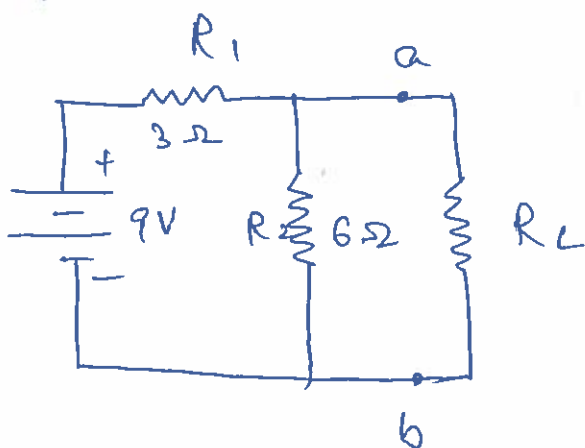
$$\Rightarrow V = 9 \times \frac{15}{8} = 16.875V.$$

The current passing through the 3Ω resistor is equal to $V/3$

$$I = \frac{16.875}{3} = 5.625A.$$

Superposition theorem is valid.

11(a)



$$R_{eq} = 9\Omega$$

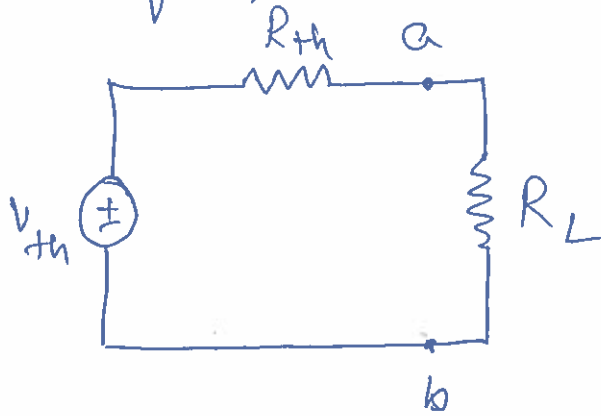
$$V_{AB} = V_{6\Omega} = 9 \times \frac{6}{9} = 6V.$$

Therefore $V_{ab} = V_{th} = 6V.$

R_{th} can be found by applying V_{ab} as source

$$\therefore R_{th} = 3 // 6 = \frac{3 \times 6}{3+6} = 2\Omega.$$

Therefore, thevenin's equivalent is.

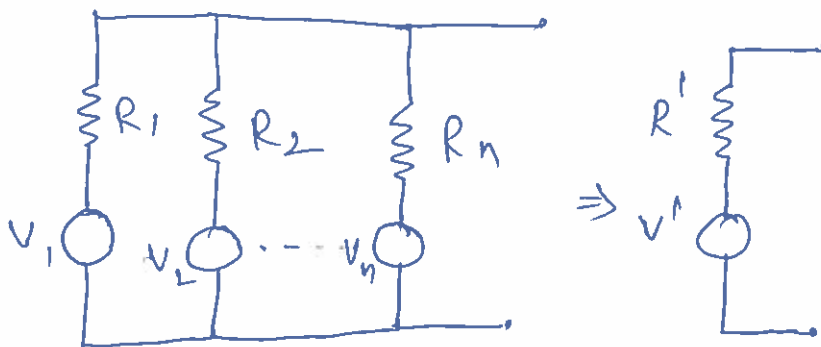


$$V_{th} = 6V$$

$$R_{th} = 2\Omega.$$

11(b) Milliman's theorem:

Milliman's theorem states that in any network if the voltage sources $V_1, V_2, \dots, V_n$ are in series with internal resistances $R_1, R_2, \dots, R_n$ respectively, are in parallel, then these sources may be replaced by a single voltage source ' V' ' in series with R' as shown below.

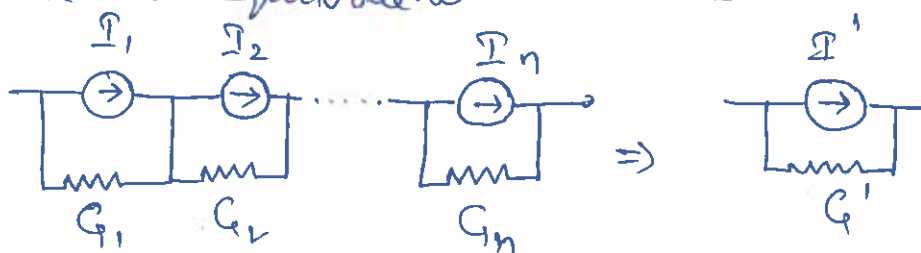


$$V' = \frac{V_1 G_1 + V_2 G_2 + \dots + V_n G_n}{G_1 + G_2 + \dots + G_n}$$

where G_n is the conductance of the n^{th} branch.

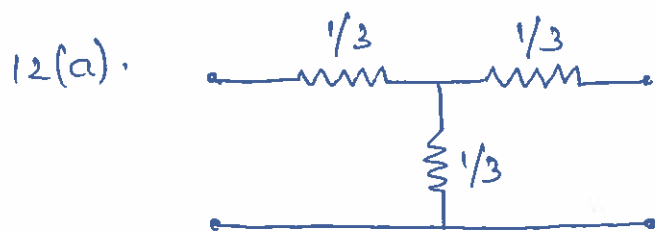
$$R' = \frac{1}{G_1 + G_2 + \dots + G_n}$$

The same theorem can be stated for n current sources having internal conductances which can be replaced by a single current source I' in parallel with an equivalent conductance.



where $I' = \frac{I_1 R_1 + I_2 R_2 + \dots + I_n R_n}{R_1 + R_2 + \dots + R_n}$

and $G' = \frac{1}{R_1 + R_2 + \dots + R_n}$

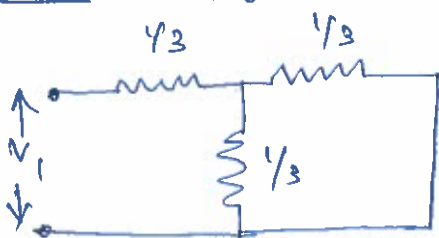


$$I_1 = Y_{11} V_1 + Y_{12} V_2$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

$$Y = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$$

Case-1: Short circuit 2-2' $\Rightarrow V_2 = 0$



$$Y_{11} = \frac{I_1}{V_1} \Big|_{V_2=0}$$

$$Y_{21} = \frac{I_2}{V_1} \Big|_{V_2=0}$$

$$R_{eq} = \left[\frac{1}{3} // \frac{1}{3} \right] + \frac{1}{3} = \frac{1}{2} \Omega$$

$$\therefore \frac{V_1}{I_1} = \frac{1}{2}$$

$$\Rightarrow \frac{I_1}{V_1} = 2 \text{ S}$$

$$\therefore Y_{11} = \frac{I_1}{V_1} = 2 \text{ S}$$

$$\therefore I_1 = V_1 \times 2$$

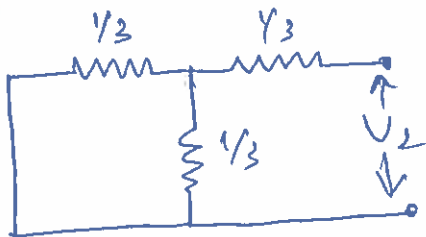
$$Y_{21} = \frac{I_2}{V_1} \Big|_{V_2=0}$$

$$\begin{aligned} I_2 &= I_{1/3} = I_1 \times \frac{1/3}{2/3} \\ &= V_1 \times 2 \times \frac{1}{2} \\ &= V_1 \end{aligned}$$

$$\therefore \frac{I_2}{V_1} = 1 \text{ S}$$

$$\therefore Y_{21} = \frac{I_2}{V_1} \Big|_{V_2=0} = 1 \text{ S}$$

Case (ii) Short circuit $4-1' \Rightarrow V_1 = 0$



$$Y_{12} = \frac{I_1}{V_2} \Big|_{V_1=0}$$

$$Y_{22} = \frac{I_2}{V_2} \Big|_{V_1=0}$$

$$R_{eq} = \frac{1}{2} \Omega$$

$$\frac{V_2}{I_2} = \frac{1}{2}$$

$$\Rightarrow \frac{I_2}{V_2} = 2 \text{ } \Omega$$

$$\therefore Y_{22} = 2 \text{ } \Omega$$

$$\therefore I_2 = 2 V_2$$

$$Y_{12} = \left. \frac{I_1}{V_2} \right|_{V_1=0}$$

$$I_1 = I_{V_3} = I_2 \times \frac{V_3}{2V_2}$$

$$= \beta V_2 \times \frac{1}{2}$$

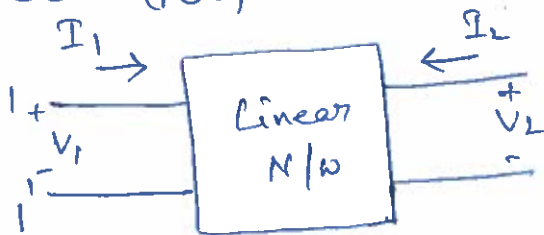
$$I_1 = V_2$$

$$\therefore \frac{I_1}{V_2} = 1 \text{ } \Omega$$

$$\therefore Y_{12} = 1 \text{ } \Omega$$

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

12(b). Consider,



The Y parameters of a two-port for the positive directions of voltages and currents may be defined by expressing

the port currents I_1 and I_2 in terms of the voltages V_1 and V_2 . Here I_1, I_2 are dependent variables and V_1 and V_2 are independent variables. I_1 may be considered to be the superposition of two components, one caused by V_1 and the other by V_2 .

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

Similarly, Y_{11}, Y_{12}, Y_{21} & Y_{22} are the n/w In and also called the admittance (Y) parameters.

$$[I] = [Y][V]$$
$$I = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}; Y = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}; V = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Case (i) : $V_2 = 0 \Rightarrow$ short circuiting port 2-2', then

$$Y_{11} = \frac{I_1}{V_1} \Big|_{V_2=0}$$

Y_{11} is the driving point admittance at port 1-1', with 2-2' short circuited. Also called the short ckt i/p admitt.

$$Y_{21} = \frac{I_2}{V_1} \Big|_{V_2=0}$$

Y_{21} is the transfer admittance at port 1-1' with port 2-2' short circuited. It is also called short circuited forward transfer admittance. If we let V_1 be zero by short circuiting port 1-1', then

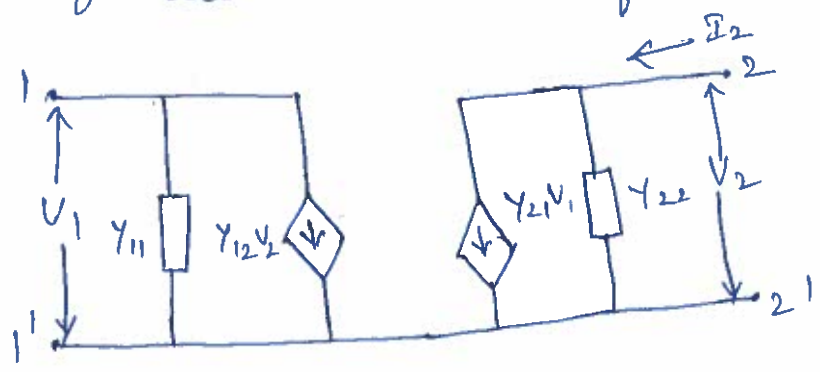
$$Y_{12} = \frac{I_1}{V_2} \Big|_{V_1=0}$$

Y_{12} is the transfer admittance at 2-2', with 1-1' short circuited. It is also called the short circuit reverse transfer admittance.

$$Y_{22} = \frac{I_2}{V_2} \bigg|_{V_1=0}$$

Y_{22} is the short circuit driving point admittance at port 2-2' with port 1-1' short circuited. It is also called the short circuit output admittance.

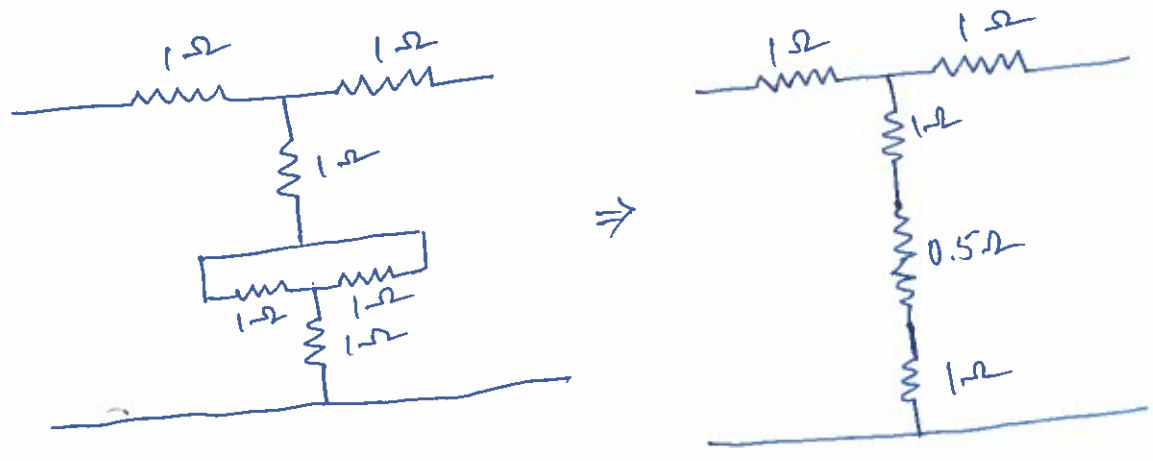
The equivalent circuit of the network is

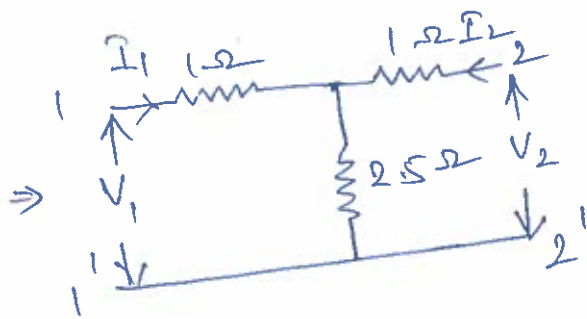


If the network under study is reciprocal, or bilateral, then.

$$\frac{I_1}{V_2} \bigg|_{V_1=0} = \frac{I_2}{V_1} \bigg|_{V_2=0}$$
$$Y_{12} = Y_{21}$$

13(a)





Z-parameters:

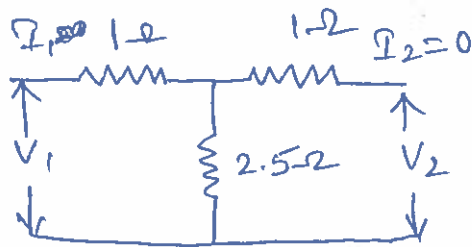
$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

$$Z_{11} = \left. \frac{V_1}{I_1} \right|_{I_2=0} \quad ; \quad Z_{21} = \left. \frac{V_2}{I_1} \right|_{I_2=0}$$

$$Z_{12} = \left. \frac{V_1}{I_2} \right|_{I_1=0} \quad ; \quad Z_{22} = \left. \frac{V_2}{I_2} \right|_{I_1=0}$$

Case-i : $I_2 = 0$, $\Rightarrow$ Open 2-2' port.



$$\begin{aligned} R_{eq} = \frac{V_1}{I_1} &= (1 \parallel 2.5) + 1 \\ &= \frac{2.5}{3.5} + 1 \\ &= 1.71 \Omega. \end{aligned}$$

$$\therefore \frac{V_1}{I_1} = 1.71$$

$$\Rightarrow V_1 = 1.71 I_1$$

$$\text{Also, } Z_{11} = 1.71 \Omega.$$

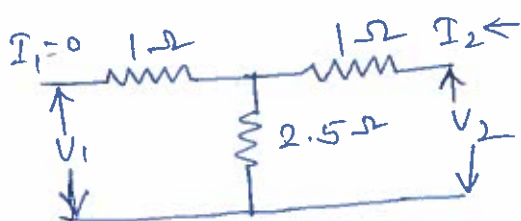
$$Z_{21} = \frac{V_2}{I_1}$$

$$\begin{aligned} V_2 = V_{2.5\Omega} &= V_1 \times \frac{2.5}{3.5} \\ &= 1.71 \times 1.71 \times I_1 \\ &= 2.92 I_1 \end{aligned}$$

$$\therefore \frac{V_2}{I_1} = 2.92 \Omega$$

$$\therefore Z_{21} = 2.92 \Omega$$

Case-(ii) $I_1 = 0 \Rightarrow$ Open 1-1' port



$$R_{eq} = (1 \parallel 2.5) + 1 = 1.71 \Omega$$

$$\frac{V_2}{I_2} = R_{eq} = 1.71 \Omega$$

$$V_2 = 1.71 I_2$$

$$\therefore Z_{22} = 1.71 \Omega$$

$$Z_{12} = \frac{V_1}{I_2}$$

$$\begin{aligned} V_1 = V_{2.5\Omega} &= V_2 \times \frac{2.5}{3.5} \\ &= 1.71 I_2 \times 1.71 \\ &= 2.92 I_2 \end{aligned}$$

$$\therefore \frac{V_1}{I_2} = 2.92 \Omega$$

$$\therefore Z_{12} = 2.92 \Omega$$

$$\therefore \boxed{Z_{12} = Z_{21}}$$

13(b) Z-parameters in terms of Y-parameters

(30)

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \rightarrow (1)$$

$$I_1 = Y_{11} V_1 + Y_{12} V_2 \rightarrow (3)$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \rightarrow (2)$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2 \rightarrow (4)$$

On solving (1) & (2) for I_1 & I_2 , we get,

$$I_1 = \frac{\begin{vmatrix} V_1 & Z_{12} \\ V_2 & Z_{22} \end{vmatrix}}{\Delta_Z} ; I_2 = \frac{\begin{vmatrix} Z_{11} & V_1 \\ V_{21} & V_2 \end{vmatrix}}{\Delta_Z}$$

where $\Delta_Z = \begin{vmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{vmatrix}$

$$I_1 = \frac{Z_{22}}{\Delta_Z} V_1 - \frac{Z_{12}}{\Delta_Z} V_2 \rightarrow (5)$$

$$I_2 = -\frac{Z_{21}}{\Delta_Z} V_1 + \frac{Z_{11}}{\Delta_Z} V_2 \rightarrow (6)$$

On comparing (3) & (4) with (5) & (6)

$$Y_{11} = \frac{Z_{22}}{\Delta_Z} ; Y_{12} = -\frac{Z_{12}}{\Delta_Z}$$

$$Y_{21} = -\frac{Z_{21}}{\Delta_Z} ; Y_{22} = \frac{Z_{11}}{\Delta_Z}$$

h-parameters:

$$V_1 = h_{11} I_1 + h_{12} V_2 \rightarrow (7)$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \rightarrow (8)$$

We know that,

$$h_{11} = \frac{V_1}{I_1} \bigg|_{V_2=0} ; h_{21} = \frac{I_2}{I_1} \bigg|_{V_2=0}$$

$$h_{12} = \left. \frac{V_1}{V_2} \right|_{I_1=0} ; h_{22} = \left. \frac{I_2}{V_2} \right|_{I_1=0}$$

$$h_{12} = \frac{V_1}{I_2} \bigg/ \frac{V_2}{I_2} \bigg|_{I_1=0}$$

$$= \frac{Z_{12}}{Z_{22}} \bigg|_{I_1=0}$$

$$\therefore h_{12} = \frac{Z_{12}}{Z_{22}}$$

$$\underline{Z_{12} = \frac{h_{22}}{h_{22}}}$$

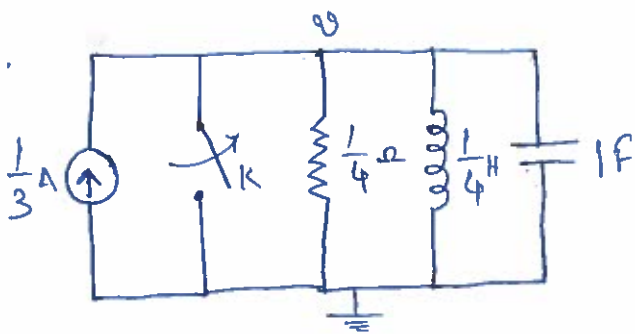
$$Z_{11} = \frac{V_1}{I_1}$$

$$Z_{11} = \frac{\Delta h}{h_{22}}$$

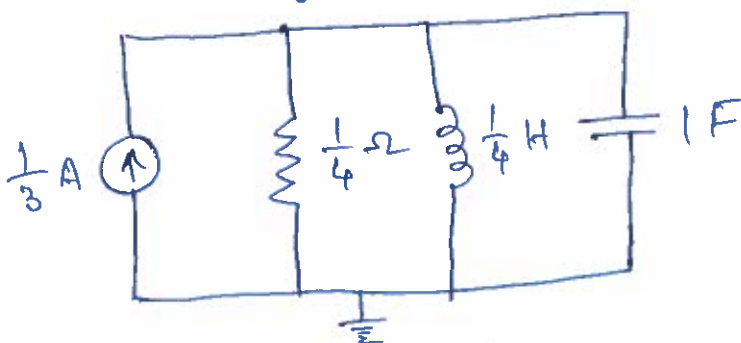
$$Z_{21} = -\frac{h_{21}}{h_{22}}$$

$$\underline{Z_{22} = \frac{1}{h_{22}}}$$

14(a).



At $t=0$, switch is opened.



Apply nodal analysis,

$$\frac{1}{3} = \frac{v}{\frac{1}{4}} + \frac{v}{X_L} + \frac{v}{X_C}$$

$$\begin{aligned} X_L &= 2\pi fL \\ &= 2 \times 3.14 \times 50 \times \frac{1}{4} \\ &= 78.5 \Omega \end{aligned}$$

$$\begin{aligned} X_C &= \frac{1}{2\pi fC} \\ &= \frac{1}{2 \times 3.14 \times 50 \times 1} \\ &= 0.00318 \Omega \end{aligned}$$

$$\frac{1}{3} = \frac{V}{\frac{1}{4}} + \frac{V}{78.5} + \frac{V}{0.0031}$$

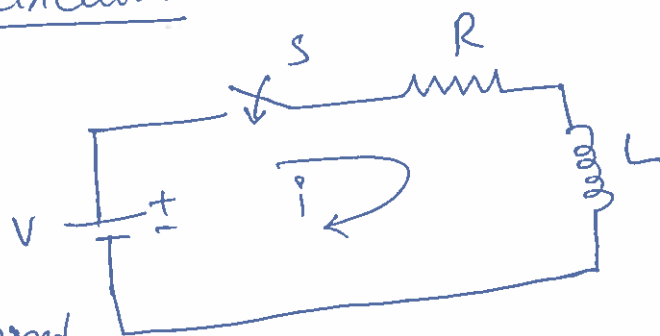
$$\Rightarrow \frac{1}{3} = V \left[4 + \frac{1}{78.5} + \frac{1}{0.0031} \right]$$

$$\Rightarrow \frac{1}{3} = V (326.59)$$

$$V = 0.00102 \text{ V} = \underline{\underline{1.02 \text{ mV}}}$$

14(b) DC Response of R-L circuit:

Consider the circuit,
The inductor in the circuit is initially uncharged.



and is in series with the resistor. When the switch S is closed, we can find complete solution for the current.

Apply the KVL to the circuit.

$$V = Ri + L \frac{di}{dt}$$

$$\Rightarrow \frac{di}{dt} + \frac{R}{L} i = \frac{V}{L}$$

In the above equation, the current i is the solution to be found and V is the applied constant voltage. The voltage V is applied to the circuit only when the switch S is closed.

Comparing with non-homogeneous DE,

$$\frac{dx}{dt} + Px = K$$

whose solution is

$$x = e^{-Pt} \int K e^{+Pt} dt + C e^{-Pt}$$

where C is an arbitrary constant.

$$i = C e^{-(R/L)t} + e^{-(R/L)t} \int \frac{V}{L} e^{(R/L)t} dt$$

$$i = C e^{-(R/L)t} + \frac{V}{R}$$

To find the value of C , we use initial conditions

@ $t=0$, $i=0$.

$$\text{Therefore, } 0 = C e^{(0)} + \frac{V}{R}$$

$$\Rightarrow 0 = C + \frac{V}{R}$$

$$\Rightarrow C = -\frac{V}{R}$$

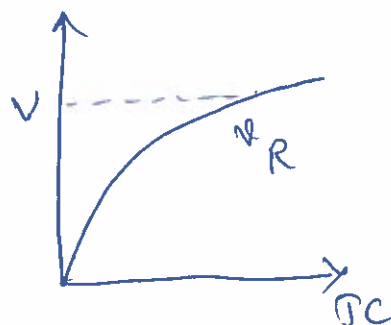
$$\text{Therefore, } i = -\frac{V}{R} e^{-(R/L)t} + \frac{V}{R}$$

$$i = \frac{V}{R} (1 - e^{-(R/L)t})$$

Voltage Transients

$$V_R = Ri = R \times \frac{V}{R} [1 - e^{-(R/L)t}]$$

$$V_R = V (1 - e^{-(R/L)t})$$

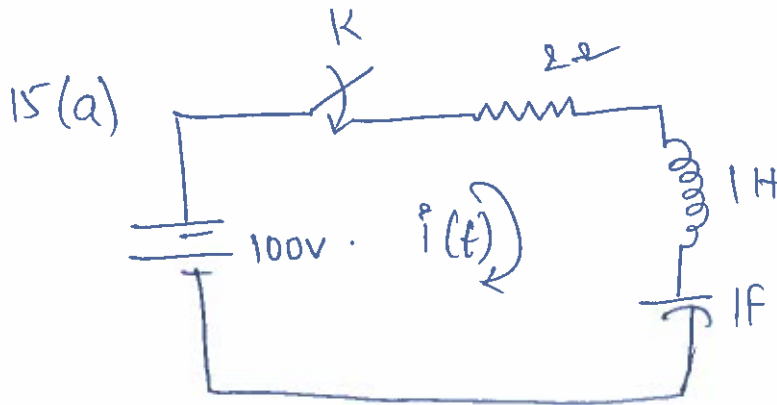
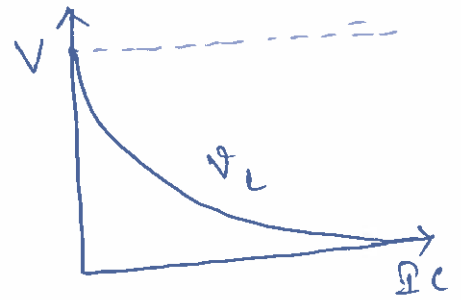


Voltage across inductor

$$v_L = L \frac{di}{dt}$$

$$= \Delta \frac{V}{R} \times \frac{R}{\Delta} e^{-(R/L)t}$$

$$v_L = V e^{-(R/L)t}$$



Apply KVL,

$$100 = 2i(t) + L \frac{d}{dt} i(t) + \frac{1}{C} \int i dt$$

Apply Laplace Transform,

$$100 = 2I(s) + L s I(s) + \frac{1}{C} \cdot \frac{1}{s} I(s)$$

$$100 = I(s) \left[2 + 1s + \frac{1}{s} \right]$$

$$100 = I(s) \left[\frac{2s + s^2 + 1}{s} \right]$$

$$I(s) = 100 \cdot \frac{s}{s^2 + 2s + 1}$$

$$I(s) = 100 \cdot \frac{s}{(s+1)^2}$$

Apply P.F to $\frac{s}{(s+1)^2}$

$$\frac{s}{(s+1)^2} = \frac{A}{s+1} + \frac{B}{(s+1)^2}$$

$$s = A(s+1) + B$$

$$s = As + A + B$$

On comparing like terms,

$$A = 1 \quad \& \quad A + B = 0$$

$$\Rightarrow A = -B$$

$$\therefore B = -1$$

$$\therefore \frac{s}{(s+1)^2} = \frac{1}{s+1} - \frac{1}{(s+1)^2}$$

$$\therefore \mathcal{I}(s) = 100 \cdot \left[\frac{1}{s+1} - \frac{1}{(s+1)^2} \right]$$

$$\mathcal{I}(s) = 100 \left[\frac{1}{s+1} - \frac{1}{(s+1)^2} \right]$$

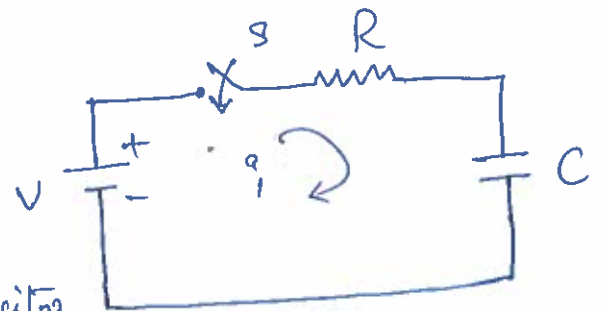
$$\underline{i(t) = 100 (e^{-t} - t e^{-t})}$$

15(b). DC Response of R-C ckt:

Consider a circuit consisting of resistance and capacitance

as shown in figure. The capacitor

in the circuit is initially uncharged, and is in series with a resistor. When the switch s is closed at $t=0$, we can determine the complete solution for the current.



Apply KVL,

$$V = Ri + \frac{1}{C} \int i dt$$

Differentiating the above equation, we get,

$$0 = R \frac{di}{dt} + \frac{i}{C}$$

$$\Rightarrow \frac{di}{dt} + \frac{1}{RC} i = 0$$

The above is a linear differential equation with only the complementary function.

$$i = c e^{-t/RC}$$

To find value of c , we use initial conditions.

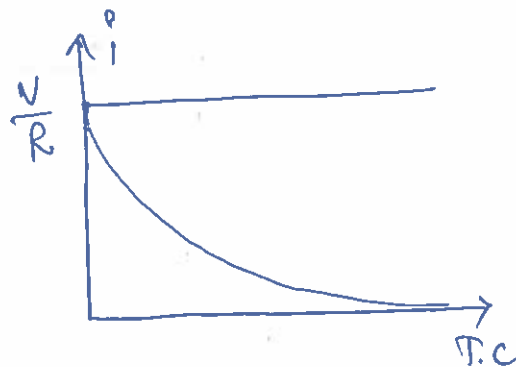
@ $t=0$, the capacitor act as short circuit,

$$i = \frac{V}{R}$$

$$\therefore \frac{V}{R} = c \cdot e^{(0)}$$

$$\therefore c = \frac{V}{R}$$

$$\therefore i(t) = \frac{V}{R} e^{-t/RC}$$



Course Instructor

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EEE, NSRIT (A).

Head of the Dept.

Arshad Ali

23/2/22.

Semester End Examination, February, 2022

Degree	B. Tech. (U. G.)	Program	EEE	Academic Year	2021 - 2022
Course Code	20EE305	Test Duration	3 Hrs.	Max. Marks	70
Course	POWER GENERATION AND TRANSMISSION			Semester	III

Part A (Short Answer Questions 5 x 2 = 10 Marks)

No.	Questions (1 through 5)	Learning Outcome (s)	DoK
1	Define a) Economizer b) Preheater	20EE305.1	L2
2	List the types of tariffs to calculate energy rate.	20EE305.2	L1
3	List any four types of conductors used in power systems	20EE305.3	L2
4	Define a) Skin Effect b) Corona Loss	20EE305.4	L1
5	Write down the expression for insulation resistance of a single core cable.	20EE305.5	L2

Part B (Long Answer Questions 5 x 12 = 60 Marks)

No.	Questions (6 through 15)	Marks	Learning Outcome (s)	DoK
6	With neat sketch, explain the construction and operation of different components of a hydroelectric power plant.	12M	20EE305.1	L2
OR				
7 (a)	What are the main components of a nuclear power station? Explain the functions of each component in detail.	9M	20EE305.1	L1
7 (b)	What are the factors to be considered for selection of site for a thermal power station?	3M	20EE305.1	L1
8	Discussion the following: (i) Connected load, (ii) Demand factor, (iii) Load factor, (iv) Plant use factor, (v) Base and peak load plants, (vi) Spinning reserve.	12M	20EE305.2	L1
OR				
An electrical supply company having a maximum load of 50 MW generates 18×10^7 units per annum and the supply consumers have an aggregate demand of 75 MW. The annual expenses including capital charges are:				
9 (a)	- For Fuel = Rs. 90 lakhs. - Fixed charges concerning generation = Rs. 28 lakhs; - Fixed charges concerning transmission and distribution = Rs. 32 lakhs;	10M	20EE305.2	L3
Assuming 90% of the fuel cost is essential to running charges and the loss in transmission and distribution as 15% of kWh generated. Deduce a two-part tariff to find the actual cost of supply to the consumers.				
9 (b)	What are desirable characteristics of a tariff?	2M	20EE305.2	L1
10 (a)	Derive an expression for inductance of a 3-phase transmission line with unsymmetrical spacing.	8M	20EE305.3	L3
10 (b)	A conductor is composed of seven identical copper strands each having a radius r . Find the self-GMD of the conductor.	4M	20EE305.3	L3
OR				
11 (a)	Derive an expression for capacitance of a 3-phase line when the conductors are symmetrically placed.	8M	20EE305.3	L3
11 (b)	Calculate the capacitance of a 100 km long 3-phase, 50 Hz overhead transmission line consisting of 3 conductors each of diameter 2 cm and spaced 2.5 m at the corners of an equilateral triangle.	4M	20EE305.3	L3
12	The A, B, C, D constants of a 3-phase transmission line are $A=D=(0.936+j0.016)$, $B=(33.5+j1.138) \Omega$, $C=(-0.928+j901.223) \times 10^{-6} S$. The load at the receiving end is 40 MW at 220 kV with power factor at 0.86 lagging. Find the magnitude of the sending end voltage, current,	12M	20EE305.4	L3

power, line efficiency and the voltage regulation. Assume the magnitude of sending end voltage remains constant.

OR

13 (a)	Explain the classification of lines based on their length of transmission.	6M	20EE305.4	L3
13 (b)	Explain the surge impedance loading with respect to an overhead transmission line.	6M	20EE305.4	L3

14	Calculate sag and tension of a conductor when, (i) supports are at equal levels and (ii) supports are at unequal levels. Analyze with and without the effect of ice loading and wind.	12M	20EE305.5	L3

OR

15 (a)	An insulator string for 66 kV line has 4 discs. The shunt capacitance between each joint and metal work is 10 % of the capacitance of each disc. Find the voltage across the different discs and string efficiency.	9M	20EE305.5	L3
15 (b)	A 3-phase overhead transmission line is being supported by three discs insulator. The potential across top unit (i.e. near the tower) and the middle unit are 8kV and 11kV respectively. Calculate the ratio of capacitance between pin and earth to the self-capacitance of each unit.	3M	20EE305.5	L3

Semester End Examination, February, 2022

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2	List the types of tariffs to calculate energy rate.	20EE305.2	L1
3	List any four types of conductors used in power systems	20EE305.3	L2
4	Define a) Skin Effect b) Corona Loss	20EE305.4	L1
5	Write down the expression for insulation resistance of a single core cable.	20EE305.5	L2

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OR				
9 (a)	An electrical supply company having a maximum load of 50 MW generates 18x10 ⁷ units per annum and the supply consumers have an aggregate demand of 75 MW. The annual expenses including capital charges are: - For Fuel = Rs. 90 lakhs. - Fixed charges concerning generation = Rs. 28 lakhs; - Fixed charges concerning transmission and distribution = Rs. 32 lakhs; Assuming 90% of the fuel cost is essential to running charges and the loss in transmission and distribution as 15% of kWh generated. Deduce a two-part tariff to find the actual cost of supply to the consumers.	10M	20EE305.2	L3
9 (b)	What are desirable characteristics of a tariff?	2M	20EE305.2	L1
10 (a)	Derive an expression for inductance of a 3-phase transmission line with unsymmetrical spacing.	8M	20EE305.3	L3
10 (b)	A conductor is composed of seven identical copper strands each having a radius r. Find the self-GMD of the conductor.	4M	20EE305.3	L3
OR				
11 (a)	Derive an expression for capacitance of a 3-phase line when the conductors are symmetrically placed.	8M	20EE305.3	L3
11 (b)	Calculate the capacitance of a 100 km long 3-phase, 50 Hz overhead transmission line consisting of 3 conductors each of diameter 2 cm and spaced 2.5 m at the corners of an equilateral triangle.	4M	20EE305.3	L3
12	The A, B, C, D constants of a 3-phase transmission line are A=D=(0.936+j0.016), B=(33.5+j1.138) Ω, C=(-0.928+j901.223)x10 ⁻⁶ S. The load at the receiving end is 40 MW at 220 kV with power factor at 0.86 lagging. Find the magnitude of the sending end voltage, current,	12M	20EE305.4	L3

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9 (b)	Assuming 90% of the fuel cost is essential to running charges and the loss in transmission and distribution as 15% of kWh generated. Deduce a two-part tariff to find the actual cost of supply to the consumers. What are desirable characteristics of a tariff?	2M	20EE305.2	L1
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OR				
15 (a)	An insulator string for 66 kV line has 4 discs. The shunt capacitance between each joint and metal work is 10 % of the capacitance of each disc. Find the voltage across the different discs and string efficiency.	9M	20EE305.5	L3
15 (b)	A 3-phase overhead transmission line is being supported by three discs insulator. The potential across top unit (i.e. near the tower) and the middle unit are 8kV and 11kV respectively. Calculate the ratio of capacitance between pin and earth to the self-capacitance of each unit.	3M	20EE305.5	L3

Semester End Examination, Feb, 2022

Course Code : ZDEE305

Course : Power generation and Transmission.

Scheme of valuation

- ① (a) Economizer — 1M
(b) Preheater — 1M
- ② Any 4 types of tariffs — 2M
- ③ types of conductors — 2M
- ④ (a) Skin Effect — 1M
(b) Corona loss — 1M
- ⑤ Insulation resistance Expression — 2 marks
- ⑥ Sketch — 4 marks
Components Explanation — 6 marks } 12M
operation — 2 marks
- ⑦ (a) List of components — 3 marks
Function of Each Component — 6M } 9M
- ⑦ (b) Any four factors — 3M
- ⑧ (a) Each one having 2 marks Equally — 12M
- ⑧ (b) Any 2 desirable characteristics — 2M
- ⑩ (a) for Fig — 2 marks
Expression — 6 marks } 8M

⑩ (b) For right Answer — 4M

⑪ (a) For Figure — 2M
Derivation & Expression — 6M } 8M

⑪ (b) Right Answer — 4M

⑫ For sending end voltage — 3
Current — 3
Power — 2
Efficiency — 2
Regulation — 2 } 12M

⑬ (a) Three Types of Transmission lines — 6M

⑬ (b) Surge impedance loading direct derivation — 6M } 12M.

⑭ Sag when equal level poles — 4M
unequal poles — 4M
wind & ice effect — 4M } 12M

⑮ (a) Voltage across Each disc — 6 marks
String Efficiency — 3M } 9M

⑮ (b) For right Answer — 3M



**N S RAJU INSTITUTE OF TECHNOLOGY
(AUTONOMOUS)
SONTYAM , ANANDAPURAM, VISAKHAPATNAM – 531 173**

ANSWER KEY AND SCHEME OF EVALUATION

Short questions and answers:

1) define (a) economizer (b) pre heater

Economiser: An economiser is essentially a feed water heater and derives heat from the flue gases for this purpose. The feed water is fed to the economiser before supplying to the boiler. The economiser extracts a part of heat of flue gases to increase the feed water temperature.

Air preheater. An air pre heater increases the temperature of the air supplied for coal burning by deriving heat from flue gases. Air is drawn from the atmosphere by a forced draught fan and is passed through air pre heater before supplying to the boiler furnace. The air pre heater extracts heat from flue gases and increases the temperature of air used for coal combustion. The principal benefits of preheating the air are : increased thermal efficiency and increased steam capacity per square meter of boiler surface.

2) List the types of tariffs to calculate energy rate.

- A Simple tariff.
- B Flat rate tariff
- C Block rate tariff
- D Two-part tariff
- E Maximum demand tariff
- F Power factor tariff
- G Three-part tariff

3) List any four types of conductors used in power systems

- A solid conductors
- B standard conductors
- C bundle conductors
- D hollow conductors

4) Define (a)Skin effect (b) corona loss

Skin effect:

The tendency of alternating current to concentrate near the surface of a conductor is known as skin effect.

Corona loss

This phenomenon of electric discharge occurring in high voltage transmission lines is known as the corona loss.

5) Write down the expression for insulation resistance of a single core cable.

Consider a very small layer of insulation of thickness dx at a radius x . The length through which leakage current tends to flow is dx and the area of X-section offered to this flow is $2\pi x l$.

$\therefore$ Insulation resistance of considered layer

$$= \rho \frac{dx}{2\pi x l}$$

Insulation resistance of the whole cable is

$$R = \int_{r_1}^{r_2} \rho \frac{dx}{2\pi x l} = \frac{\rho}{2\pi l} \int_{r_1}^{r_2} \frac{1}{x} dx$$

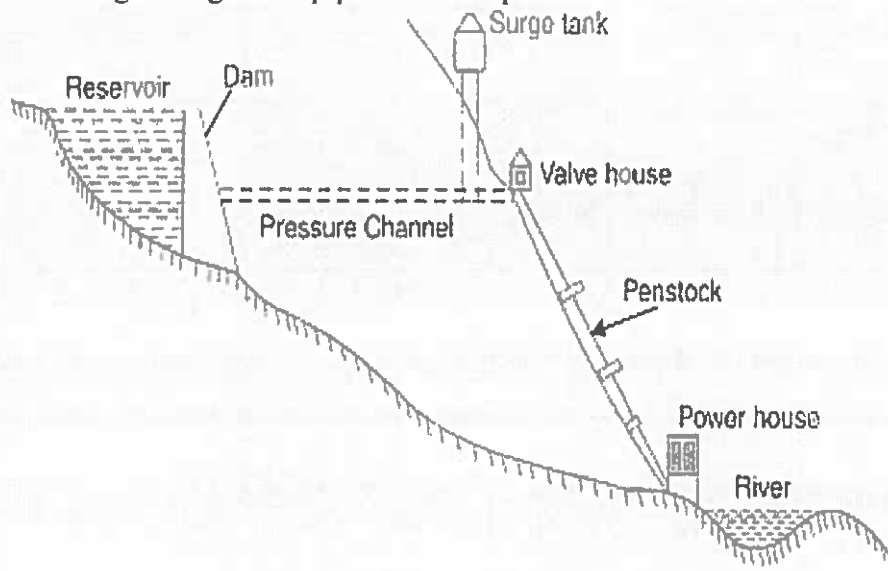
$$\therefore R = \frac{\rho}{2\pi l} \log_e \frac{r_2}{r_1}$$

Long questions and answers:

6) With neat sketch, explain the construction and operation of different components of a hydroelectric power plant.

CONSTRUCTION:

A hydro-electric power station simply involves the conversion of hydraulic energy into electrical energy, yet it embraces many arrangements for proper working and efficiency. The schematic arrangement of a modern hydro-electric plant is shown in below fig.2 The dam is constructed across a river or lake and water from the catchment area collects at the back of the dam to form a reservoir. A pressure tunnel is taken off from the reservoir and water brought to the valve house at the start of the penstock. The valve house contains main sluice valves and automatic isolating valves. The former controls the water flow to the power house and the latter cuts off supply of water when the penstock bursts. From the valve house, water is taken to water turbine through a huge steel pipe known as penstock.



Schematic arrangement of a Hydro-electric plant

Fig. 2.2

The water turbine converts hydraulic energy into mechanical energy. The turbine drives the alternator which converts mechanical energy into electrical energy. A surge tank (open from top) is built just before the valve house and protects the penstock from bursting in case the turbine gates suddenly close* due to electrical load being thrown off. When the gates close, there is a sudden stopping of water at the lower end of the penstock and consequently the penstock can burst like a paper log. The surge tank absorbs this pressure swing by increase in its level of water.

OPERATION:

A generating station which utilises the potential energy of water at a high level for the generation of electrical energy is known as a hydro-electric power station.

Hydro-electric power stations are generally located in hilly areas where dams can be built conveniently and large water reservoirs can be obtained. In a hydro-electric power station, water head is created by constructing a dam across a river or lake. From the dam, water is led to a water turbine. The water turbine captures the energy in the falling water and changes the hydraulic energy (*i.e.* product of head and flow of water) into mechanical energy at the turbine shaft. The turbine drives the alternator which converts mechanical energy into electrical energy. Hydro-electric power stations are becoming very popular because the reserves of fuels (*i.e.*, coal and oil) are depleting day by day. They have the added importance for flood control, storage of water for irrigation and water for drinking purposes.

COMPONENTS:

The constituents of a hydro-electric plant are

- (1) hydraulic structures
- (2) water turbines and
- (3) electrical equipment.

We shall discuss these items in turn.

1. **Hydraulic structures.** Hydraulic structures in a hydro-electric power station include dam, spillways, head works, surge tank, penstock and accessory works.

(i) **Dam.** A dam is a barrier which stores water and creates water head. Dams are built of concrete or stone masonry, earth or rock fill. The type and arrangement depends upon the topography of the site. A masonry dam may be built in a narrow canyon. An earth dam may be best suited for a wide valley. The type of dam also depends upon the foundation conditions, local materials and transportation available, occurrence of earthquakes and other hazards. At most of sites, more than one type of dam may be suitable and the one which is most economical is chosen.

(ii) **Spillways.** There are times when the river flow exceeds the storage capacity of the reservoir. Such a situation arises during heavy rainfall in the catchment area. In order to discharge the surplus water from the storage reservoir into the river on the down-stream side of the dam, spillways are used. Spillways are constructed of concrete piers on the top of the dam. Gates are provided between these piers and surplus water is discharged over the crest of the dam by opening these gates.

(iii) **Headworks.** The headworks consists of the diversion structures at the head of an intake. They generally include booms and racks for diverting floating debris, sluices for by-passing debris and sediments and valves for controlling the flow of water to the turbine. The flow of water into and through head works should be as smooth as possible to avoid head loss and cavitation. For this purpose, it is necessary to avoid sharp corners and abrupt contractions or enlargements.

(iv) **Surge tank.** Open conduits leading water to the turbine require no protection. However, when closed conduits are used, protection becomes necessary to limit the abnormal pressure in the conduit. For this reason, closed conduits are always provided with a surge tank. A surge tank is a small reservoir or tank (open at the top) in which water level rises or falls to reduce the pressure swings in the conduit. A surge tank is located near the beginning of the conduit. When the turbine is running at a steady load, there are no surges in the flow of water through the conduit *i.e.*, the quantity of water flowing in the conduit is just sufficient to meet the turbine requirements.

(v) *Penstocks*. Penstocks are open or closed conduits which carry water to the turbines. They are generally made of reinforced concrete or steel. Concrete penstocks are suitable for low heads (< 30 m) as greater pressure causes rapid deterioration of concrete. The steel penstocks can be designed for any head; the thickness of the penstock increases with the head or working pressure.

2. **Water turbines**. Water turbines are used to convert the energy of falling water into mechanical energy. The principal types of water turbines are :

(i) Impulse turbines

(ii) Reaction turbines

(i) *Impulse turbines*. Such turbines are used for high heads. In an impulse turbine, the entire pressure of water is converted into kinetic energy in a nozzle and the velocity of the jet drives the wheel. The example of this type of turbine is the Pelton wheel. It consists of a wheel fitted with elliptical buckets along its periphery. The force of water jet striking the buckets on the wheel drives the turbine. The quantity of water jet falling on the turbine is controlled by means of a *needle* or *spear* (not shown in the figure) placed in the tip of the nozzle. The movement of the needle is controlled by the governor. If the load on the turbine decreases, the governor pushes the needle into the nozzle, thereby reducing the quantity of water striking the buckets. Reverse action takes place if the load on the turbine increases.

(ii) *Reaction turbines*. Reaction turbines are used for low and medium heads. In a reaction turbine, water enters the runner partly with pressure energy and partly with velocity head. The important types of reaction turbines are :

(a) Francis turbines (b) Kaplan turbines

A Francis turbine is used for low to medium heads. It consists of an outer ring of stationary guide blades fixed to the turbine casing and an inner ring of rotating blades forming the runner. The guide blades control the flow of water to the turbine.

3. **Electrical equipment**. The electrical equipment of a hydro-electric power station includes alternators, transformers, circuit breakers and other switching and protective devices.

7) (a) **What are the main components of a nuclear power station? Explain the functions of each component in detail.**

The whole arrangement of nuclear power station can be divided into the following main stages :

(i) Nuclear reactor (ii) Heat exchanger (iii) Steam turbine (iv) Alternator

(i) **Nuclear reactor**. It is an apparatus in which nuclear fuel (U_{235}) is subjected to nuclear fission. It controls the chain reaction that starts once the fission is done. If the chain reaction is not controlled, the result will be an explosion due to the fast increase in the energy released. A nuclear reactor is a cylindrical stout pressure vessel and houses fuel rods of Uranium, moderator and control rods. The fuel rods constitute the fission material and release huge amount of energy when bombarded with slow moving neutrons. The moderator consists of graphite rods which enclose the fuel rods. The moderator slows down the neutrons before they bombard the fuel rods. The control rods are of cadmium and are inserted into the reactor. Cadmium is strong neutron absorber and thus regulates the supply of neutrons for fission. When the control rods are pushed in deep enough, they absorb most of fission neutrons and hence few are available for chain reaction which, therefore, stops. However, as they are being withdrawn, more and more of these fission neutrons cause fission and hence the *intensity* of chain reaction (or heat produced) is increased. Therefore, by pulling out the control rods, power of the nuclear reactor is increased, whereas by pushing them in, it is reduced. In actual practice, the lowering or raising of control rods is accomplished automatically according to the requirement of load. The heat produced in the reactor is removed by the coolant, generally a sodium metal. The coolant carries the heat to the heat exchanger.

(ii) **Heat exchanger**. The coolant gives up heat to the heat exchanger which is utilised in raising the steam. After giving up heat, the coolant is again fed to the reactor.

(iii) **Steam turbine.** The steam produced in the heat exchanger is led to the steam turbine through a valve. After doing a useful work in the turbine, the steam is exhausted to condenser. The condenser condenses the steam which is fed to the heat exchanger through feed water pump.

(iv) **Alternator.** The steam turbine drives the alternator which converts mechanical energy into electrical energy. The output from the alternator is delivered to the bus-bars through transformer, circuit breakers and isolators.

7) (b) What are the factors to be considered for selection of site for a thermal power station?

In order to achieve overall economy, the following points should be considered while selecting a site for a steam power station :

(i) **Supply of fuel.** The steam power station should be located near the coal mines so that transportation cost of fuel is minimum. However, if such a plant is to be installed at a place where coal is not available, then care should be taken that adequate facilities exist for the transportation of coal.

(ii) **Availability of water.** As huge amount of water is required for the condenser, therefore, such a plant should be located at the bank of a river or near a canal to ensure the continuous supply of water.

(iii) **Transportation facilities.** A modern steam power station often requires the transportation of material and machinery. Therefore, adequate transportation facilities must exist *i.e.*, the plant should be well connected to other parts of the country by rail, road. etc.

(iv) **Cost and type of land.** The steam power station should be located at a place where land is cheap and further extension, if necessary, is possible. Moreover, the bearing capacity of the ground should be adequate so that heavy equipment could be installed.

(v) **Nearness to load centres.** In order to reduce the transmission cost, the plant should be located near the centre of the load. This is particularly important if *d.c.* supply system is adopted. However, if *a.c.* supply system is adopted, this factor becomes relatively less important. It is because *a.c.* power can be transmitted at high voltages with consequent reduced transmission cost. Therefore, it is possible to install the plant away from the load centres, provided other conditions are favourable.

(vi) **Distance from populated area.** As huge amount of coal is burnt in a steam power station, therefore, smoke and fumes pollute the surrounding area. This necessitates that the plant should be located at a considerable distance from the populated areas.

Conclusion. It is clear that all the above factors cannot be favourable at one place. However, keeping in view the fact that now-a-days the supply system is *a.c.* and more importance is being given to generation than transmission, a site away from the towns may be selected. In particular, a site by river side where sufficient water is available, no pollution of atmosphere occurs and fuel can be transported economically, may perhaps be an ideal choice.

8) Discussion the following:

(i) Connected load, (ii) Demand factor, (iii) Load factor, (iv) Plant use factor, (v) Base and peak load plants, (vi) Spinning reserve

1

(i) **Load factor.** The ratio of average load to the maximum demand during a given period is known as load factor *i.e.*,

Load factor = $\frac{\text{Average load}}{\text{Max. demand}}$

plant is in operation for T hours,

Load factor = $\frac{\text{Average load} \times T}{\text{Max. demand} \times T}$

Units generated in T hours = $\text{Max. demand} \times T$

load factor may be daily load factor, monthly load factor or annual load factor if the time period considered is day or month or year. Load factor is always less than 1 because average load is smaller than the maximum demand.

(ii) **diversity factor** The ratio of the sum of individual maximum demands to the maximum demand on power station is known as **diversity factor** i.e.,

$$\text{Diversity factor} = \frac{\text{Sum of individual max. demands}}{\text{Max. demand on power station}}$$

A power station supplies load to various types of consumers whose maximum demands generally do not occur at the same time. Therefore, the maximum demand on the power station is always less than the sum of individual maximum demands of the consumers

iii) **Plant capacity factor.** It is the ratio of actual energy produced to the maximum possible energy that could have been produced during a given period

$$\text{Plant capacity factor} = \frac{\text{Actual energy produced}}{\text{Max. energy that could have been produced}}$$

iv) **Plant use factor.** It is ratio of kWh generated to the product of plant capacity and the number of hours for which the plant was in operation i.e.

$$\text{Plant use factor} = \frac{\text{Station output in kWh}}{\text{Plant capacity} \times \text{Hours of use}}$$

5) **Utilization factor:** It is the ratio of maximum demand on the power station to the rated capacity of the power station. **Plant capacity factor:** It is the ratio of actual energy produced to the maximum possible energy that could have been produced during a given period.

6) **Spinning reserve :** Spinning reserve is defined as the total amount of generation available from all units synchronised to the grid (or system) minus the load demand and the losses being supplied. Spinning reserve is very essential in an interconnected system as loss of a large unit in the system may result in under frequency problem. However, if sufficient spinning reserve is available in the system, the loss of one of the units can be taken care of by the remaining units in the system.

9) (a) An electrical supply company having a maximum load of 50 MW generates 18×10^7 units per annum and the supply consumers have an aggregate demand of 75 MW. The annual expenses including capital charges are:

For Fuel = Rs. 90 lakhs.

Fixed charges concerning generation = Rs. 28 lakhs;

Fixed charges concerning transmission and distribution = Rs. 32 lakhs;

Assuming 90% of the fuel cost is essential to running charges and the loss in transmission and distribution is 15% of kWh generated. Deduce a two-part tariff to find the actual cost of supply to the consumers.

For generation = in Rs 28×10^5

For transmission and distribution = In Rs 32×10^5

For fuel (10% only) = Rs $0.1 \times 90 \times 10^5$

= Rs 9×10^5
Total annual fixed charge = Rs $(28 + 32 + 9) \times 10^5 =$ Rs 69×10^5

This cost has to be spread over the aggregate maximum demand of all the consumers i.e. 75 MW.

Cost per kW of maximum demand = $(69 \times 10^5) / (75 \times 10^3) =$ Rs. 92

Annual running charges:

Cost of fuel (90%) = Rs $0.9 \times 90 \times 10^5 =$ Rs 81×10^5

Units delivered to consumers = 85% of units generated
= $0.85 \times 18 \times 10^7 = 15.3 \times 10^7$ kWh

This cost is to be spread over the units delivered to the consumers.

$\therefore$ Cost/kWh = Rs $(81 \times 10^5) / (15.3 \times 10^7) = 0.053 =$ Re 5.3 paise

$\therefore$ Tariff is Rs 92 per kW of maximum demand plus 5.3 paise per kWh.

9)(b) What are desirable characteristics of a tariff?

A tariff must have the following desirable characteristics :

(i) *Proper return* : The tariff should be such that it ensures the proper return from each consumer. In other words, the total receipts from the consumers must be equal to the cost of producing and supplying electrical energy plus reasonable profit. This will enable the electric supply company to ensure continuous and reliable service to the consumers.

(ii) *Fairness* : The tariff must be fair so that different types of consumers are satisfied with the rate of charge of electrical energy. Thus a big consumer should be charged at a lower rate than a small consumer. It is because increased energy consumption spreads the fixed charges over a greater number of units, thus reducing the overall cost of producing electrical energy. Similarly, a consumer whose load conditions do not deviate much from the ideal should be charged at a lower rate than the one whose load conditions change appreciably from the ideal.

(iii) *Simplicity* : The tariff should be simple so that an ordinary consumer can easily understand it. A complicated tariff may cause an opposition from the public which is generally distrustful of supply companies.

(iv) *Reasonable profit* : The profit element in the tariff should be reasonable. An electric supply company is a public utility company and generally enjoys the benefits of monopoly. Therefore, the investment is relatively safe due to non-competition in the market. This calls for the profit to be restricted to 8% or so per annum.

(v) *Attractive* : The tariff should be attractive so that a large number of consumers are encouraged to use electrical energy. Efforts should be made to fix the tariff in such a way so that consumers can pay easily.

10)(a) Derive an expression for inductance of a 3-phase transmission line with Un symmetrical spacing.

Unsymmetrical spacing:

When 3-phase line conductors are not equidistant from each other, the conductor spacing is said to be unsymmetrical. Under such conditions, the flux linkages and inductance of each phase are not the same.

A different inductance in each phase results in unequal voltage drops in the three phases even if the currents in the conductors are balanced. Therefore, the voltage at the receiving end will not be the same for all phases. In order that voltage drops are equal in all conductors, we generally interchange the positions of the conductors at regular intervals along the line so that each conductor occupies the original position of every other conductor over an equal distance. Such an exchange of positions is known as transposition.

transposed line. The phase conductors are designated as *A*, *B* and *C* and the positions occupied are numbered 1, 2 and 3. The effect of transposition is that each conductor has the same average inductance.

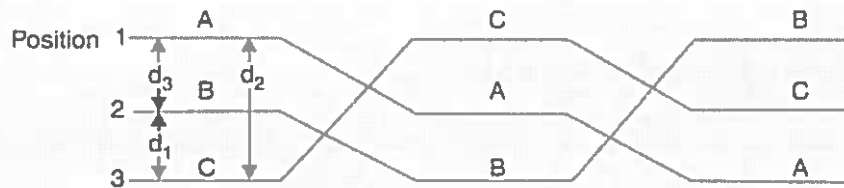


Fig. 9.9

Fig. 9.9 shows a 3-phase transposed line having unsymmetrical spacing. Let us assume that each of the three sections is 1 m in length. Let us further assume balanced conditions i.e., $I_A + I_B + I_C = 0$. Let the line currents be :

$$I_A = I(1 + j0)$$

$$I_B = I(-0.5 - j0.866)$$

$$I_C = I(-0.5 + j0.866)$$

As proved above, the total flux linkages per metre length of conductor *A* is

$$\psi_A = \frac{\mu_0}{2\pi} \left[\left(\frac{1}{4} - \log_e r \right) I_A - I_B \log_e d_3 - I_C \log_e d_2 \right]$$

Putting the values of I_A , I_B and I_C , we get.

$$\begin{aligned} \psi_A &= \frac{\mu_0}{2\pi} \left[\left(\frac{1}{4} - \log_e r \right) I - I(-0.5 - j0.866) \log_e d_3 - I(-0.5 + j0.866) \log_e d_2 \right] \\ &= \frac{\mu_0}{2\pi} \left[\frac{1}{4} I - I \log_e r + 0.5 I \log_e d_3 + j0.866 \log_e d_3 + 0.5 I \log_e d_2 - j0.866 I \log_e d_2 \right] \end{aligned}$$

$$\begin{aligned}
&= \frac{\mu_0}{2\pi} \left[\frac{1}{4} I - I \log_e r + 0.5 I (\log_e d_3 + \log_e d_2) + j 0.866 I (\log_e d_3 - \log_e d_2) \right] \\
&= \frac{\mu_0}{2\pi} \left[\frac{1}{4} I - I \log_e r + I \log_e \sqrt{d_2 d_3} + j 0.866 I \log_e \frac{d_3}{d_2} \right] \\
&= \frac{\mu_0}{2\pi} \left[\frac{1}{4} I + I \log_e \frac{\sqrt{d_2 d_3}}{r} + j 0.866 I \log_e \frac{d_3}{d_2} \right] \\
&= \frac{\mu_0 I}{2\pi} \left[\frac{1}{4} + \log_e \frac{\sqrt{d_2 d_3}}{r} + j 0.866 \log_e \frac{d_3}{d_2} \right]
\end{aligned}$$

∴ Inductance of conductor A is

$$\begin{aligned}
L_A &= \frac{\Psi_A}{I_A} = \frac{\Psi_A}{I} \\
&= \frac{\mu_0}{2\pi} \left[\frac{1}{4} + \log_e \frac{\sqrt{d_2 d_3}}{r} + j 0.866 \log_e \frac{d_3}{d_2} \right] \\
&= \frac{4\pi \times 10^{-7}}{2\pi} \left[\frac{1}{4} + \log_e \frac{\sqrt{d_2 d_3}}{r} + j 0.866 \log_e \frac{d_3}{d_2} \right] \text{ H/m} \\
&= 10^{-7} \left[\frac{1}{2} + 2 \log_e \frac{\sqrt{d_2 d_3}}{r} + j 1.732 \log_e \frac{d_3}{d_2} \right] \text{ H/m}
\end{aligned}$$

Similarly inductance of conductors B and C will be :

$$\begin{aligned}
L_B &= 10^{-7} \left[\frac{1}{2} + 2 \log_e \frac{\sqrt{d_3 d_1}}{r} + j 1.732 \log_e \frac{d_1}{d_3} \right] \text{ H/m} \\
L_C &= 10^{-7} \left[\frac{1}{2} + 2 \log_e \frac{\sqrt{d_1 d_2}}{r} + j 1.732 \log_e \frac{d_2}{d_1} \right] \text{ H/m}
\end{aligned}$$

Inductance of each line conductor

$$\begin{aligned}
&= \frac{1}{3} (L_A + L_B + L_C) \\
&= \left[\frac{1}{2} + 2 \log_e \frac{\sqrt[3]{d_1 d_2 d_3}}{r} \right] \times 10^{-7} \text{ H/m} \\
&= \left[0.5 + 2 \log_e \frac{\sqrt[3]{d_1 d_2 d_3}}{r} \right] \times 10^{-7} \text{ H/m}
\end{aligned}$$

If we compare the formula of inductance of an unsymmetrically spaced transposed line with that of symmetrically spaced line, we find that inductance of each line conductor in the two cases will be equal if $d = \sqrt[3]{d_1 d_2 d_3}$. The distance d is known as *equivalent equilateral spacing* for unsymmetrically transposed line.

10) (b) A conductor is composed of seven identical copper strands each having a radius r . Find the self-GMD of the conductor.

Self GMD of the seven strand conductor is 49th root of the distances. Thus

$$\begin{aligned} D_s &= [(r')^7 \times (D_{12}^2 \times D_{26}^2 \times D_{14} \times D_{17})^6 \times (2)^6]^{1/49} \\ &= [(0.7788 r)^7 \times (2^2 r^2 \times 3 \times 2^2 r^2 \times 2^2 r \times 2r)^6]^{1/49} \\ &= \frac{2r[3(0.7788)]^{1/7}}{6^{1/49}} = 2.177 r \end{aligned}$$

11) (a) Derive an expression for capacitance of a 3-phase line when the conductors are symmetrically.

In a 3-phase transmission line, the capacitance of each conductor is considered instead of capacitance from conductor to conductor. Here, again two cases arise viz., symmetrical spacing and unsymmetrical spacing.

(i) **Symmetrical Spacing.** Fig. 9.24 shows the three conductors A, B and C of the 3-phase overhead transmission line having charges Q_A , Q_B and Q_C per metre length respectively. Let the conductors be equidistant (d metres) from each other. We shall find the capacitance from line conductor to neutral in this symmetrically spaced line. Referring to Fig. 9.24, overall potential difference between conductor A and infinite neutral plane is given by (Refer to Art. 9.9):

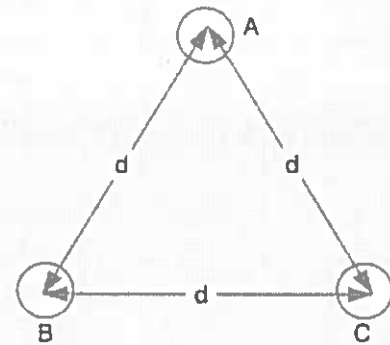


Fig. 9.24

$$\begin{aligned} V_A &= \int_r^\infty \frac{Q_A}{2\pi\lambda\epsilon_0} dx + \int_d^\infty \frac{Q_B}{2\pi\lambda\epsilon_0} dx + \int_d^\infty \frac{Q_C}{2\pi\lambda\epsilon_0} dx \\ &= \frac{1}{2\pi\epsilon_0} \left[Q_A \log_e \frac{1}{r} + Q_B \log_e \frac{1}{d} + Q_C \log_e \frac{1}{d} \right] \\ &= \frac{1}{2\pi\epsilon_0} \left[Q_A \log_e \frac{1}{r} + (Q_B + Q_C) \log_e \frac{1}{d} \right] \end{aligned}$$

Assuming balanced supply, we have, $Q_A + Q_B + Q_C = 0$

$$\therefore Q_B + Q_C = -Q_A$$

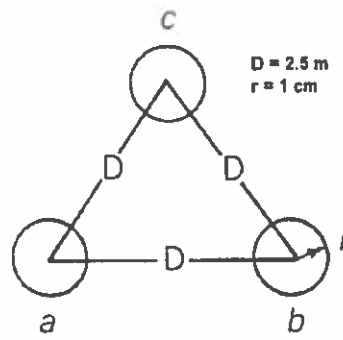
$$\therefore V_A = \frac{1}{2\pi\epsilon_0} \left[Q_A \log_e \frac{1}{r} - Q_A \log_e \frac{1}{d} \right] = \frac{Q_A}{2\pi\epsilon_0} \log_e \frac{d}{r} \text{ volts}$$

$\therefore$ Capacitance of conductor A w.r.t neutral,

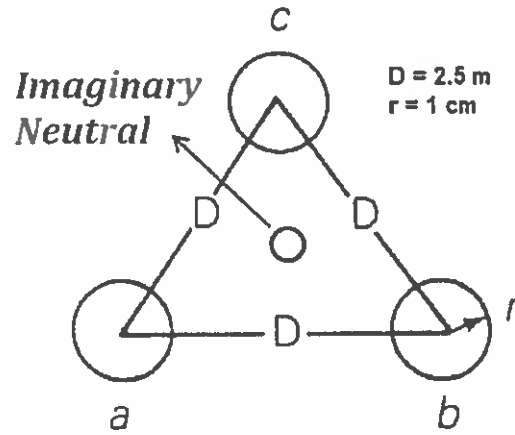
$$C_A = \frac{Q_A}{V_A} = \frac{Q_A}{\frac{Q_A}{2\pi\epsilon_0} \log_e \frac{d}{r}} \text{ F/m} = \frac{2\pi\epsilon_0}{\log_e \frac{d}{r}} \text{ F/m}$$

$$\therefore C_A = \frac{2\pi\epsilon_0}{\log_e \frac{d}{r}} \text{ F/m}$$

11)(b) Calculate the capacitance of a 100 km long 3-phase, 50 Hz overhead transmission line consisting of 3 conductors each of diameter 2 cm and spaced 2.5 m at the corners of an equilateral triangle.



As you see there is no neutral line in this TL, we assume that there is an imaginary one (Not in real), so the arrangement is like this:



Now we can calculate the capacitance between phase and this imaginary neutral line as:

$$C_{aN} = C_{bN} = C_{cN} = \frac{2\pi\epsilon_0}{\ln\left(\frac{GMD}{GMR}\right)}$$

As you can note that capacitance between any phase and neutral is equal because the phases placed in a symmetrical configuration.

Now to know:

1. GMD is called the geometric mean distance, and it depends on the distance between phases, and it can be calculated as,

$GMD = \sqrt[3]{D_{ab} * D_{bc} * D_{ac}}$, as you can see it's normal GMD as we know in mathematics, now

$$GMD = \sqrt[3]{D * D * D} = \sqrt[3]{D^3} = D = 2.5 \text{ m}$$

2. GMR is called the geometric mean radius, and it depend on the radius of conductors, and it's equal to

$$GMR = \sqrt[3]{r_a * r_b * r_c} = \sqrt[3]{r * r * r} = r = 1 \text{ cm}$$

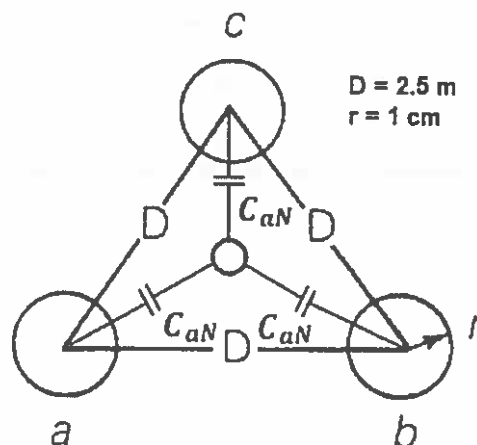
Now you can substitute and calculate it simply,

$$C_{aN} = \frac{2\pi\epsilon_0}{\ln\left(\frac{2.5}{0.01}\right)} = 10.07 * 10^{-12} \text{ F/m}$$

Now you can calculate the total capacitance by multiplying in the length of your

$$\text{TL: } C_{aN} = 10.07 * 10^{-12} \frac{\text{F}}{\text{m}} * 100 * 10^3 \text{ m} = 1.007 * 10^{-6} \text{ F}$$

Now calculate the real capacitance between phases, knowing that:



So to get the C_{ab} it's something like two capacitors of capacitance C_{aN} in series,

then the $C_{ab} = \frac{C_{aN}}{2} = 0.504 \times 10^{-6} \text{ F}$ - This is the answer, Line to line capacitance of your TL.

12) The A, B, C, D constants of a 3-phase transmission line are $A=D=(0.936+j0.016)$, $B=(33.5+j1.138) \Omega$, $C=(-0.928+j901.223) \times 10^{-6} \text{ S}$. The load at the receiving end is 40 MW at 220 kV with power factor at 0.86 lagging. Find the magnitude of the sending end voltage, current, power, Efficiency & Voltage regulation.

Solution

$$A=D=0.936+j0.016$$

$$B=33.5+j1.138 \Omega$$

$$C=(-0.928+j901.223) \times 10^{-6} \text{ S}$$

$$V_{RL-L}=220 \text{ kV} \quad P_R=40 \text{ MW}$$

$$V_{Rph} = \frac{220}{\sqrt{3}} = 127.017 \text{ kV}$$

$$I_R = \frac{40 \times 10^6}{\sqrt{3} \times 220 \times 10^3 \times 0.86} = 122.0613 \text{ Amps} \quad \left[P_R = \sqrt{3} V_L I_L \cos \phi \right]$$

Taking V_R as Reference, We have

$$\vec{V}_R = (127.017 + j0)$$

$$\vec{I} = \dots$$

$$\vec{I}_R = 122.0613 \angle -\cos^{-1} 0.86 = 122.0613 (0.86 - j0.510)$$

$$= 104.972 - j62.25 \text{ Amps}$$

(i) Sending end Voltage!

$$\vec{V}_S = A\vec{V}_R + B\vec{I}_R$$

$$= (0.936 + j0.016)(127.017 + j0) + (33.5 + j1.138)(104.972 - j62.25)$$

$$= 122523.9321 - j123.9435$$

$$= 122523.9948 \angle -0.0579^\circ$$

$$\boxed{V_S = 122.523 \text{ kV}}$$

(ii) Sending end Current

$$\vec{I}_S = C\vec{V}_R + D\vec{I}_R$$

$$\rightarrow = (-0.928 + j901.223) \times 10^{-6} (127.017 + j0) + (0.936 + j0.016)(104.972 - j62.25)$$

$$= -0.118 - j0.118$$

$$I_s = (1143.5227 + j0) + (99.249 - j56.58)$$

$$I_s = 1241.7767 \angle -2.60^\circ \text{ Amps}$$

$$(iii) \text{ Sending Power} = 3 V_s I_s \cos \phi \quad \text{pf/cor} = (-2.60 - 0.0579)$$

$$= 3 \times 122.523 \times 10^3 \times 1241.776 \times \cos(-2.65)$$

$$= 48.7 \text{ MW}$$

$$(iv) \text{ Efficiency} = \frac{\text{Output power}}{\text{Input power}} \times 100$$

$$= \frac{40 \times 10^6}{48.7 \times 10^6} \times 100$$

$$= 82.135 \%$$

$$(v) \text{ Regulation} = \frac{\frac{V_s}{A} - V_R}{V_R} \times 100$$

$$= \frac{122.523}{0.936} - 220$$

$$= \frac{220}{220} \times 100$$

$$= 27\%$$

13) (a) Explain the classification of lines based on their length of transmission.

The transmission line is the part of the power system, it is the medium for transmitting the electrical power generated at generating station to load centers. The transmission lines are classified based on the voltage level, length of the conductor, and types of current flow. They are mainly classified into two types,

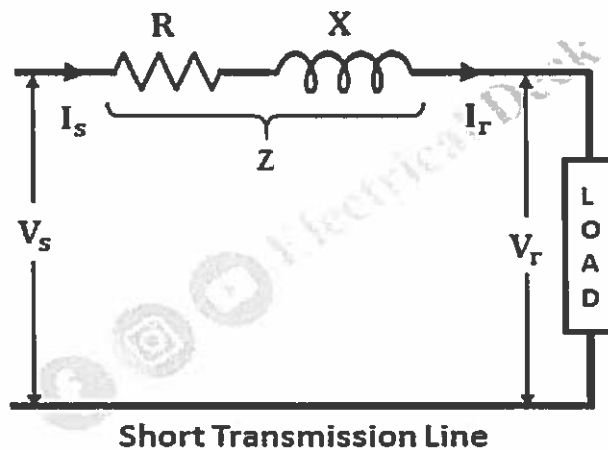
- 1 AC transmission line
- 2 DC transmission line

1. AC Transmission Line: An ac transmission line uses an alternating current to transmit the power from generating stations to load centers. It is a set of conductors being run from one place to another supported on transmission towers. Such lines will have three constants resistance, inductance, and capacitance distributed uniformly along the whole length of the line. The resistance and inductance form the series impedance and capacitance forms the shunt element throughout the length of the line. Depending upon the operating voltage and length, the overhead ac transmission lines are classified as,

1. Short transmission lines
2. Medium transmission lines
3. Long transmission lines

1. Short Transmission Line :

If the overhead transmission line is less than 50km, then this type of transmission line is referred to as Short Transmission Line. Generally, the short transmission lines carry operating voltages of less than 20kV. In these lines, the effect of capacitance is neglected due to smaller length and low operating voltage. Hence, the resistance and inductance effects of the line are considered while determining the performance of the short transmission line as shown below.



2 Medium Transmission Line

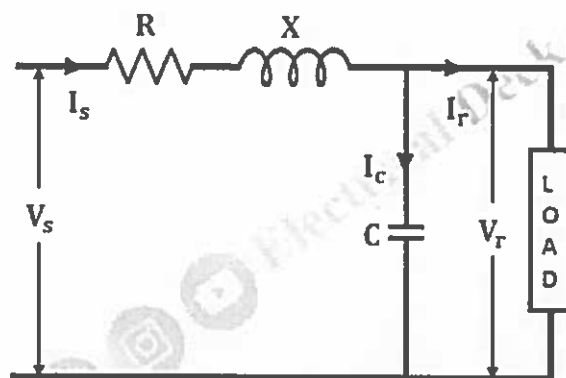
When the length of the overhead transmission line is in the range of 50-150km, and the operating voltage is greater than 20kV, then such lines are called medium transmission lines. As the length of the medium transmission lines is more, the charging current will be significant and hence the effect of capacitance will be taken into consideration.

The capacitance of the transmission lines forms the shunt element and is distributed throughout the length of the line. But, for analysis and calculations, it will be assumed that the capacitance is lumped and concentrated at different strategic places. Based on the location of the capacitance at different places, the medium transmission lines have different configurations. These configurations show the different ways in which the effect of capacitance is taken into consideration. The three configurations based on the location of capacitance are,

1. End condenser representation of medium transmission lines
2. Nominal-T representation of medium transmission lines
3. Nominal- π representation of medium transmission lines

1. End Condenser Representation of Medium Transmission Line :

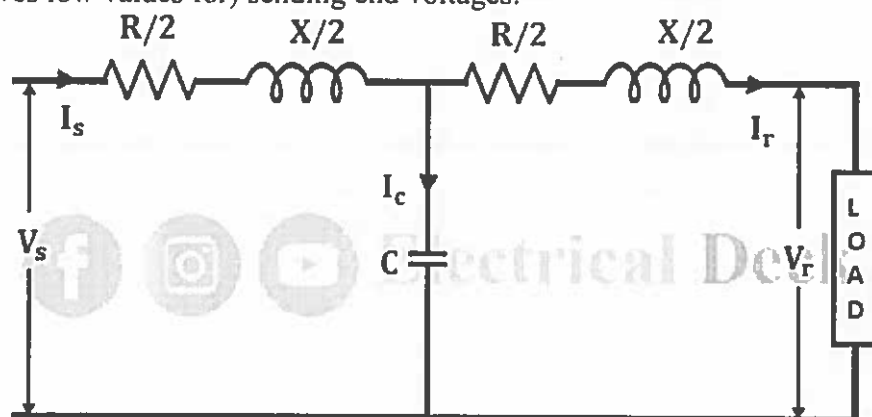
In this configuration, the total capacitance of the line is assumed to be concentrated at the receiving end side of the transmission line. The below shows the end condenser medium transmission line configuration



End Condenser Method of Medium Transmission Line

2. Nominal-T Representation of Medium Transmission Line :

The nominal-T method is also called as middle condenser method and it is one of the localized capacitance methods used to calculate the performance of the transmission line. This method overcompensates (gives low values for) sending end voltages.

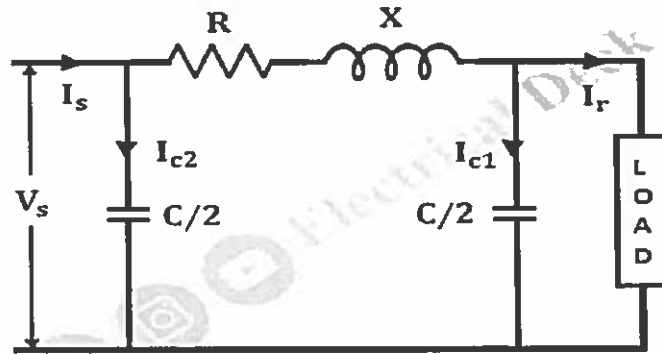


Nominal-T Method of Medium Transmission Line

In the nominal-T method, the total capacitance of the line is assumed to be concentrated at the mid-point (center) of the entire transmission line. Since the total line capacitance is assumed at the center, the charging current flows through half of the line, and the series impedance (resistance and inductive reactance) of the line is divided into half on either side of the capacitance as shown above.

3. Nominal- π Representation of Medium Transmission Line :

The nominal- π method is also called as split condenser method. and it is one of the localized capacitance methods, using which the performance of the transmission line can be calculated efficiently. This method under compensates (gives higher values for) sending end voltages.

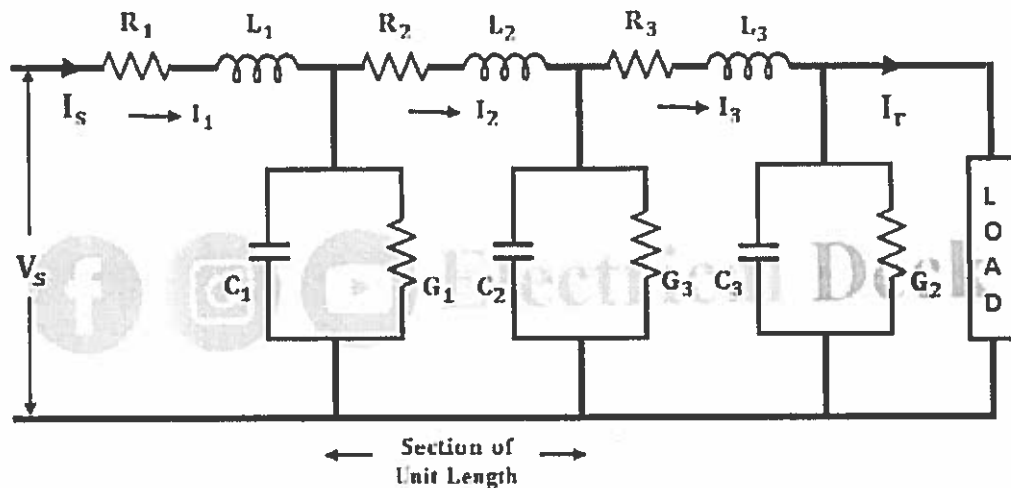


Nominal- π Method of Medium Transmission Line

In the nominal- π method, the capacitance of the transmission line is divided into halves placed at the receiving end and sending end respectively as shown above.

3 Long Transmission Line :

The overhead transmission lines whose length is more than 150km are referred to as long transmission lines. The operating voltage of these lines is more than 100kV. For treatment of this type of line, their line constants are assumed to be distributed in a uniform manner along the length of the line, and also rigorous methods are used for their solution.



Long Transmission Line

DC Transmission Line :

In ac transmission lines, for transmission of power over long distances at higher voltages, the cost of transmission line and loss increases. Also, the ac long transmission line suffers from problems like stability

limits, voltage control, line compensation, interconnection of lines, ground impedance, etc due to an increase in voltage levels and distance.

The various problems associated with long-distance ac transmission have led to the development of HVDC (high voltage direct current) transmission nothing but a dc transmission line. The use of dc power for long transmission lines has various advantages like no stability problem, absence of charging current, no skin effect, need for reactive compensation, bulk power transfer, economic power transmission, etc.

13)(b) Explain the surge impedance loading with respect to an overhead transmission line.

Where,

f = Frequency of the system

L = Inductance per unit length of the line

l = Length of the line

Hence we get,

$$\frac{V}{I} = \sqrt{\frac{L}{C}} = Z_s$$

Surge Impedance Loading is a very essential parameter when it comes to the study of power systems as it is used in the prediction of maximum loading capacity of transmission lines.

However before understanding SIL, we first need to have an idea of what is Surge Impedance (Z_s). It can be defined in two ways one a simpler one and other a bit rigorous.

Method 1

It is a well known fact that a long transmission lines (> 250 km) have distributed inductance and capacitance as its inherent property. When the line is charged, the capacitance component feeds reactive power to the line while the inductance component absorbs the reactive power. Now if we take the balance of the two reactive powers we arrive at the following equation

Capacitive VAR = Inductive VAR

$$\frac{V^2}{X_C} = I^2 X_L$$

Where,

V = Phase voltage

I = Line Current

X_C = Capacitive reactance per phase

X_L = Inductive reactance per phase

Upon simplifying

$$\frac{V}{I} = \sqrt{X_L X_C} = \sqrt{\frac{2\pi f L l}{2\pi f C l}}$$

14) Calculate sag and tension of a conductor when, (i) supports are at equal levels and (ii) supports are at unequal levels. Analyze with and without the effect of ice loading and wind.

In an overhead line, the sag should be so adjusted that tension in the conductors is within safe limits. The tension is governed by conductor weight, effects of wind, ice loading and temperature variations. It is a standard practice to keep conductor tension less than 50% of its ultimate tensile strength i.e., minimum factor of safety in respect of conductor tension should be 2. We shall now calculate sag and tension of a conductor when (i) supports are at equal levels and (ii) supports are at unequal levels

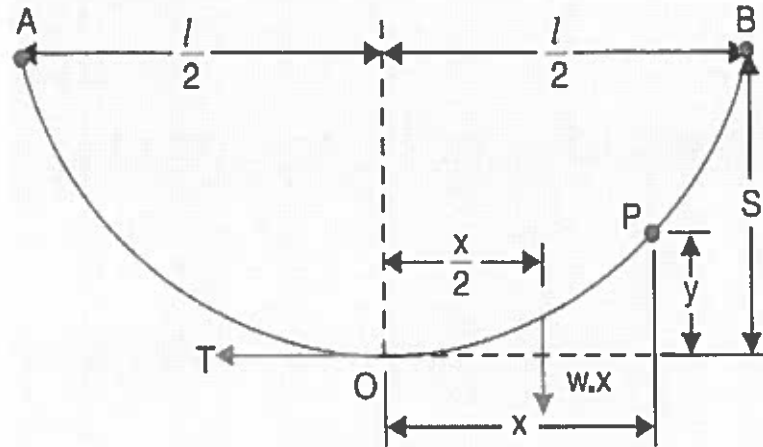
(i) When supports are at equal levels. Consider a conductor between two equilevel supports A and B with O as the lowest point as shown in below Fig. It can be proved that lowest point will be at the mid-span.

Let

l = Length of span

w = Weight per unit length of conductor

T = Tension in the conductor.



Consider a point P on the conductor. Taking the lowest point O as the origin, let the co-ordinates of point P be x and y . Assuming that the curvature is so small that curved length is equal to its horizontal projection (i.e., $OP = x$), the two forces acting on the portion OP of the conductor are :

(a) The weight $w x$ of conductor acting at a distance $x/2$ from O .

(b) The tension T acting at O .

Equating the moments of above two forces about point O , we get.

$$T y = w x \times \frac{x}{2}$$

or
$$y = \frac{w x^2}{2 T}$$

The maximum dip (sag) is represented by the value of y at either of the supports A and B .

At support A , $x = l/2$ and $y = S$

$\therefore$ Sag.
$$S = \frac{w(l/2)^2}{2T} = \frac{w l^2}{8 T}$$

- (ii) When supports are at unequal levels. In many areas, we generally come across conductors suspended between supports at unequal levels. Fig. 8.25 shows a conductor suspended between two supports A and B which are at different levels. The lowest point on the conductor is O .

Let

l = Span length

h = Difference in levels between two supports

x_1 = Distance of support at lower level (i.e., A) from O

x_2 = Distance of support at higher level (i.e., B) from O

T = Tension in the conductor

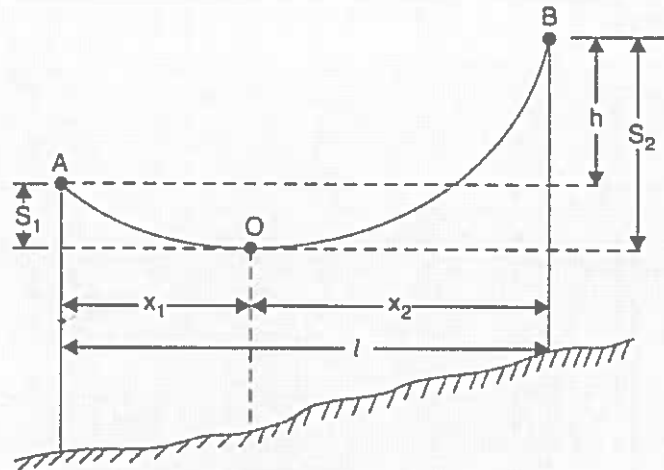


Fig. 8.25

If w is the weight per unit length of the conductor, then,

$$\text{Sag } S_1 = \frac{w x_1^2}{2T}$$

and $\text{Sag } S_2 = \frac{w x_2^2}{2T}$

Also

$$x_1 + x_2 = l \quad \dots(i)$$

Now

$$S_2 - S_1 = \frac{w}{2T} [x_2^2 - x_1^2] = \frac{w}{2T} (x_2 + x_1)(x_2 - x_1)$$

$\therefore$

$$S_2 - S_1 = \frac{w l}{2T} (x_2 - x_1) \quad [\because x_1 + x_2 = l]$$

But

$$S_2 - S_1 = h$$

$\therefore$

$$h = \frac{w l}{2T} (x_2 - x_1)$$

or

$$x_2 - x_1 = \frac{2Th}{wl} \quad \dots(ii)$$

Solving exs. (i) and (ii), we get,

$$x_1 = \frac{l}{2} - \frac{Th}{wl}$$

$$x_2 = \frac{l}{2} + \frac{Th}{wl}$$

Having found x_1 and x_2 , values of S_1 and S_2 can be easily calculated.

Effect of wind and ice loading. The above formulae for sag are true only in still air and at normal temperature when the conductor is acted by its weight only. However, in actual practice, a conductor may have ice coating and simultaneously subjected to wind pressure. The weight of ice acts vertically downwards *i.e.*, in the same direction as the weight of conductor. The force due to the wind is assumed to act horizontally *i.e.*, at right angle to the projected surface of the conductor. Hence, the total force on the conductor is the vector sum of horizontal and vertical forces as shown in Fig. 8.26 (iii).

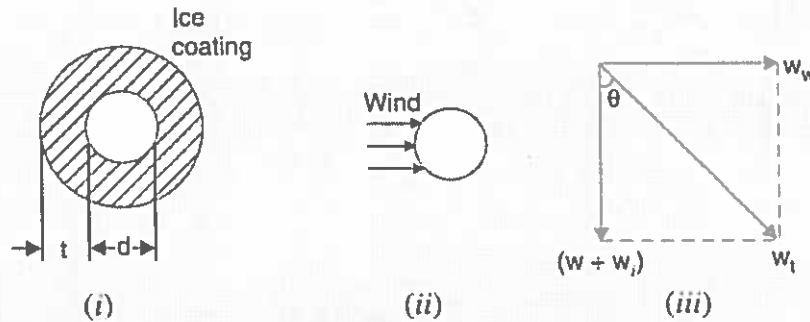


Fig. 8.26

Total weight of conductor per unit length is

$$w_t = \sqrt{(w + w_i)^2 + (w_w)^2}$$

where

w = weight of conductor per unit length

= conductor material density $\times$ volume per unit length

w_i = weight of ice per unit length

= density of ice $\times$ volume of ice per unit length

= density of ice $\times \frac{\pi}{4} [(d + 2t)^2 - d^2] \times 1$

= density of ice $\times \pi t (d + t)^*$

w_w = wind force per unit length

= wind pressure per unit area $\times$ projected area per unit length

= wind pressure $\times [(d + 2t) \times 1]$

When the conductor has wind and ice loading also, the following points may be noted :

(i) The conductor sets itself in a plane at an angle θ to the vertical where

$$\tan \theta = \frac{w_w}{w + w_i}$$

(ii) The sag in the conductor is given by :

$$S = \frac{w_t l^2}{2T}$$

Hence S represents the slant sag in a direction making an angle θ to the vertical. *If no specific mention is made in the problem, then slant sag is calculated by using the above formula.*

(iii) The vertical sag = $S \cos \theta$

15) (a) An insulator string for 66 kV line has 4 discs. The shunt capacitance between each joint and metal work is 10 % of the capacitance of each disc. Find the voltage across the different discs and string efficiency.

From above data $K = 0.1$ $\left[K_f = \frac{10}{100} \right]$

$$\text{Voltage across string} = \frac{66}{\sqrt{3}} = 38.105 \text{ kV} \quad K = 0.1$$

In 4-disc insulator string

$$V_2 = 1.11 V_1$$

$$V_3 = 1.342 V_1$$

$$V_4 = 1.6872 V_1$$

$$V = V_1 + V_2 + V_3 + V_4$$

$$38.105 = V_1 + 1.11 V_1 + 1.342 V_1 + 1.6872 V_1$$

$$38.105 \times 10^3 = 5.1392 V_1$$

$$V_1 = 7.414 \text{ kV}$$

$$V_2 = 1.11 \times 7.414 = 8.2301 \text{ kV}$$

$$V_3 = 1.342 \times 7.414 = 9.949 \text{ kV}$$

$$V_4 = 1.6872 \times 7.414 = 12.508 \text{ kV}$$

$$\begin{aligned} \text{String Efficiency, } \eta &= \frac{\text{Voltage across the string}}{n \times V_f} \times 100 \\ &= \frac{38.105}{4 \times 12.508} \times 100 \\ &= 76.16 \% \end{aligned}$$

15) (b) A 3-phase overhead transmission line is being supported by three discs insulator. The potential across top unit (i.e. near the tower) and the middle unit are 8kV and 11kV respectively. Calculate the ratio of capacitance between pin and earth to the self-capacitance of each unit.

Solution. The equivalent circuit of string insulators is the same as shown in Fig. 8.14. It is given that $V_1 = 8$ kV and $V_2 = 11$ kV.

(i) Let K be the ratio of capacitance between pin and earth to self capacitance. If C farad is the self capacitance of each unit, then capacitance between pin and earth = KC .

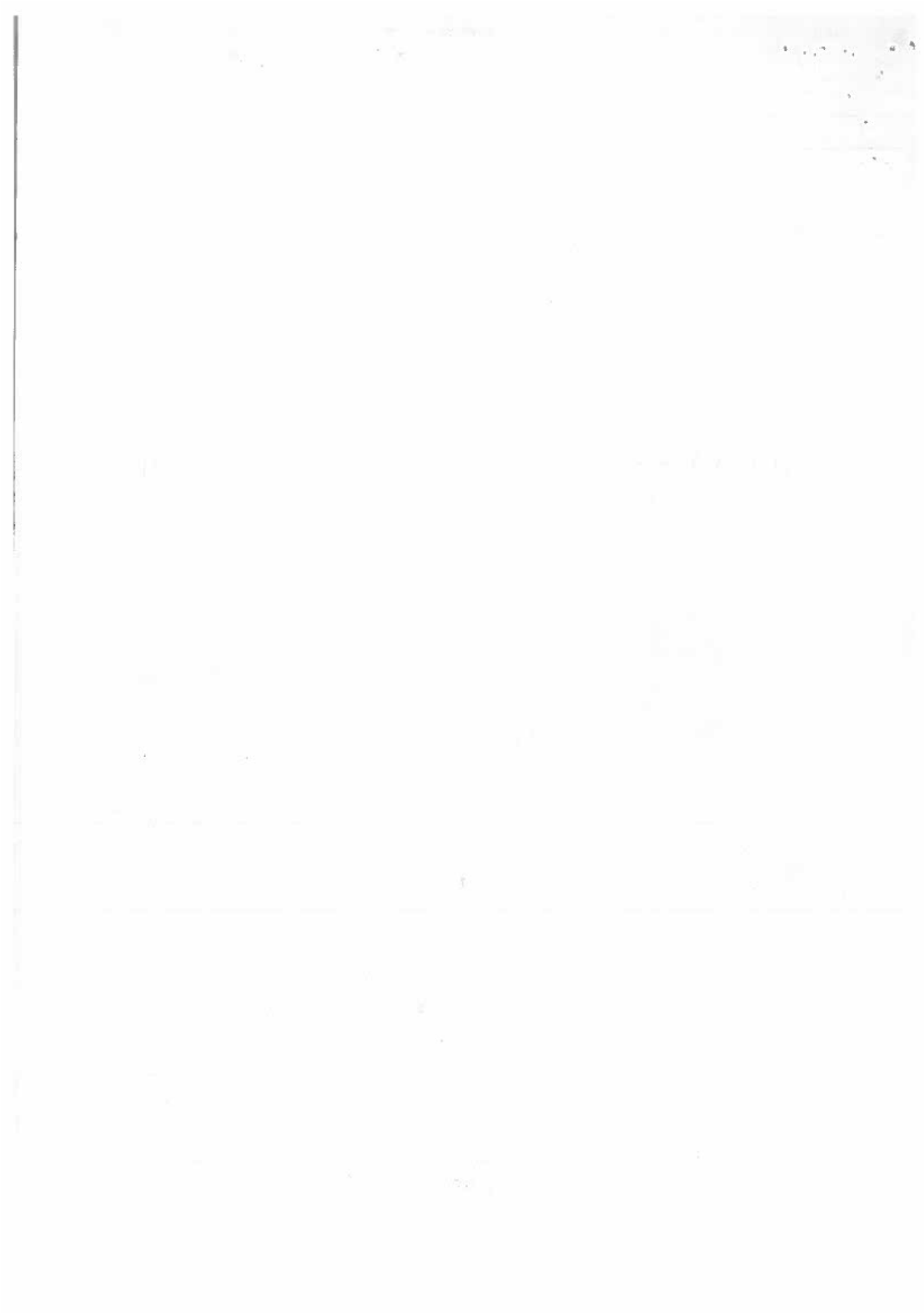
Applying Kirchoff's current law to Junction A.

$$\begin{aligned} I_2 &= I_1 + i_1 \\ \text{or } V_2 \omega C &= V_1 \omega C + V_1 K \omega C \\ \text{or } V_2 &= V_1 (1 + K) \\ \therefore K &= \frac{V_2 - V_1}{V_1} = \frac{11 - 8}{8} = 0.375 \end{aligned}$$



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EEE


PRINCIPAL



Semester End Examination, February, 2022

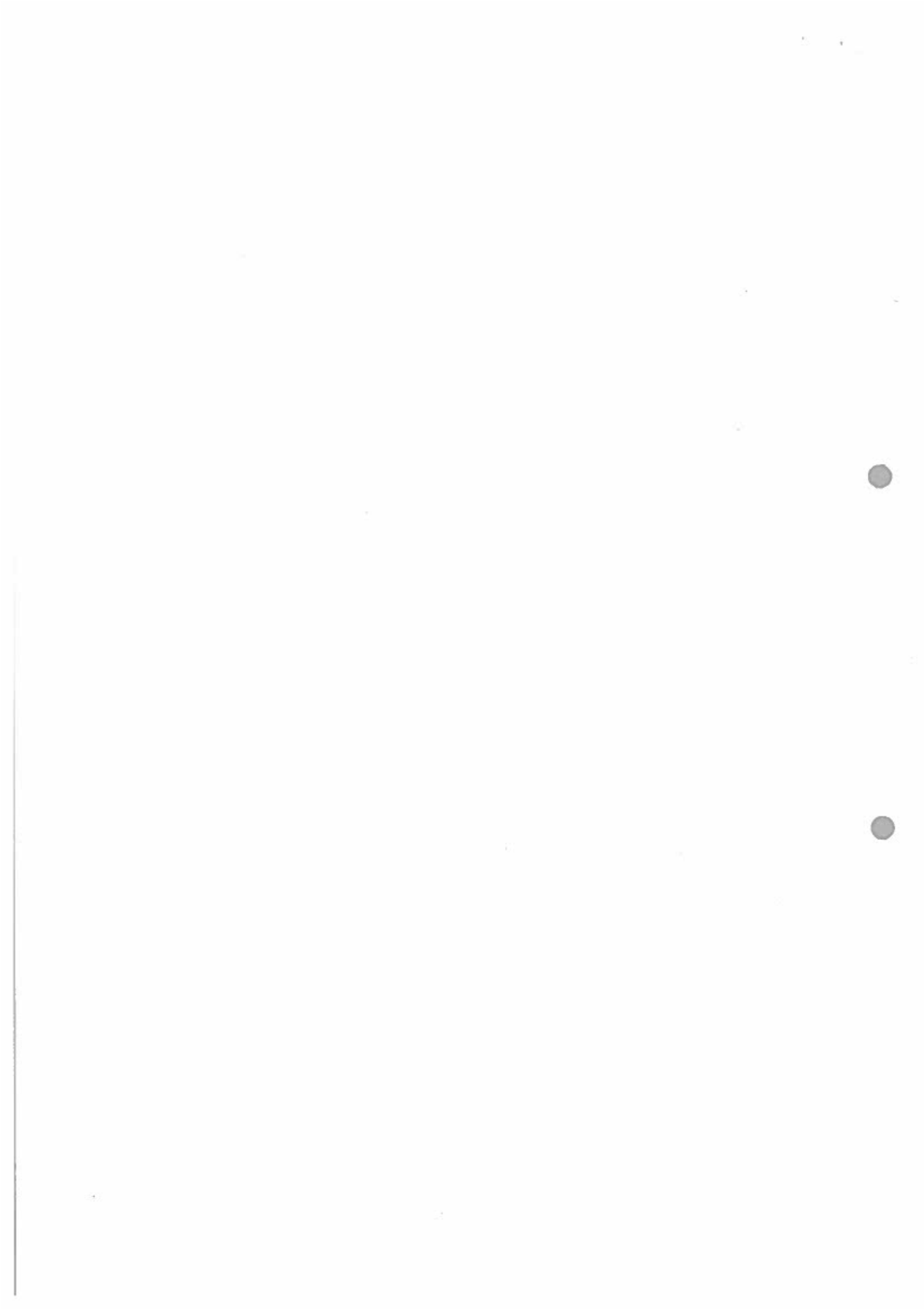
Degree	B. Tech. (U. G.)	Program	EEE	Academic Year	2021 - 2022
Course Code	20EE304	Test Duration	3 Hrs.	Max. Marks	70
Course	DC MACHINES & TRANSFORMERS	Semester	III		

Part A (Short Answer Questions 5 x 2 = 10 Marks)

No.	Questions (1 through 5)	Learning Outcome (s)	DoK
1	Identify any two features of multiply excited magnetic field system	20EE304.1	L1
2	Define the term commutation used in dc machine	20EE304.2	L1
3	Differentiate 3-point starter with 4-point starter	20EE304.3	L2
4	Define all day efficiency of a transformer	20EE304.4	L2
5	List any four conditions to connect transformer in parallel	20EE304.5	L2

Part B (Long Answer Questions 5 x 12 = 60 Marks)

No.	Questions (6 through 15)	Marks	Learning Outcome (s)	DoK
6	Show that the torque in a doubly excited magnetic system is equal to the rate of increase of the field energy with respect to displacement at constant currents	12M	20EE304.1	L2
OR				
7	Derive the expression of the flux, reluctance of the magnetic material with air gap	12M	20EE304.1	L2
8 (a)	Derive an expression for the emf generated in the armature winding of a dc generator	6M	20EE304.2	L2
8 (b)	An 8 pole lap wound armature having 40 slots with 12 conductors/ slot generates 500 V. Determine speed at which machine is running if the flux per pole is 50 mWb	6M	20EE304.1	L3
OR				
9 (a)	Explain demagnetizing & cross magnetizing effects of armature reaction	6M	20EE304.2	L2
9 (b)	Draw the internal and external characteristics of DC series generator in detail	6M	20EE304.2	L2
10 (a)	Discuss the speed torque characteristics of DC shunt, series and compound motors with necessary diagrams	12M	20EE304.3	L2
OR				
11 (a)	Identify the role of starter in DC Motor operation. Also explain the working of three point starter with neat diagram.	12M	20EE304.3	L2
12 (a)	Derive the EMF equation of transformer	6M	20EE304.4	L2
12 (b)	The voltage per turn of a single-phase transformer is 1.1 V. When the primary winding is connected to a 220 V, 50 Hz A.C. supply, the secondary voltage is found to be 550 V. Find: i) Primary and secondary turns ii) Core area if the maximum flux density is 1.1 Wb/m ²	6M	20EE304.4	L3
OR				
13 (a)	Draw and explain the equivalent Circuit diagram of single phase Transformer	6M	20EE304.4	L3
13 (b)	Compare the features of shell type and core type transformers	6M	20EE304.4	L2
14	Draw the various types of three phase transformer connections and brief each one of them	12M	20EE304.5	L2
OR				
15	A 10 kVA, 500/250 V, 50 Hz single-phase transformer gave the following test Data: OC Test (LV side): 250 V, 1.0 A, 80 W, SC Test (HV side): 25 V, 12 A, 100 W, Where LV refers to the low voltage and HV refers to high voltage side. Determine the following: (i) Equivalent circuit referred to LV side, (ii) Secondary load voltage at 0.8 p.f. lagging with full-load current	12M	20EE304.5	L3





N S RAJU INSTITUTE OF TECHNOLOGY

(AUTONOMOUS)

SONTYAM , ANANDAPURAM, VISAKHAPATNAM – 531 173

DCMT ANSWER KEY AND SCHEME OF EVALUATION

1. Identify any two features of multiply excited magnetic field system

- Singly-excited devices are generally employed for motion through a limited distance or rotation through a prescribed angle.
- Electro-mechanical transducers have the special requirement of producing an electrical signal proportional to forces or velocities or producing force proportional to electrical signal (current or voltage).
- Such transducers require two excitations—one excitation establishes a magnetic field of specified strength while the other excitation produces the desired signal (electrical or mechanical). Also, continuous energy conversion devices—motors and generators require multiple excitation

2. Define the term commutation used in dc machine

Commutation in DC machines is the process by which the reversal of current takes place. In DC generator this process is used to convert the induced AC in the conductors to a DC output. In DC motors commutation is used to reverse the directions of DC current before being applied to the coils of the motor

3. Differentiate 3-point starter with 4-point starter

Basis for Comparison	Three-point Starter	Four Point Starter
Definition	The starter which uses the three terminals for starting the motor.	The four-point starter uses the four terminals for acceleration of motor
Terminals	Armature terminal, field terminal and the Line terminal	Armature terminal, Field terminal, Line terminal and the additional terminal that directly connects the supply to the no voltage coil (NVC).
No Volts Coil (NVC)	Connects in series with the field coil.	Connects in parallel with the field coil

4. Define all day efficiency of a transformer

The ratio of output in kWh to the input in kWh of a transformer over a 24-hour period is known as all day efficiency.

$$\eta_{\text{all-day}} = \text{kWh output in 24 hours} / \text{kWh input in 24 hours}$$

All day efficiency is of special importance for those transformers whose primaries are never open-circuited but the secondaries carry little or no load much of the time during the day.

In the design of such transformers, efforts should be made to reduce the iron losses which continuously occur during the whole day.

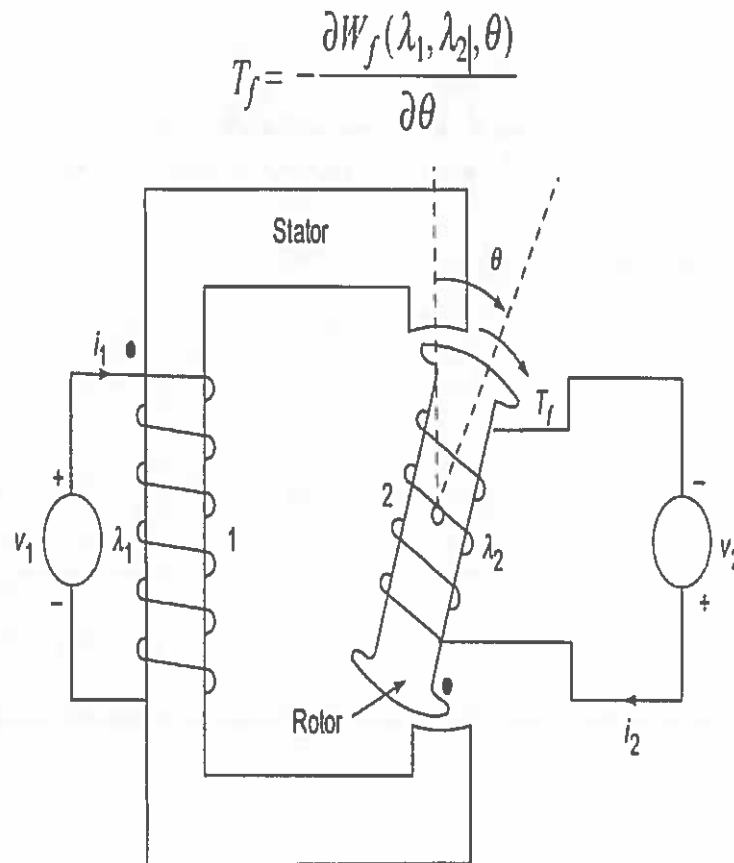
5. List any four conditions to connect transformer in parallel

For parallel connection of transformers, primary windings of the transformers are connected to source bus-bars and secondary windings are connected to the load bus-bars. Various conditions that must be fulfilled for the successful parallel operation of transformers are

- Same voltage ratio and turns ratio (both primary and secondary voltage rating is same) – Same percentage impedance and X/R ratio.
- Identical position of tap changer
- Same KVA ratings
- Same phase angle shift (vector group are same)
- Same frequency rating
- Same polarity
- Same phase sequence

6. Show that torque in doubly excited magnetic system is equal to the rate of increase of the field energy with respect to displacement at constant currents

Below Figure shows a magnetic field system with two electrical excitations—one on stator and the other on rotor. The system can be described in either of the two sets of three independent variables; $(\lambda_1, \lambda_2, q)$ or (i_1, i_2, q) . In terms of the first set



in terms of the first set

$$T_f = \frac{\partial W_f(\lambda_1, \lambda_2, \theta)}{\partial \theta} \quad \text{--- (1)}$$

where the field energy is given by

$$W_f(\lambda_1, \lambda_2, \theta) = \int_0^{\lambda_1} i_1 d\lambda_1 + \int_0^{\lambda_2} i_2 d\lambda_2 \quad \text{--- (2)}$$

we know that $i = \frac{\partial W_f(\lambda, \theta)}{\partial \lambda}$ Like this

$$i_1 = \frac{\partial W_f(\lambda_1, \lambda_2, \theta)}{\partial \lambda_1}$$

$$i_2 = \frac{\partial W_f(\lambda_1, \lambda_2, \theta)}{\partial \lambda_2}$$

Assuming linearity

$$\lambda_1 = L_{11}i_1 + L_{12}i_2$$

$$\lambda_2 = L_{21}i_1 + L_{22}i_2$$

$$(L_{12} = L_{21})$$

∴ The flux produced by C_1 links with its own coil C_1 & secondary coil C_2
 $\Phi_1 = L_{11}i_1 + M_{12}i_2$
 Similarly $\Phi_2 = L_{21}i_1 + M_{21}i_2$

solving for i_1 & i_2 in terms of λ_1, λ_2 and substituting in eqn (2)

$$W_f(\lambda_1, \lambda_2, \theta) = \frac{1}{2} \beta_{11} \lambda_1^2 + \beta_{12} \lambda_1 \lambda_2 + \frac{1}{2} \beta_{22} \lambda_2^2$$

where $\beta_{11} = \frac{L_{22}}{L_{11}L_{22} - L_{12}^2}$

$$\beta_{12} = \beta_{21} = \frac{-L_{12}}{(L_{11}L_{22} - L_{12}^2)}$$

$$\beta_{22} = \frac{L_{11}}{L_{11}L_{22} - L_{12}^2}$$

The self & mutual inductance of two exciting coils are functions of angle θ

If currents are used to describe the system state

$$T_f = \frac{\partial W_f}{\partial \theta}(i_1, i_2, \theta)$$

Coenergy $W_f'(i_1, i_2, \theta) = \int_0^{i_1} \lambda_1 di_1 + \int_0^{i_2} \lambda_2 di_2$

in linear case $W_f'(i_1, i_2, \theta) = \frac{1}{2} L_{11} i_1^2 + L_{12} i_1 i_2 + \frac{1}{2} L_{22} i_2^2$

where inductances are functions of angle θ

$$\begin{cases} i_1 = P_{11} \lambda_1 + P_{12} \lambda_2 \\ i_2 = P_{21} \lambda_1 + P_{22} \lambda_2; P_{21} = P_{12} \end{cases}$$

$$W_f(\lambda_1, \lambda_2, \theta) = \int_0^{\lambda_1} (P_{11} \lambda_1 + P_{12} \lambda_2) d\lambda_1 + \int_0^{\lambda_2} (P_{12} \lambda_1 + P_{22} \lambda_2) d\lambda_2$$

$$= P_{11} \int_0^{\lambda_1} \lambda_1 d\lambda_1 + P_{12} \left[\int_0^{\lambda_1} \lambda_2 d\lambda_1 + \int_0^{\lambda_2} \lambda_1 d\lambda_2 \right] + P_{22} \int_0^{\lambda_2} \lambda_2 d\lambda_2$$

$$= \left[P_{11} \int_0^{\lambda_1} \lambda_1 d\lambda_1 + P_{12} \int_0^{\lambda_1} \lambda_1 d\lambda_2 + P_{12} \int_0^{\lambda_2} \lambda_2 d\lambda_1 + P_{22} \int_0^{\lambda_2} \lambda_2 d\lambda_2 \right]$$

$$= \left[\frac{1}{2} P_{11} \lambda_1^2 + P_{12} \lambda_1 \lambda_2 + \frac{1}{2} P_{22} \lambda_2^2 \right]$$

7. Derive the expression of the flux, reluctance of the magnetic material with air gap

8(a). Derive the expression for the emf generated in the armature winding of a dc generator

Derivation for Induced EMF of One Armature Conductor

For one revolution of the conductor,

Let,

Φ = Flux produced by each pole in weber (Wb)

and

P = number of poles in the DC generator

therefore,

Total flux produced by all the poles = $\Phi * P$

And,

Time taken to complete one revolution = $60/N$

Where,

N = speed of the armature conductor in rpm.

Now, according to Faraday's law of induction, the induced emf of the armature conductor is denoted by "e" which is equal to rate of cutting the flux.

Therefore $e = d\Phi/dt$ and $e = \text{total flux/time take}$

Induced emf of one conductor is $e = (\Phi * P) / (60/N) = \Phi P N / 60$

Derivation for Induced EMF for DC Generator

Let us suppose there are Z total numbers of conductor in a generator, and arranged in such a manner that all parallel paths are always in series.

Here,

Z = total numbers of conductor

A = number of parallel paths

Then,

Z/A = number of conductors connected in series

We know that induced emf in each path is same across the line

Therefore,

Induced emf of DC generator

E = emf of one conductor $\times$ number of conductors connected in series.

Induced emf of DC generator is

$$e = \phi P \frac{N}{60} \times \frac{Z}{A} \text{ volts}$$

8(b). An 8-pole lap wound armature having 40 slots with 12 conductors/slot generates 500 v. Determine speed at which machine is running if the flux per pole is 50 mWb

$$P=8$$

Lap wound $A=P$

Slots 40

$$\text{Total conductors } Z = 12 \times 40 = 480$$

Flux $\Phi = 50 \text{ mWb}$

$$E_b = (\Phi P Z N) / (60 A)$$

$$N = (E_b 60 A) / (\Phi P Z)$$

$$= (500 \times 60 \times 8) / (50 \times 10^{-3} \times 8 \times 480)$$

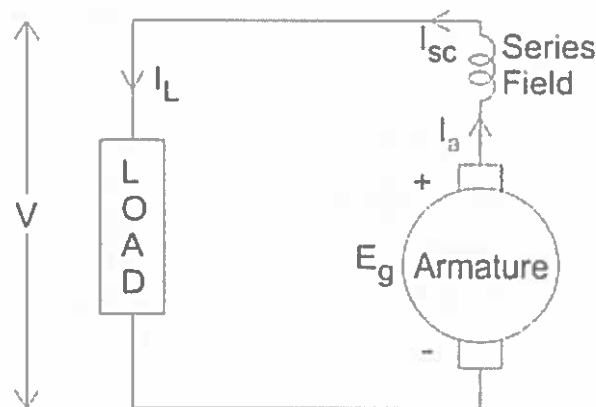
$$= 240000 / (192000 \times 10^{-3})$$

$$N = 1250 \text{ RPM}$$

9(a). Explain demagnetizing & cross magnetizing effects of armature reaction

9(b). Draw the internal and external characteristics of DC series generator in detail

In these types of generators, the field windings, armature windings and external load circuit all are connected in series as shown in figure below.



Series Wound Generator

Therefore, the same current flows through armature winding, field winding and the load.

$$\text{Let, } I = I_a = I_{sc} = I_L$$

Here, I_a = armature current

I_{sc} = series field current

I_L = load current

There are generally three most important characteristics of series wound DC generator which show the relation between various quantities such as series field current or excitation current, generated voltage, terminal voltage and load current.

Magnetic or Open Circuit Characteristic of Series-Wound DC Generator

The curve which shows the relation between no load voltage and the field excitation current is called magnetic or open circuit characteristic curve. As during no load, the load terminals are open circuited, there will be no field current in the field since, the armature, field and load are series connected and these three make a closed loop of circuit. So, this curve can be obtained practically by separating the field winding and exciting the DC generator by an external source.

Here in the diagram below AB curve is showing the magnetic characteristic of series wound DC generator. The linearity of the curve will continue till the saturation of the poles. After that there will be no further significant change of terminal voltage of DC generator for

increasing field current. Due to residual magnetism there will be a small initial voltage across the armature that is why the curve started from a point A which is a little way up to the origin O

Internal Characteristic of Series-Wound DC Generator

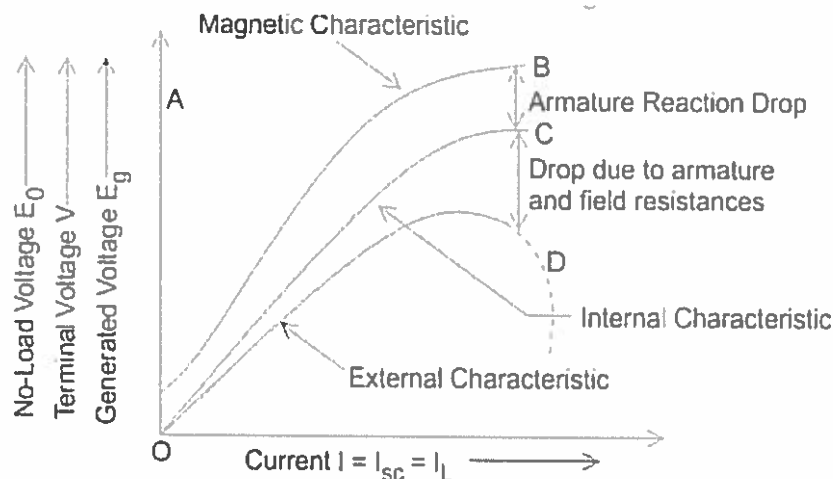
The internal characteristic curve gives the relation between voltage generated in the armature and the load current. This curve is obtained by subtracting the drop due to the demagnetizing effect of armature reaction from the no load voltage. So, the actual generated voltage (E_g) will be less than the no load voltage (E_0). That is why the curve is slightly dropping from the open circuit characteristic curve. Here in the diagram below OC curve is showing the internal characteristic or total characteristic of the series wound DC generator

Characteristic of Series-Wound DC Generator

The external characteristic curve shows the variation of terminal voltage (V) with the load current (I_L). Terminal voltage of this type of generator is obtained by subtracting the ohmic drop due to armature resistance (R_a) and series field resistance (R_{sc}) from the actually generated voltage (E_g).

$$\text{Terminal voltage } V = E_g - I(R_a + R_{sc})$$

The external characteristic curve lies below the internal characteristic curve because the value of terminal voltage is less than the generated voltage. Here in the figure OD curve is showing the external characteristic of the series wound DC generator.



Characteristic Curves of Series Wound DC Generators

10(a). Discuss the speed torque characteristics of DC shunt, series and compound motors

Speed-Torque Characteristics of DC Shunt Motor

This characteristic is also called as mechanical characteristic

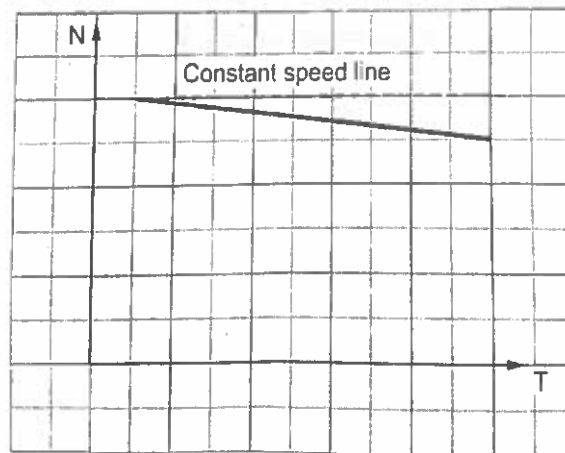
Torque is directly proportional to I_a armature current

Speed almost remains constant though torque changes from no load to full load conditions

$$\frac{0.7958 \phi N Z}{60 A} \times I_a = T_a \times \frac{2 \pi N}{60}$$

$$T_a = \frac{1}{2 \pi} \phi I_a \times \frac{P Z}{A} = 0.159 \phi I_a \times \frac{P Z}{A} \text{ Nm}$$

This is the torque equation of a d.c. motor.



Speed-Torque Characteristics Of Dc Series Motor

$$T \propto I_a^2 \text{ and } N \propto 1/I_a$$

Hence we can write $N \propto (1/\sqrt{T})$

Thus torque increases when load increases, the speed decreases

On no load, torque is very less and hence speed increases to dangerously high value

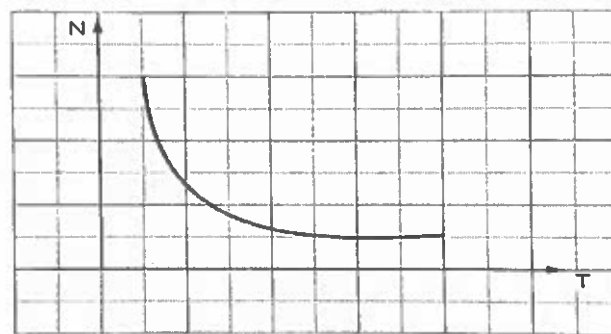
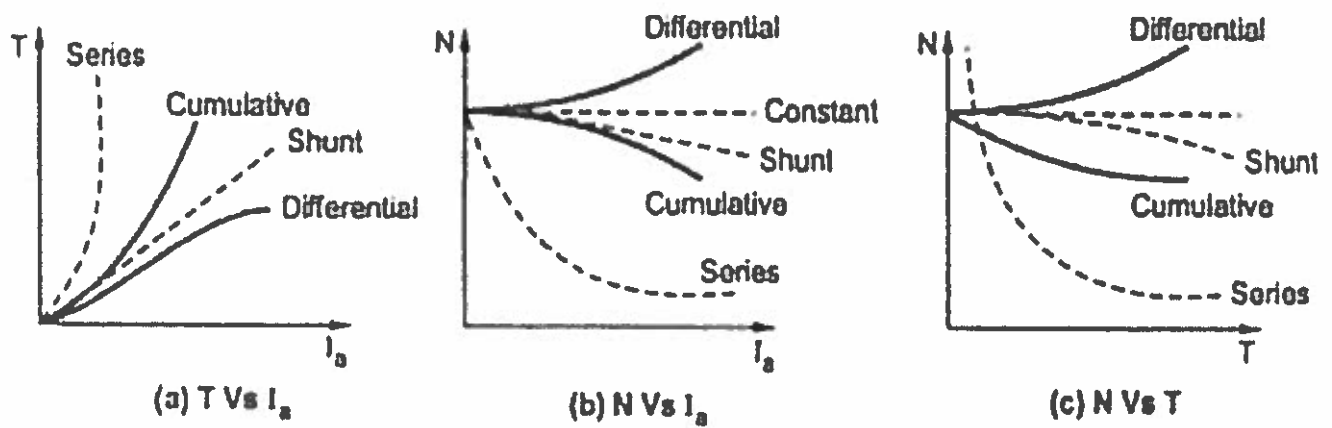


Fig. 2.40 N Vs T for series motor

Speed-Torque Characteristics Of Dc Compound Motor

- Compound motor characteristics basically depends on the fact whether the motor is cumulatively compound or differential compound.
- All the characteristics of the compound motor are the combination of the shunt and series characteristic.
- Cumulative compound motor is capable of developing large amount of torque at low speeds just like series motor.
- However it is not having a disadvantages of series motor even at light or no load.
- The shunt field winding produces the definite flux and series flux helps the shunt field flux to increase the total flux level
- So cumulative compound motor can run at reasonable speed and will not run with dangerously high speed like series motor, on light or no load condition.
- In differential compound motor, as two fluxes oppose each other, the resultant flux decreases as load increases, thus the machine runs at a higher speed with increase in the load
- This property is dangerous as on full load, the motor may try to run with dangerously high speed. So differential compound motor is generally not used in practice.
- The exact shape of these characteristics depends on the relative contribution of series and shunt field windings.
- If the shunt field winding is more dominant then the characteristics take the shape of the shunt motor characteristics.
- While if the series field winding is more dominant then the characteristics take the shape of the series characteristics.



11(a). Identify the role of starter in DC Motor operation. Also explain the working of three-point starter with neat diagram

Role of starter in DC Motor

Starter is not required to start the DC motor but it enables us to start the motor in a desired , safe way.

At the starting instant the speed of the motor is zero , ($N = 0$). As speed is zero, there cannot be any back emf as $E_b \propto N$ and N is zero at start.

E_b at start = 0

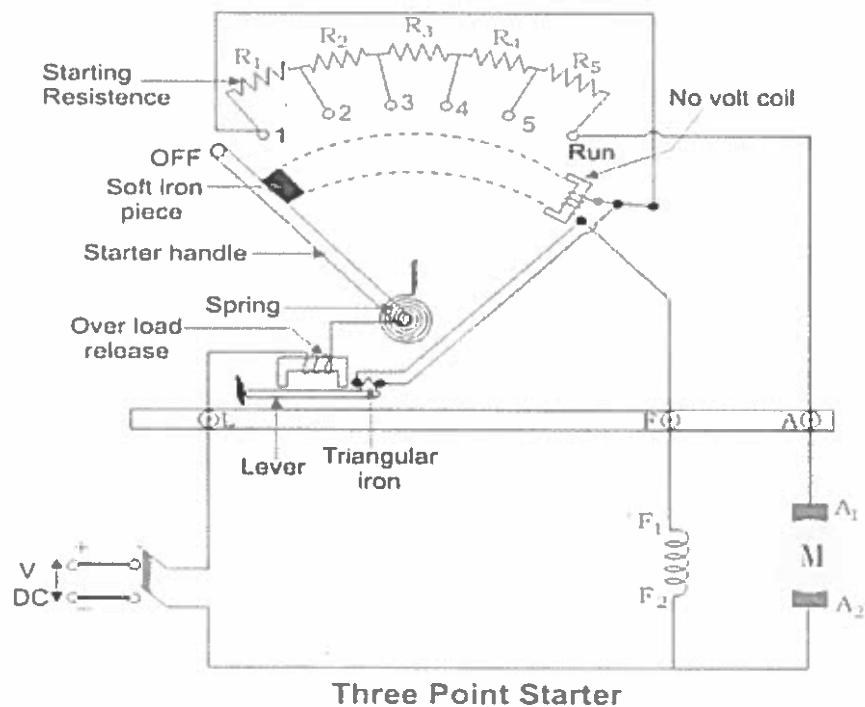
The voltage equation of DC motor $V = E_b + I_a R_a$

So at start , $V = I_a R_a$ as $E_b = 0$

$I_a = V / R_a$

Generally motor is switched on with normal voltage and as armature resistance is very small , the armature current at start is very high.

3-POINT STARTER



Working of Three Point Starter

- let us now go into the **working of the 3 point starter**. To start with the handle is in the OFF position when the supply to the DC motor is switched on.
- Then handle is slowly moved against the spring force to make contact with stud No. 1.
- At this point, field winding of the shunt or the compound motor gets supply through the parallel path provided to starting the resistance, through No Voltage Coil.
- While entire starting resistance comes in series with the armature. The high starting armature current thus gets limited as the current equation at this stage becomes
- As the handle is moved further, it goes on making contact with studs 2, 3, 4, etc., thus gradually cutting off the series resistance from the armature circuit as the motor gathers speed.
- Finally, when the starter handle is in 'RUN' position, the entire starting resistance is eliminated, and the motor runs with normal speed.
- This is because back emf is developed consequently with speed to counter the supply voltage and reduce the armature current.
- So the external electrical resistance is not required anymore and is removed for optimum operation.
- The handle is moved manually from OFF to the RUN position with the development of speed.
- Now the obvious question is once the handle is taken to the RUN position how it is supposed to stay there, as long as the motor is running.

Working of No Voltage Coil of 3 Point Starter

- The supply to the field winding is derived through no voltage coil. So when field current flows, the NVC is magnetized.
- Now when the handle is in the 'RUN' position, a soft iron piece is connected to the handle and gets attracted by the magnetic force produced by NVC, because of flow of current through it.
- The NVC is designed in such a way that it holds the handle in 'RUN' position against the force of the spring as long as supply is given to the motor.
- Thus NVC holds the handle in the 'RUN' position and hence also called **hold on coil**.
- Now when there is any kind of supply failure, the current flow through NVC is affected and it immediately loses its magnetic property and is unable to keep the soft iron piece on the handle, attracted.
- At this point under the action of the spring force, the handle comes back to OFF position, opening the circuit and thus switching off the motor.
- So due to the combination of NVC and the spring, the starter handle always comes back to OFF position whenever there is any supply problem.
- Thus it also acts as a protective device safeguarding the motor from any kind of abnormality.

12(a) Derive the EMF equation of transformer

Let,

N_1 = Number of turns in primary winding

N_2 = Number of turns in secondary winding

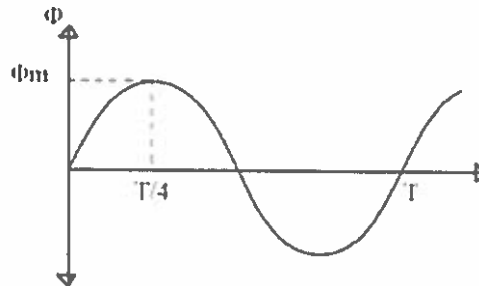
Φ_m = Maximum flux in the core (in Wb) = $(B_m \times A)$

f = frequency of the AC supply (in Hz)

As, shown in the fig., the flux rises sinusoidally to its maximum value Φ_m from 0. It reaches to the maximum value in one quarter of the cycle i.e in $T/4$ sec (where, T is time period of the sin wave of the supply = $1/f$).

Therefore,

average rate of change of flux = $\Phi_m / (T/4) = \Phi_m / (1/4f)$



Therefore,

average rate of change of flux = $4f \Phi_m$ (Wb/s).

Now,

Induced emf per turn = rate of change of flux per turn

Therefore, average emf per turn = $4f \Phi_m$ (Volts).

Now, we know, Form factor = RMS value / average value

Therefore, RMS value of emf per turn = Form factor X average emf per turn.

As, the flux Φ varies sinusoidally, form factor of a sine wave is 1.11

Therefore, RMS value of emf per turn = $1.11 \times 4f \Phi_m = 4.44f \Phi_m$.

RMS value of induced emf in whole primary winding (E_1) = RMS value of emf per turn X Number of turns in primary winding

$$E_1 = 4.44f N_1 \Phi_m \quad \text{..... eq 1}$$

Similarly, RMS induced emf in secondary winding (E_2) can be given as

$$E_2 = 4.44f N_2 \Phi_m. \quad \text{..... eq 2}$$

from the above equations 1 and 2

$$\frac{E_1}{N_1} = \frac{E_2}{N_2} = 4.44f \Phi_m$$

This is called the emf equation of transformer, which shows, emf / number of turns is same for both primary and secondary winding.

12(b). The voltage per turn of a single phase transformer is 1.1 V. when primary winding is connected to a 220 V, 50 Hz A.C. Supply. the secondary voltage is found to be 550 V. find i) primary and secondary turns. ii) core area if the maximum flux density is 1.1 Wb/m^2

$$\text{EMF per Turn} = 1.1 \text{ V}$$

primary induced voltage

$$E_1 = V_1 = 220 \text{ V}$$

secondary induced emf

$$E_2 = V_2 = 550 \text{ V}$$

(1) Number of primary Turns

$$N_1 = \frac{E_1}{\text{EMF per Turn}} = \frac{220}{1.1} = 200$$

$$N_2 = \frac{E_2}{\text{EMF per Turn}} = \frac{550}{1.1} = 500$$

maximum value of flux

$$\phi_{\max} = \frac{\text{EMF per Turn}}{4.44 f}$$

$$= \frac{1.1}{4.44 \times 50} = 0.00495 \text{ wb}$$

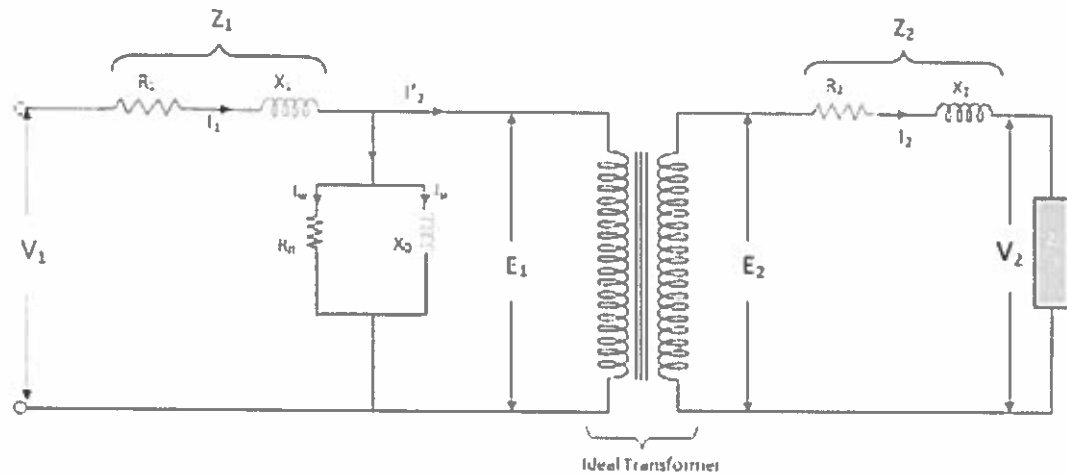
Net cross sectional area

$$\text{of core} \Rightarrow a = \frac{\phi_{\max}}{B_{\max}}$$

$$= \frac{0.00495 \text{ wb}}{1.1 \text{ wb/m}^2} = 0.0045 \text{ m}^2$$

13(a) Draw and explain the equivalent circuit diagram of single-phase transformer

Equivalent circuit diagram of a transformer is basically a diagram which can be resolved into an equivalent circuit in which the resistance and leakage reactance of the transformer are imagined to be external to the winding. The equivalent circuit diagram of transformer is given below



****Equivalent Circuit diagram of Transformer****

Where,

R_1 = Primary Winding Resistance.

R_2 = Secondary winding Resistance.

I_0 = No-load current.

I_μ = Magnetizing Component,

I_w = Working Component,

This I_μ & I_w are connected in parallel across the primary circuit.

The value of E_1 (Primary e.m.f) is obtained by subtracting vectorially $I_1 Z_1$ from V_1 .

The value of $X_0 = E_1 / I_0$ and

$R_0 = E_1 / I_w$.

We know that the relation of E_1 and E_2 is $E_2 / E_1 = N_2 / N_1 = K$, (transformation Ratio)

From the equivalent circuit, we can easily calculate the total impedance of to transfer voltage, current, and impedance either to the primary or the secondary.

The secondary circuit is shown in fig-1. and its equivalent primary value is shown in fig- 2,

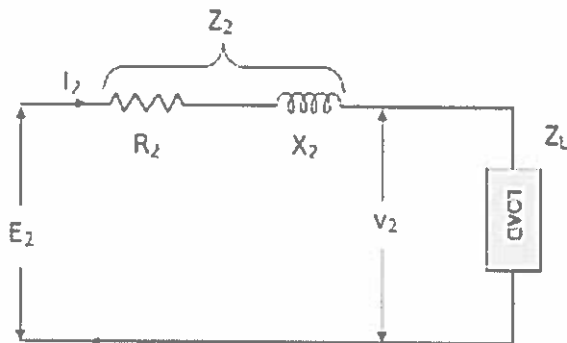


FIG.1

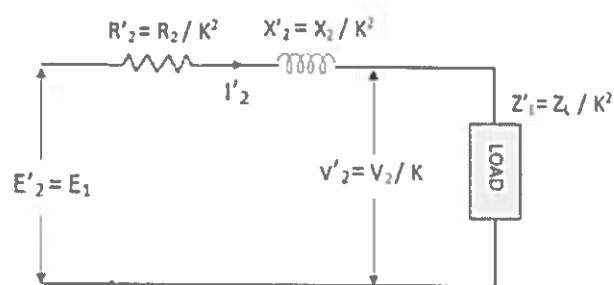
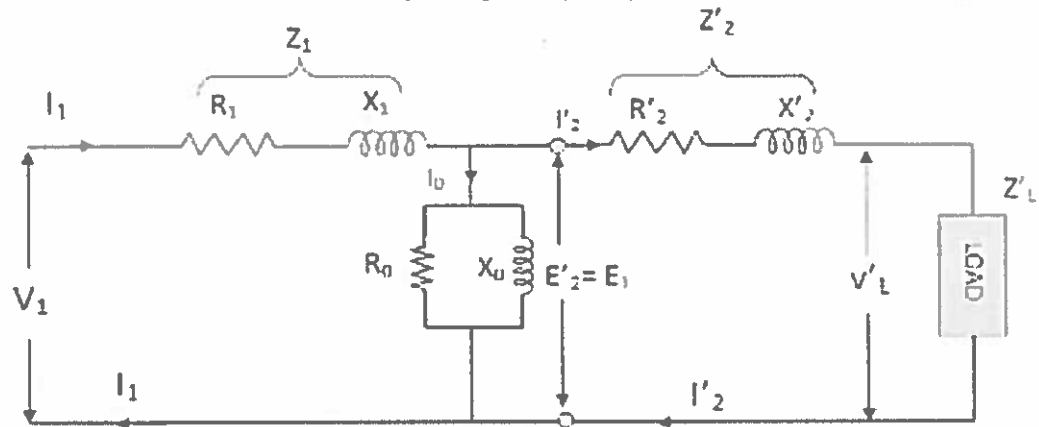
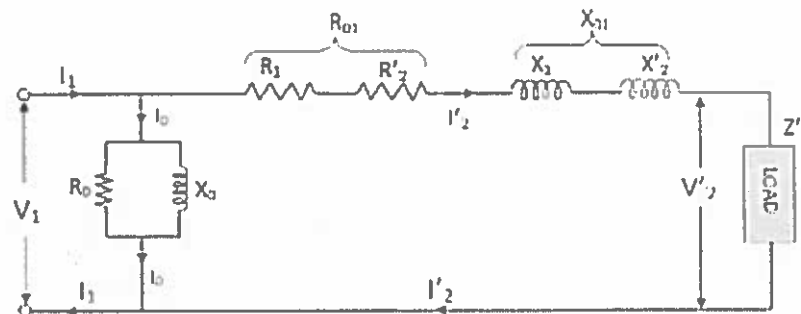


FIG.2

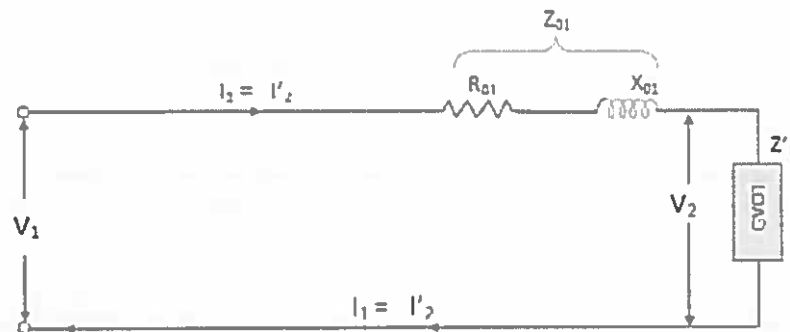
The total equivalent circuit of the transformer is obtained by adding in the primary impedance as shown in – Fig-3



And It can be simplified the terminals shown in fig – 4 & further simplify the equivalent circuit is shown in fig.- 5



At last, the circuit is simplified by omitting I_0 altogether as shown in fig- 5



13(b). Compare features of shell type and core type transformers

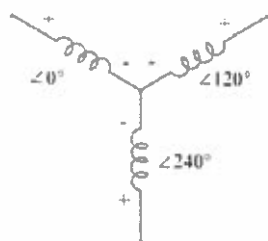
Core Type Transformer	Shell Type Transformer
In Core type transformer, the core consists of two limbs are two yoke.	In Shell type transformer, the core have three limbs and two yokes.
Total magnetic flux Φ flows through the entire core. This means that there exists a single magnetic	Half of the total flux Φ flows through the yoke and outer limbs i.e. there are two magnetic circuits.

circuit.	Total flux Φ only flows in the central limb.
The HV and LV winding in Core type transformer are concentric. LV winding is placed on the core. This LV winding is then surrounded by HV winding.	HV and LV winding in shell type transformer is interleaved or sandwiched on the central limb.
This type of transformer is used for high voltage and high power applications.	It is suitable for low voltage and low power applications.
The conductor material requirement for winding is more as compared to shell type transformer.	The conductor material requirement is less
Iron requirement for core construction is less.	The iron requirement for core construction is more as compared to core type transformer.
<u>Core loss</u> is more due to flow of total flux through the entire core.	Core loss is less as compared to shell type transformer. This is due to the fact that only half of the total flux flows in the core.
More copper conductor requirement leads to more <u>ohmic loss</u> .	Ohmic loss is less as conductor requirement is less
Total loss in core type transformer is thus more than the shell type transformer.	Total losses (Core+Ohmic loss) is less.
The overall efficiency is less due to more losses.	The overall efficiency of shell type transformer is more than the core type transformer.

14. Draw the various types of three phase transformer connections and brief each one of them

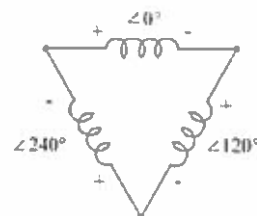
Three phase transformer connections In three phase system, the three phases can be connected in either star or delta configuration. In case you are not familiar with those configurations, study the following image which explains star and delta configuration. In any of these configurations, there will be a phase difference of 120° between any two phases.

Star



Center point must tie either all '-' or all '+' ends together.

Delta



The winding polarities must stack together in a complementary manner (+ to -)

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Three Phase Transformer Connections

Windings of a three phase transformer can be connected in various configurations as (i) star-star, (ii) delta-delta, (iii) star-delta, (iv) delta-star, (v) open delta and (vi) Scott connection. These configurations are explained below.

Star-Star (Y-Y)

- Star-star connection is generally used for small, high-voltage transformers. Because of star connection, number of required turns/phase is reduced (as phase voltage in star connection is $1/\sqrt{3}$ times of line voltage only). Thus, the amount of insulation required is also reduced.
- The ratio of line voltages on the primary side and the secondary side is equal to the transformation ratio of the transformers.
- Line voltages on both sides are in phase with each other.
- This connection can be used only if the connected load is balanced.

Delta-Delta (Δ - Δ)

- This connection is generally used for large, low-voltage transformers. Number of required phase/turns is relatively greater than that for star-star connection.
- The ratio of line voltages on the primary and the secondary side is equal to the transformation ratio of the transformers.
- This connection can be used even for unbalanced loading.
- Another advantage of this type of connection is that even if one transformer is disabled, system can continue to operate in open delta connection but with reduced available capacity.

Star-Delta OR Wye-Delta (Y- Δ)

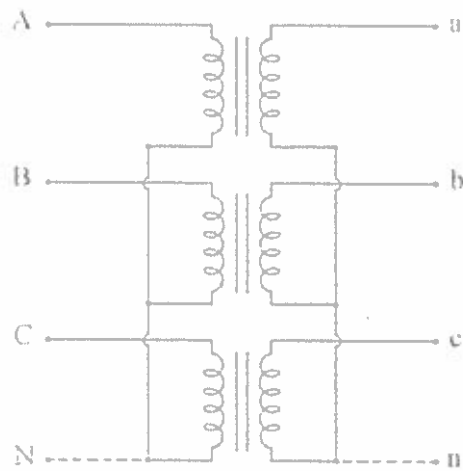
- The primary winding is star star (Y) connected with grounded neutral and the secondary winding is delta connected.
- This connection is mainly used in step down transformer at the substation end of the transmission line.
- The ratio of secondary to primary line voltage is $1/\sqrt{3}$ times the transformation ratio.
- There is 30° shift between the primary and secondary line voltages.

Delta-Star OR Delta-Wye (Δ -Y)

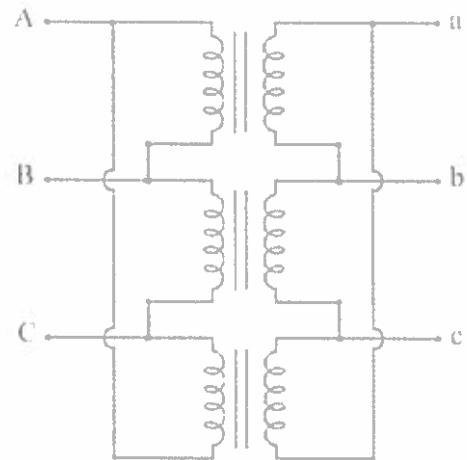
- The primary winding is connected in delta and the secondary winding is connected in star with neutral grounded. Thus it can be used to provide 3-phase 4-wire service.
- This type of connection is mainly used in step-up transformer at the beginning of transmission line.

- The ratio of secondary to primary line voltage is $\sqrt{3}$ times the transformation ratio.
- There is 30° shift between the primary and secondary line voltages.

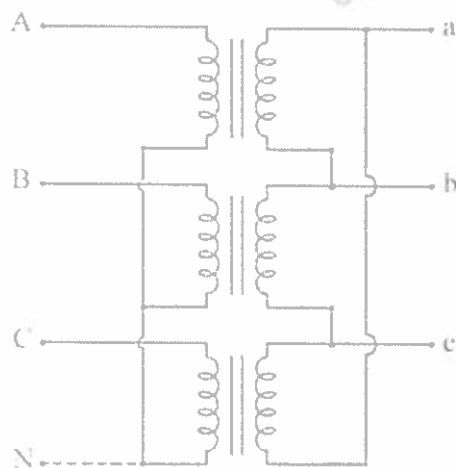
Above transformer connection configurations are shown in the following figure.



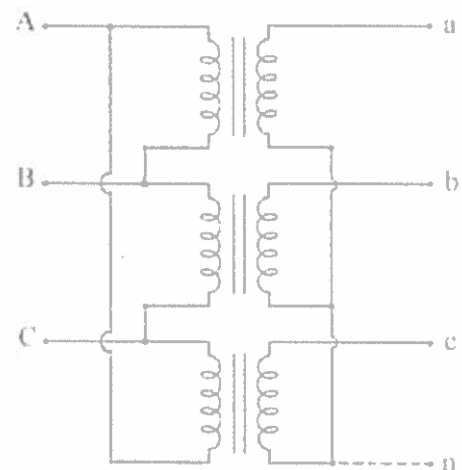
Y - Y



Δ - Δ



Y - Δ

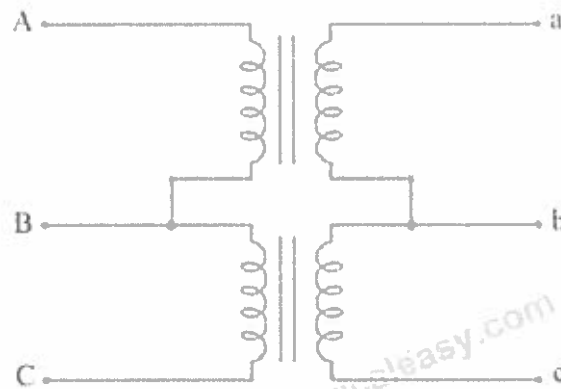


Δ - Y

Open Delta (V-V) Connection

Two transformers are used and primary and secondary connections are made as shown in the figure below. Open delta connection can be used when one of the transformers in Δ - Δ bank is disabled and the service is to be continued until the faulty transformer is repaired or replaced. It can also be used for small three phase

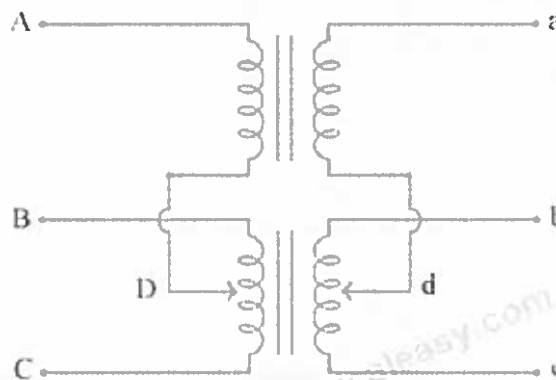
loads where installation of full three transformer bank is un-necessary. The total load carrying capacity of open delta connection is 57.7% than that would be for delta-delta connection.



Open Delta (V-V) Connection

Scott (T-T) Connection

Two transformers are used in this type of connection. One of the transformers has centre taps on both primary and secondary windings (which is called as main transformer). The other transformer is called as teaser transformer. Scott connection can also be used for three phase to two phase conversion. The connection is made as shown in the figure below.



Scott (T-T) Connection

15.A 10 KVA, 500/250 V, 50HZ Single-phase transformer gave the following test

Data: OC Test (LV Side):250 V, 1.0 A, 80 W,

SC Test (HV Side) : 25 V,12 A,100 W, Where LV refers to the low voltage and HV refers to high voltage side. Determine the following

- (i) Equivalent circuit referred to LV Side
- (ii) Secondary load voltage at 0.8 pf lagging with full-load current

Transformer rating = 10 kVA;

$E_1=500$ V; $E_2=250$ V

As per data, open circuit test is performed on LV side and short circuit test is performed on HV side.

Open circuit test (LV side); $V_2=250$ V; $I_0=1.0$ A; $W_0=80$ W

Short circuit test (HV side); $V_1(sc)=25$ V; $I_1(sc)=12$ A; $W_c=100$ W

(a) Open circuit test performed on LV side given;

$$I_0=1.0 \text{ A}; I_w=W_0/V_2=80/250=0.32 \text{ A}$$

$$I_{mag} = \sqrt{(I_0^2 - I_w^2)} = \sqrt{(1.0)^2 - (0.32)^2} = 0.9474 \text{ A}$$

Exciting resistance, $R_0 = V_2/I_w = 250/0.32 = 781.25 \Omega$

Exciting reactance, $X_0 = V_2/I_{mag} = 250/0.9474 = 263.88 \Omega$

Short circuit test performed on HV side gives;

$$Z_{ep} = V_1(sc)/I_1(sc) = 25/12 = 2.083 \Omega$$

$$R_{ep} = W_c / (I_1(sc))^2 = 100/(12)^2 = 0.694 \Omega$$

$$X_{ep} = \sqrt{(Z_{ep}^2 - R_{ep}^2)} = \sqrt{(2.083)^2 - (0.694)^2} = 1.961 \Omega$$

Transformation ratio, $K = E_2/E_1 = 250/500 = 0.5$

Parameters, when referred to LV side;

$$R_{es} = K^2 * R_{ep} = (0.5)^2 * 0.694 = 0.1735 \Omega$$

$$X_{es} = K^2 * X_{ep} = (0.5)^2 * 1.961 = 0.490 \Omega$$

(II) When p.f. $\cos\phi=0.8$ lagging; $\sin\phi=\sin\cos^{-1}0.8=0.6$

$$V_2 = E_2 - I_2 R_{es} \cos\phi - I_2 X_{es} \sin\phi$$

$$I_{2(n)} = (10 * 1000) / 250 = 40 \text{ A}$$

$$V_2 = 250 - (40 * 0.1735 * 0.8) - (40 * 0.490 * 0.6)$$

$$= 250 - 5.552 - 23.1432$$

$$= 221.3048 \text{ V}$$

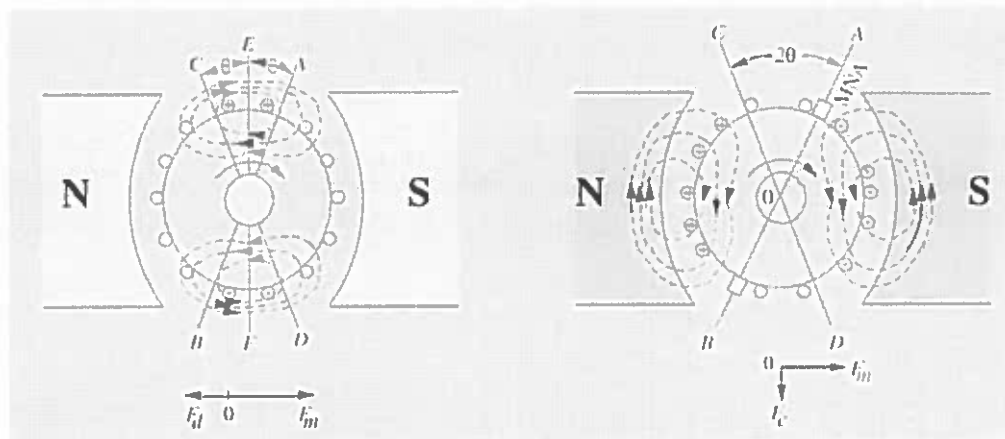
9(a). Explain demagnetizing & cross magnetizing effects of armature reaction

When the field winding of a dc generator is excited, a flux with uniform distribution in the air gap of the dc generator is produced. Once the generator is loaded, load current flows through the armature conductors and set up its own flux. This armature flux will be always in the 180° phase apart from the field flux and distorts or opposes the uniformly distributed main field flux

The opposition of armature flux on the main field flux is called armature reaction. This armature reaction has two undesirable effects, either it weakens (reduces) the main field flux called 'Demagnetizing Effect' or it distorts the main field flux called 'Cross-magnetizing Effect'.

Phenomenon of Demagnetization and Cross-magnetisation

Assume, for example, the brushes are lying at the new MNA position (to avoid sparking during commutation). Let θ be the angle between GNA and MNA. The armature MMF will now lie along with the new position of the brush axis as shown in the below diagram.

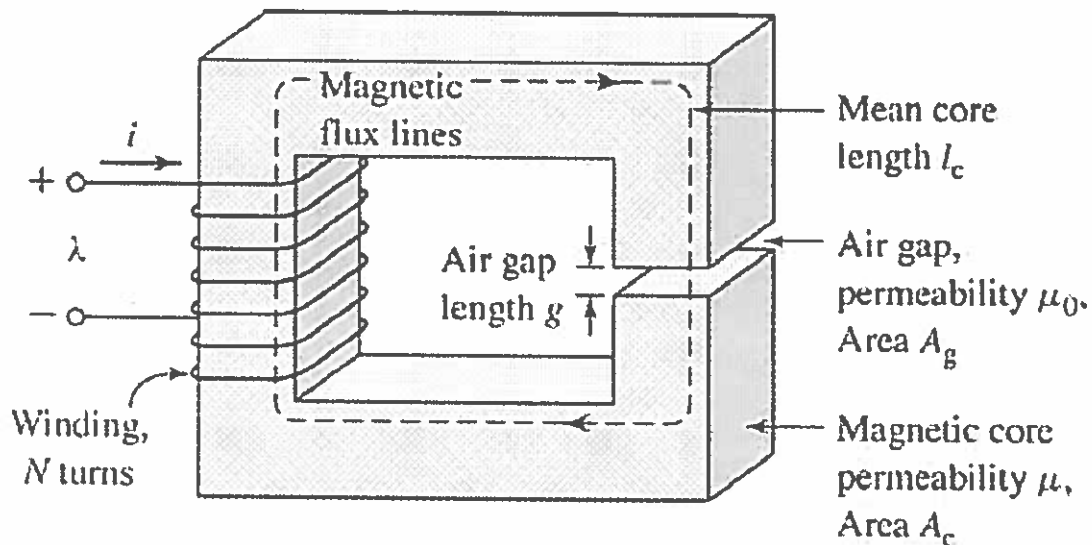


Suppose the field winding flux is in the direction from N to S i.e., top to bottom. We can see that the current carrying armature conductors in the region AOC and BOD are in the opposite direction to the field winding flux. Thus armature field F_a by the conductors in the region AOC ($\theta + \theta = 2\theta$) and BOD ($\theta + \theta = 2\theta$) exerts demagnetising influence on the main field F_f (reduces field strength) and called demagnetising armature conductors or weakening component of the armature reaction.

Similarly, the remaining armature conductors i.e., in the region AOD and BOC, produce a flux F_a in such a direction that right angle to the field flux F_f as shown above. Thus component $O F_a$ is at right angles to the vector $O F_f$ as shown in the vector representation. These armature conductors (in the region AOD and BOC) produce a distorted effect on flux distribution in the air gap produced by the main field. This distortion effect by the armature conductors is known as cross-magnetizing armature conductors or a distorting component of the armature reaction.

7. Derive the expression of the flux, reluctance of the magnetic material with air gap

Consider a magnetic circuit with an air gap as below. Let's discuss different reasons why air gaps are crucial in practical applications. In this circuit, there is only one path for the magnetic circuit and thus it can be called as a series magnetic circuit.



One of the main reasons for an Air Gap is to increase the reluctance of the magnetic circuit. The amount of air or another non-magnetic material like a fibre plate or fibre board increases the reluctance of the circuit, thereby increasing the amount of current that we could put in a coil before we reach saturation. Also, the air gaps help the magnetic flux to expand outside the magnetic circuit. This flux fringes out into the neighbouring air path and such paths for flux are called flux fringing resulting in a non-uniform flux density in the air gap. As we increase the air gap, it increases flux fringing and vice versa. With small air gaps, fringing can be neglected unless otherwise noted. However, with a larger air gap we might have to take that change in area into consideration when we start doing flux density calculation.

How to Calculate Flux Density in an Air Gap?

Let's look at the basic formula to calculate flux density.

$$\text{Flux density, } B_g = \Phi_g / A_g$$

where, B is magnetic flux density in Teslas (T)

Φ is magnetic flux in Webers (Wb)

A is area in square meters (m²)

As we know that permeability of any material can be stated as the ratio of the magnetic flux density to the magnetic field intensity of a material. It can be expressed as below,

$$\mu = B / H$$

where, μ is permeability of a material, in henrys/meter

H is magnetic field strength, in ampere-turns/meter

B is flux density, in teslas

Solving the above equation for B, we get

$$B = \mu \times H \quad \dots\dots \text{(Equation 1)}$$

Now, magnetizing force of the air gap is determined as below,

$$H = F_m / l \quad \dots\dots \text{(Equation 2)}$$

where, F_m is Magnetomotive force (mmf), in ampere-turns, l is length of material (gap), in meters

Substituting the value of H from Equation 2 into Equation 1, we get

$$B = \mu \times (F_m / l)$$

Solving the above equation for magnetomotive force, we get

$$F_m = Bl / \mu \quad \dots\dots \text{(Equation 3)}$$

Let's assume that we have an air gap, and the permeability of air (free space) is a constant

$$\mu_{air} = 4\pi \times 10^{-7} \text{ H/m}$$

Replacing the value for μ in Equation 3, we come up with a very basic formula for determining the magnetomotive force required to set up a particular flux in an air gap.

$$F_m = BL / (4\pi \times 10^{-7})$$

What is Reluctance?

Magnetic reluctance (also known as reluctance, magnetic resistance, or a magnetic insulator) is defined as the opposition offered by a magnetic circuit to the production of magnetic flux. It is the property of the material that opposes the creation of magnetic flux in a magnetic circuit.

In an electrical circuit, the resistance opposes the flow of current in the circuit and it dissipates the electric energy. The magnetic reluctance in a magnetic circuit is analogous to the resistance in an electric circuit as it opposes the production of magnetic flux in a magnetic circuit but it does not give rise to the dissipation of energy rather it stores magnetic energy.

Reluctance is directly proportional to the length of the magnetic circuit and inversely proportional to the area of the cross-section of the magnetic path. It is a scalar quantity and denoted by S . Note that a scalar quantity is one which is fully described by a magnitude (or numerical value) only. No direction is required to define the scalar quantity.

Mathematically it can be expressed as

$$S = \frac{l}{\mu_0 \mu_r A}$$

where, l = length of the magnetic path in meters

μ_0 = permeability of free space (vacuum) = $4\pi \times 10^{-7}$ Henry/meter

μ_r = relative permeability of a magnetic material

A = Cross sectional area in square meters (m^2)

In AC as well as DC magnetic fields, the reluctance is the ratio of the magnetomotive force (m.m.f) to the magnetic flux in a magnetic circuit. In a pulsating AC or DC field, the reluctance is also pulsating.


Prepared By

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HOD-EEE

Dr.R.S.R.Krishnam Naidu

Scheme of Evaluation (DCMT)

- ① any two features of multiply excited magnetic field system (2M)
- ② Definition of commutation in DC machine (2M)
- ③ Differences between 3-point & 4-point starter (2M)
- ④ Definition of all day efficiency (2M)
- ⑤ four condition of parallel operation of Transformers (2M)
- ⑥ diagram (4M)
derivation (8M)
- ⑦ derivation of flux (5M)
diagram (2M)
derivation of reluctance (5M)
- ⑧ a) Derivation of generator (6M)
$$E = \frac{\Phi P Z N}{60A}$$
- 8 ⑧ b) numerical solving (4M)
Correct answer (2M)

(9) (a) Demagnetizing effect (4M)
Diagram (2M)

Cross magnetizing effect (4M)
Diagram (2M)

(9) (b) Diagram (2M)

Explanation of internal & external characteristics (4M)

(10) (a) speed-torque characteristics of DC series motor
shunt motor (2M)

Diagram (2M)

series motor (2M)

Diagram (2M)

Compound motor (2M)

Diagram (2M)

(11) (a) Need of starter (3M)

3-point starter diagram (3M)

Boosting (6M)

(12) (a) emf equation of Transformer
derivation (12M)

$$E = 4.44 f \phi_m N$$

(12) (b) Numerical methodology (4M)
Correct answer (2M)

(13) a) equivalent circuit of Transformer
diagrams (2M)
parameters solving (4M)

(13) b) comparison b/w core type, shell type
transformers (6M)

(14) Three phase Transformer Connections

$\lambda - \Delta$ - 2M

$\lambda - \lambda$ - 2M

$\Delta - \lambda$ - 2M

$\Delta - \Delta$ - 2M

Open - Δ - 2M

SCOTT-T - 2M

(15) numerical
Methodology - 6M

(i) correct answer equivalent circuit referred
to LV side (2M)

Methodology - 4M

(ii) Secondary load voltage at 0.8 pf
lagging with full load current

correct answer (2M)

Methodology - (4M)

Anurag

Pooja
23/11/22
HOD-EEE

Semester End Examination, January/February, 2022

Degree	B. Tech. (U. G.)	Program	ME	Academic Year	2021 - 2022
Course Code	20ME301	Test Duration	3 Hrs. Max. Marks 70	Semester	III
Course	Thermodynamics				

Part A (Short Answer Questions 5 x 2 = 10 Marks)

No.	Questions (1 through 5)	Learning Outcome (s)	DoK
1	What is Quasi static process?	20ME301.1	L1
2	What is a steady flow process and unsteady flow process?	20ME301.2	L2
3	What is PMM of Second kind? Why it is impossible?	20ME301.3	L2
4	What is a pure substance?	20ME301.4	L1
5	What is Specific humidity and relative humidity?	20ME301.5	L1

Part B (Long Answer Questions 5 x 12 = 60 Marks)

No.	Questions (6 through 15)	Marks	Learning Outcome (s)	DoK
6 (a)	A temperature scale of certain thermometer is given by the relation $t = a \ln p + b$ where a and b are constants and p is the thermometric property of the fluid in the thermometer. If at the ice point and steam point the thermometric properties are found to be 1.5 and 7.5 respectively what will be the temperature corresponding to the thermometric property of 3.5 on Celsius scale	6M	20ME301.1	L3
6 (b)	Explain the working of constant volume gas thermometer	6M	20ME301.1	L2
OR				
7 (a)	A piston and cylinder machine containing a fluid system has a stirring device. The piston is frictionless, and it is held down against the fluid due to atmospheric pressure of 101.3 kPa. The stirring device is turned 9500 revolutions with an average torque against the fluid of 1.25 Nm. Meanwhile the piston of 0.65 m diameter moves out 0.6 m. Find the net work transfer for the system.	5M	20ME301.1	L3
7 (b)	Example 2.20. To a closed system 150 kJ of work is supplied. If the initial volume is 0.6 m ³ and pressure of the system changes as $p = 8 - 4V$, where p is in bar and V is in m ³ , determine the final volume and pressure of the system	7M	20ME301.1	L3
8 (a)	A system receives 50 kJ of heat while expanding with volume change of 0.14 m ³ against an atmosphere of 1.2×10^5 N/m ² . A mass of 90 kg in the surroundings is also lifted through a distance of 5.5 metres. (i) Find the change in energy of the system. (ii) The system is returned to its initial volume by an adiabatic process which requires 110 kJ of work. Find the change in energy of the system. (iii) For the combined processes of (i) and (ii) determine the change in energy of the system.	6M	20ME301.2	L3
8 (b)	Explain first law of thermodynamics applied to a process	6M	20ME301.2	L2
OR				
9 (a)	A cylinder contains 0.45 m ³ of a gas at 1×10^5 N/m ² and 80°C. The gas is compressed to a volume of 0.13 m ³ , the final pressure being 5×10^5 N/m ² . Determine : (i) The mass of gas ; (ii) The value of index 'n' for compression ; (iii) The increase in internal energy of the gas ; (iv) The heat received or rejected by the gas during compression. Take $\gamma = 1.4$, $R = 294.2$ J/kg°C	6M	20ME301.2	L3
9 (b)	Explain the steady flow energy equation applied to Steam of Gas turbine with the help of a neat diagram	6M	20ME301.2	L2

10 (a)	An ideal gas is heated from temperature T_1 to T_2 by keeping its volume constant. The gas is expanded back to its initial temperature according to the law $p v^n = \text{constant}$. If the entropy change in the two processes are equal, find the value of n in terms of the adiabatic index γ .	6M	20ME301.3	L2
10 (b)	Discuss about Carnot theorem with neat diagram	6M	20ME301.3	L2
OR				
11 (a)	A reversible heat pump is used to maintain a temperature of 0°C in a refrigerator when it rejects the heat to the surroundings at 25°C . If the heat removal rate from the refrigerator is 1440 kJ/min , determine the C.O.P. of the machine and work input required.	6M	20ME301.3	L3
11 (b)	Explain about heat engine and heat pump	6M	20ME301.3	L2
OR				
12 (a)	Two boilers one with superheater and other without superheater are delivering equal quantities of steam into a common main. The pressure in the boilers and main is 20 bar. The temperature of steam from a boiler with a superheater is 350°C and temperature of the steam in the main is 250°C . Determine the quality of steam supplied by the other boiler. Take $c_{ps} = 2.25 \text{ kJ/kg}$.	6M	20ME301.4	L3
12 (b)	Steam at 140 bar has an enthalpy of 3001.9 kJ/kg , find the temperature, the specific volume and the internal energy.	6M	20ME301.4	L2
OR				
13 (a)	1000 kg of steam at a pressure of 16 bar and 0.9 dry is generated by a boiler per hour. The steam passes through a superheater via boiler stop valve where its temperature is raised to 380°C . If the temperature of feed water is 30°C , determine : (i) The total heat supplied to feed water per hour to produce wet steam. (ii) The total heat absorbed per hour in the superheater. Take specific heat for superheated steam as 2.2 kJ/kg K	7M	20ME301.4	L3
13 (b)	Find the specific volume, enthalpy and internal energy of wet steam at 18 bar, dryness fraction 0.85	5M	20ME301.4	L2
14 (a)	Derive the expressions for the internal energy and specific heats for mixtures of ideal gases.	6M	20ME301.5	L2
14 (b)	On a particular day the weather forecast states that the dry bulb temperature is 37°C , while the relative humidity is 50% and the barometric pressure is 101.325 kPa . Find the humidity ratio, dew point temperature and enthalpy of moist air on this day.	6M	20ME301.5	L3
OR				
15 (a)	Explain the following: i) Heating and dehumidification ii) Cooling and dehumidification	8M	20ME301.5	L2
15 (b)	What do you understand by dry bulb temperature and wet bulb temperatures? When do the DBT, WBT and DPT become equal?	4M	20ME301.5	L2



N S RAJU INSTITUTE OF TECHNOLOGY

(AUTONOMOUS)

SONTYAM, ANANDAPURAM, VISAKHAPATNAM – 531 173

THERMODYNAMICS

CODE: RA 17E302

ANSWER KEY AND SCHEME OF EVALUATION

1a) **quasi-static process** also known as a **quasi-equilibrium process**; is a thermodynamic process that happens slowly enough for the system to remain in internal physical thermodynamic equilibrium. (2n)

2a) Steady-state and unsteady-state processes describe the time interval that a process occurs over. Steady-state refers to the time where the **variable of interest doesn't change**. Unsteady-state is when the variable of interest **changes over time**. While all processes have a steady-state and transient phase, it is more beneficial to evaluate certain processes with steady-state or transience (2n)

3a) Perpetual motion machine of the second kind is a machine which produces work from a single heat source. In this kind of perpetual motion machine, there is only one heat reservoir, and it is being spontaneously cooled generating work without heat transfer to a cooler reservoir. This kind of machine is impossible, since it violates the Second law of thermodynamics (2n)

4a) Pure substances are substances that are made up of only one kind of particles and has a fixed or constant structure (2n)

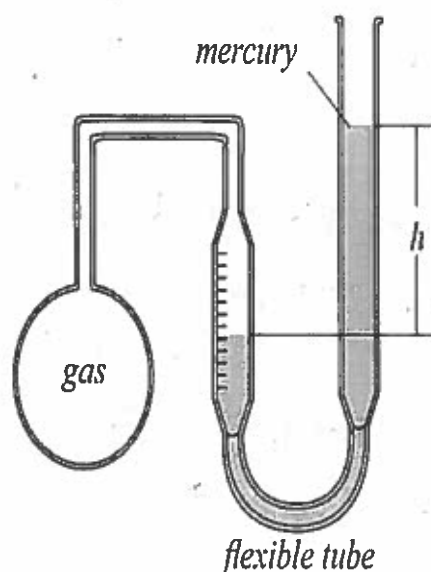
5a) Relative humidity, often expressed as a percentage, indicates a present state of absolute humidity relative to a maximum humidity given the same temperature. Specific humidity is the **ratio of water vapor mass to total moist air parcel mass**. (2n)



ANSWER KEY AND SCHEME OF EVALUATION

6b) Constant Volume Gas Thermometer:

- A constant volume gas thermometer usually consists of a bulb filled with a fixed amount of a dilute gas which is attached to a mercury manometer.
- The manometer is used to measure variation in pressure.
- This thermometer works on the principle of Law of Gay-Lussac.
- The law states that when the temperature of an ideal gas increases, there is a corresponding increase in pressure.
- Also, when the temperature decreases, the pressure too decreases correspondingly.
- This is how constant volume gas thermometers traces the increase in temperature with the change in pressure while the volume remains constant.
- These types of thermometers are used in calibration of various other thermometers.



6A) Given data

$$T = a \ln(P+b)$$

$$\text{Ice point temperature} = 0^\circ\text{C}$$

$$\text{Steam point} = 100^\circ\text{C}$$

$$0 = a \ln 1.5 + b \quad \text{--- (1)}$$

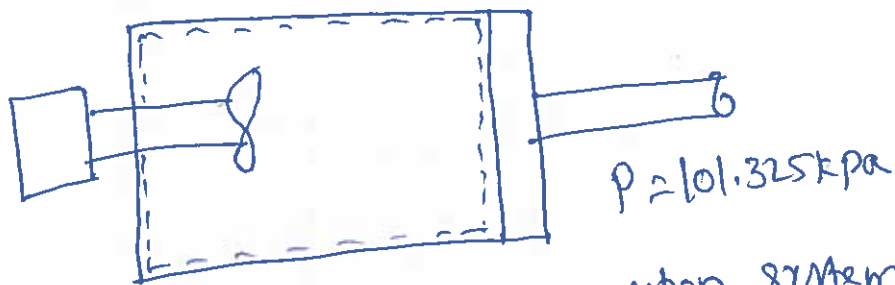
$$100 = a \ln 7.5 + b \quad \text{--- (2)}$$

$$\text{we get } a = 62.112, b = -25.18$$

$$t = 62.112 \ln(3.5) - 25.18$$

$$t = 52.63^\circ\text{C}$$

7a)



work done by piston device upon system

$$\begin{aligned} W_1 &= 2 \pi r L N \\ &= 2 \times \pi \times 1.25 \times 9500 \\ &= 74.5755 \text{ (J)} \end{aligned}$$

work done by system on surroundings

$$\begin{aligned} W_2 &= (P A) L \\ &= 101.325 \times \frac{\pi}{4} \times (0.65)^2 \times 0.6 \times 10^3 \\ &= 20.162 \text{ kJ} \end{aligned}$$

$$W = W_1 + W_2$$

$$W_{\text{network}} = -74.575 + 20.162 = -54.413 \text{ kJ}$$

(3)

8b) Explain first law of thermodynamics applied to a process

The first law of thermodynamics is the law of conservation of energy. It states that if a system absorbs heat dQ and as a result the internal energy of the system changes by dU and the system does a work dW , then $dQ = dU + dW$

But, $W = PdV$

$$dQ = dU + PdV$$

(i) First law of thermodynamics applied to isothermal process

Isothermal Process:

$T = \text{constant}$ [Boyle's law applicable] $PV = \text{constant}$ There is exchange of heat between system and surroundings.

System should be compressed or expanded very slowly so that there is sufficient time for exchange of heat to keep the temperature constant. Hence, for an isothermal process, the first law of thermodynamics becomes

$$dQ = 0 + PdV \text{ or } dQ = PdV \dots (i)$$

When a gas undergoes isothermal compression. In such a case, dV and hence PdV is negative and likewise dQ will also be negative. Therefore, if a gas is to be compressed isothermally than an amount of heat equivalent to the work done on the gas will have to be removed from it.

7b) Given data

work is supplied $= -150 \text{ kJ}$

Initial volume $V_1 = 0.6 \text{ m}^3$

$$P = 8 - 4V$$

$$W = \int_{V_1}^{V_2} P dV = \int_{V_1}^{V_2} (8 - 4V) dV$$

$$-150 \text{ kJ} = \int_{V_1}^{V_2} (8 - 4V) dV$$

$$-150 \text{ kJ} = \left[8V - \frac{4V^2}{2} \right]_{V_1}^{V_2}$$

$$-150 \text{ kJ} = [8V_2 - 2V_2^2] - [8V_1 - 2V_1^2]$$

$$-150 \text{ kJ} = [8V_2 - 2V_2^2] - [8 \times 0.6 - 2 \times (0.6)^2]$$

$$-150 \text{ kJ} = [8V_2 - 2V_2^2] - (4.08)$$

$$2V_2^2 - 8V_2 + 2.58 = 0$$

$$\boxed{V_2 = 0.707 \text{ m}^3}$$

$$P = 8 - 4(0.707) = 5.172 \text{ bar}$$

8a) Heat received by the system

$$Q = 50 \text{ kJ}$$

change in volume $\Delta V = 0.14 \text{ m}^3$

$$\text{pressure} = 1.2 \times 10^5 \text{ N/m}^2$$

Mass lifted in the surroundings $\approx 90 \text{ kg}$

Distance through which lifted $= 5.5 \text{ m}$

work done during adiabatic process $= -110 \text{ kJ}$

$$(1) \quad Q = \Delta E + w$$

$$w = P \Delta V + w_{\text{ext}}$$

$$= \left(\frac{1.2 \times 10^5 \times 0.14}{1000} \right) + \left(\frac{90 \times 5.5 \times 9.8}{1000} \right) \text{ kJ}$$

$$= 16.8 + 4.85 = 21.65 \text{ kJ}$$

$$\Delta E = Q - w = 50 - 21.65 = 28.35 \text{ kJ}$$

(i) since the process is adiabatic

$$Q = 0$$

$$\Delta E = -w = -(-110) = 110 \text{ kJ}$$

(ii) change in internal energy

$$\Delta E = Q - w = 50 - [-110 + 21.65] = 138.35 \text{ kJ}$$

9a) (i)

$$Q = \Delta U + w$$

$$w = \frac{P_1 V_1 - P_2 V_2}{n-1} = \frac{mR(T_1 - T_2)}{n-1}$$

$$= \frac{0.433 \times 2942 (353 - 509.7)}{1.2961}$$

$$= -67438 \text{ N-m (or) } -67438 \text{ J} = -67.44 \text{ kJ}$$

$$Q = 49.9 + (-67.44) = -17.54 \text{ kJ}$$

(5)

9a) Initial volume of gas $V_1 = 0.45 \text{ m}^3$

Initial pressure of gas $P_1 = 1 \times 10^5 \text{ N/m}^2$

Initial temperature $T_1 = 80 + 273 = 353 \text{ K}$

Final volume after compression $V_2 = 0.13 \text{ m}^3$

The final pressure $P_2 = 5 \times 10^5 \text{ N/m}^2$

1) To find mass m using the relation

$$m = \frac{P_1 V_1}{R T_1} = \frac{1 \times 10^5 \times 0.45}{294.2 \times 353} = 0.433 \text{ kg}$$

2) To find index n using the relation

$$P_1 V_1^n = P_2 V_2^n$$

$$\left(\frac{V_1}{V_2}\right)^n = \frac{P_2}{P_1}$$

$$\left(\frac{0.45}{0.13}\right)^n = \left(\frac{5 \times 10^5}{1 \times 10^5}\right) = 5$$

$$(3.46)^n = 5 \Rightarrow n \log 3.46 = \log 5$$

$$n = \log 5 / \log 3.46 = 1.296$$

3) Polytropic process.

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{n-1} = \left(\frac{0.45}{0.13}\right)^{1.296-1} = 1.44$$

$$T_2 = 353 \times 1.44 = 509.7 \text{ K}$$

Now, Increase in internal energy

$$\Delta U = m C_V (T_2 - T_1)$$

$$= 0.433 \times \frac{R}{(n-1)} (T_2 - T_1)$$

$$\left[\because C_V = \frac{R}{n-1} \right]$$

$$= 0.433 \times \frac{294.2}{(1.4-1)} (509.7 - 353)$$

$$= 49.9 \text{ kJ}$$

(6)

4)

$$Q = \Delta U + W$$

$$W = \frac{P_1 V_1 - P_2 V_2}{n-1} = \frac{nR(T_1 - T_2)}{n-1}$$

$$= \frac{0.433 \times 294.2 (353 - 509.7)}{1.296 - 1}$$

$$= -67438 \text{ N}\cdot\text{m (or)} -67438 \text{ J} = -67.44 \text{ kJ}$$

$$Q = 49.9 + (-67.44) = -17.54 \text{ kJ}$$

9b) For steady flow process, net quantity of energy contained within the system will never change with respect to time. Therefore according to the principle of conservation of energy, we will have following statement and energy equation for a steady flow process.

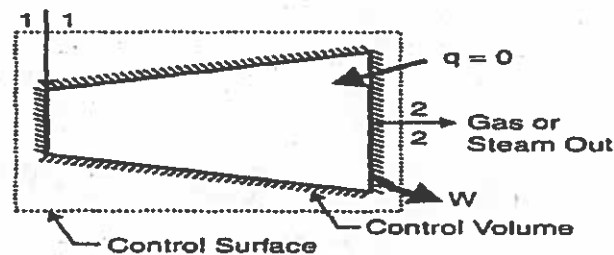
Net quantity of energy entering to the control volume = Net quantity of energy leaving the control volume

$$H_1 + \frac{mV_1^2}{2} + mgZ_1 + Q - W = H_2 + \frac{mV_2^2}{2} + mgZ_2$$

Steady flow energy equation for a turbine

Turbine is basically defined as one prime mover where thermal energy of the high pressure fluid will be converted in to mechanical energy in terms of rotation of turbine shaft. Mechanical energy developed by turbine will be finally converted in to electrical energy.

Turbine is also designated as mechanical device that will provide the work energy through the expansion of fluid. Let us see here the following figure, where high pressure fluid enters at inlet 1-1 and low pressure fluid leaves the turbine at its outlet section i.e. 2-2.



Let turbine is well insulated and hence there will no heat energy interaction between system and surrounding and therefore we can say that $Q = 0$.

When fluid flows through a turbine, change in kinetic energy could be assumed as zero as velocity of fluid flow will be small and we can neglect the small change in kinetic energy of the fluid. Therefore, we will have $\Delta KE = 0$

We have also assumed here that change in potential energy is zero or we can say that $\Delta PE = 0$

Let us implement above data that we have mentioned above in steady flow energy equation for a steady flow process and we will have following equation

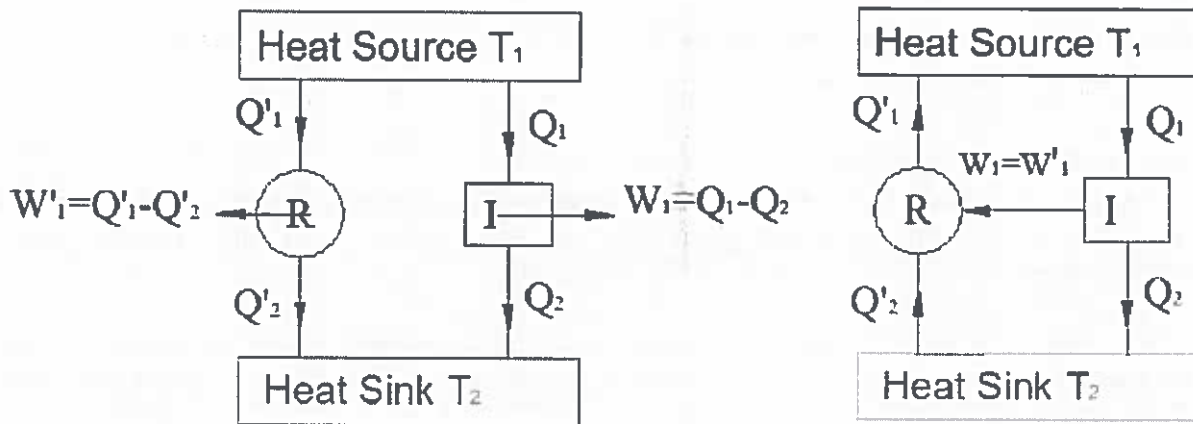
$$H_1 - W = H_2$$

We have taken work energy as positive because turbine is producing the work energy and this work is being done over the surrounding by the system. Therefore according to the "Sign convention for heat and work transfer in thermodynamic system" work energy will be positive.

$$W = H_1 - H_2$$

We can also say that, work energy produced by the turbine during the process will be the result of drop in enthalpy.

10b) The Carnot theorem states: No heat engine operating on a cycle between two heat reservoirs at different fixed temperatures can be more efficient than a reversible engine. Two reversible heat engines operating between two heat reservoirs at different fixed temperatures will have the same efficiency.



10A) change in entropy during constant volume process

$$= mc_v \ln\left(\frac{T_2}{T_1}\right)$$

change in entropy during polytropic process

$$pV^n = \text{constant}$$

$$= mc_v \left(\frac{\gamma - n}{n} \right) \ln\left(\frac{T_2}{T_1}\right)$$

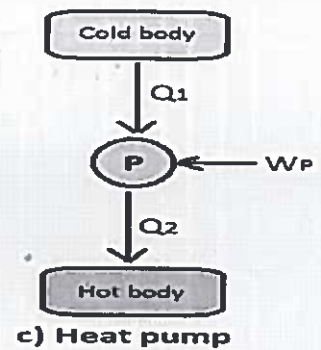
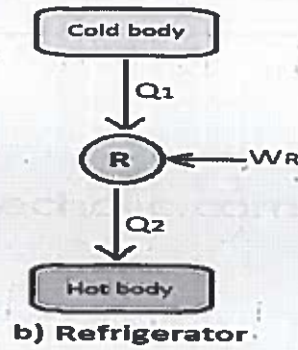
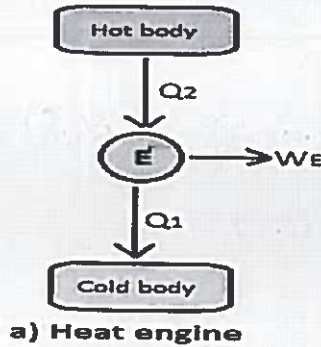
for the same entropy, equating (i) and (ii) we have

$$\frac{\gamma - n}{n} = 1 \quad \text{or} \quad (\gamma - n) = (n - 1)$$

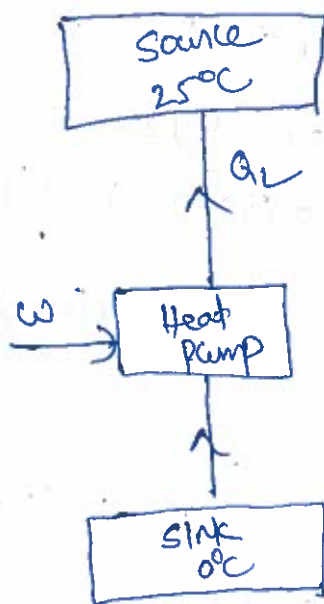
(or)

$$2n = \gamma + 1$$

11b) Heat engines are the devices which convert the thermal energy into mechanical work, while heat pumps transfer heat energy from low temperature to high temperature



11a)



given

1) Temperature $T_1 = 25 + 273 = 298\text{K}$

Temperature $T_2 = 0 + 273 = 273\text{K}$

Heat removal from refrigerator

$Q_1 = 1440\text{ kJ/min} = 24\text{ kJ/s}$

$$\text{C.O.P} = \frac{T_1}{T_1 - T_2} = \frac{298}{298 - 273} = 11.92$$

$$(\text{C.O.P})_{\text{ref}} = \frac{T_2}{T_1 - T_2} = \frac{273}{298 - 273} = 10.92$$

$$10.92 = \frac{Q_1}{W} = \frac{24}{W}$$

$$W = 2.2\text{ kW}$$

work input required $= 2.2\text{ kW (Ans)}$.

$$Q_2 = Q_1 + W = 24 + 2.2 = 26.2\text{ kJ/s}$$

12a) Boiler B₁, 20 bar, 350°C

Enthalpy $h_1 = h_{g1} + C_{p0}(T_{\text{sup}} - T_s)$

$$= 2797.2 + 2.25(350 - 212.4) = 3106.8 \text{ kJ/kg.} \quad \text{--- (1)}$$

Boiler B₂, 20 bar (Temperature not known)

$$h_2 = h_{f2} + x_2 h_{fg2}$$

$$= 908.6 + x_2 \times 1888.6 \text{ kJ/kg} \quad \text{--- (2)}$$

Main, 20 bar, 250°C

Total heat of 2 kg of steam in the steam main

$$= 2[h_g + C_{p0}(T_{\text{sup}} - T_1)]$$

$$= 2[2797.2 + 2.25(250 - 212.4)] = 5763.6 \text{ kJ} \quad \text{--- (3)}$$



Equating (1) and (2) with (iii) we get

$$3106.8 + 908.6 + x_2 \times 1888.6 = 5763.6$$

$$4015.4 + 1888.6x_2 = 5763.6$$

$$x_2 = \frac{5763.6 - 4015.4}{1888.6} = 0.925$$

12 b) Given that

pressure of steam ; $P = 140 \text{ bar}$

Enthalpy of steam $h = 3001.9 \text{ kJ/kg}$

(i) Temperature

At 140 bar , $h_g = 2642.4 \text{ kJ/kg}$, which is less than the actual enthalpy of 3001.9 kJ/kg and hence the steam is superheated

from superheat table at 140 bar ,
 $h = 3001.9 \text{ kJ/kg}$ at a temperature of 400°C

(ii) The specific volume $U = 0.01722 \text{ m}^3/\text{kg}$

$\therefore$ The internal energy

$$U = h - PV = 3001.9 - \frac{140 \times 10^5 \times 0.01722}{10^3}$$
$$= 2760.82 \text{ kJ/kg.}$$

13 a) Mass of steam generated $m = 1000 \text{ kg/h}$

Pressure of steam $P = 16 \text{ bar}$

Dryness fraction $x = 0.9$

Temperature of superheated steam

$$T_{\text{sup}} = 380 + 273 = 653 \text{ K}$$

Temperature of feed water $= 30^\circ\text{C}$

Sp. heat of superheated steam $C_p = 2.2 \text{ kJ/kgK}$

At 16 bar, from steam tables

$$t_g = 201.4^\circ\text{C} \quad (T_s = 201.4 + 273 = 474.4 \text{ K})$$

$$h_f = 858.6 \text{ kJ/kg}; \quad h_{fg} = 1933.2 \text{ kJ/kg}$$

1) Heat supplied to feed water per hour to produce wet steam is given by

$$H = m(h_f + x h_{fg}) - 1 \times 4.18 \times (30 - 0)$$

$$= 1000[858.6 + 0.9 \times 1933.2] - 4.18 \times 30$$

$$= 1000(858.6 + 1739.88 - 125.4)$$

$$= 2473.08 \times 10^3 \text{ kJ} = 2.473 \times 10^6 \text{ kJ}$$

2) Heat absorbed by superheated per hour

$$= m[(1-x)h_{fg} + C_p(T_{\text{sup}} - T_s)]$$

$$= 1000[(1-0.9) \times 1933.2 + 2.2(653 - 474.4)]$$

$$= 1000(193.32 + 392.92)$$

$$= 586.24 \times 10^3 \text{ kJ}$$

13b)

pressure of steam $p = 18 \text{ bar}$

dryness fraction $x = 0.85$

from steam tables corresponding to 18 bar

$h_f = 884.6 \text{ kJ/kg}$, $h_{fg} = 1910.3 \text{ kJ/kg}$, $v_g = 0.110 \text{ m}^3/\text{kg}$.

$u_f = 883 \text{ kJ/kg}$, $u_g = 2598 \text{ kJ/kg}$.

(1) sp. volume of wet steam

$$v = x v_g = 0.85 \times 0.110 = 0.0935 \text{ m}^3/\text{kg}$$

(2) sp. enthalpy of wet steam

$$h = h_f + x h_{fg} = 884.6 + 0.85 \times 1910.3$$

$$= 2508.35 \text{ kJ/kg}$$

3) sp. internal energy of wet steam

$$u = (1-x) u_f + x u_g$$

$$= (1-0.85) \times 883 + 0.85 \times 2598$$

$$= 2340.75 \text{ kJ/kg}$$

14b) At 37°C the saturation pressure of water vapour is obtained from steam tables as 6.279 kPa
 since relative humidity is 50% , the vapour pressure of water in air is

$$P_v = 0.5 \times P_s = 0.5 \times 6.2795 = 3.13975 \text{ kPa}$$

the humidity ratio

$$\omega = \frac{0.622 \times P_v}{P_t - P_v} = \frac{0.622 \times 3.13975}{(101.325 - 3.13975)} = 0.01989$$

enthalpy of air (h)

$$\begin{aligned} h &= 1.005t + \omega(2501 + 1.88t) \\ &= 1.005 \times 37 + 0.01989(2501 + 1.88 \times 37) \\ &= 88.31 \text{ kJ} \end{aligned}$$

14. Expression for Internal energy $U_2 - U_1$

$$\bar{U}_2 - \bar{U}_1 = \sum H_i (U_2 - U_1)_i = \sum m_i (\bar{U}_2 - \bar{U}_1)_i$$

$$M_{\min} = \sum H_i = M_{\min} \sum \frac{H_i}{M_{\min}} (U_2 - U_1)_i$$

$$= M_{\min} \sum H_i (U_2 - U_1)_i$$

$$= (U_2 - U_1)_{\min} = \sum H_i (U_2 - U_1)_i$$

$$\text{const. } C_v, C_p = M_{\min} (\sum H_i C_{v,i}) (T_2 - T_1)$$

$$U_2 - U_1 = M_{\min} \cdot C_{v, \min} \cdot (T_2 - T_1)$$

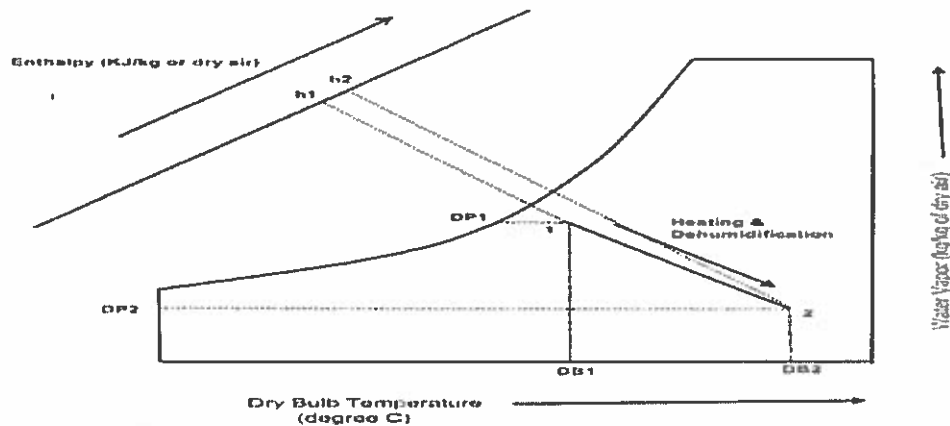
$$\text{sp heat} = M_{\min} (\sum H_i C_{v,i}) (T_2 - T_1)$$

$$C_{v, \min} = \sum H_i \bar{C}_{v,i}$$

15a) Heating and Dehumidification Process (8m)

The process in which the air is heated and at the same time moisture is removed from it is called as heating and dehumidification process. This process is obtained by passing the air over certain chemicals like alumina and molecular sieves. These elements have inherent properties due to which they keep on releasing the heat and also have the tendency to absorb the moisture. These are called as the hygroscopic chemicals.

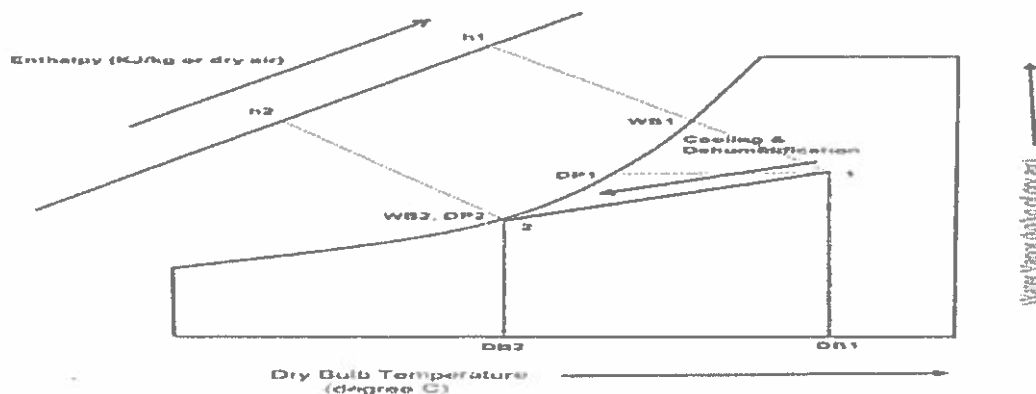
During the heating and dehumidification process dry bulb temperature of the air increases while its dew point and wet bulb temperature reduces. On the psychrometric chart, this process is represented by a straight angular line starting from the given DB temperature conditions and extending downwards towards right to the final DB temperature conditions.



cooling and dehumidification process

The process in which the air is cooled sensibly and at the same time the moisture is removed from it is called as cooling and dehumidification process. Cooling and dehumidification process is obtained when the air at the given dry bulb and dew point (DP) temperature is cooled below the dew point temperature.

During the cooling and dehumidification process the dry bulb, wet bulb and the dew point temperature of air reduces. Similarly, the sensible heat and the latent heat of the air also reduce leading to overall reduction in the enthalpy of the air. The cooling and dehumidification process is represented by a straight angular line on the psychrometric chart. The line starts from the given value of the DB temperature and extends downwards towards left.



15b)

Dry Bulb Temperature (DBT):

- It is a normal temperature measured by an ordinary thermometer.

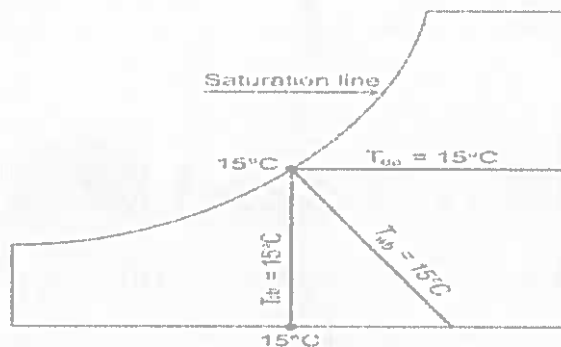
Wet Bulb Temperature (WBT):

- It is a temperature measured by thermometer when its bulb is covered with a wet cloth.

Dew point temperature (DPT):

- It is a temperature at which **moisture present in the atmosphere starts condensing** or it is the temperature at which **the first droplet of moisture is formed**.
- As we know that **for each pressure air has a specific saturation temperature**, therefore for fully saturated air DBT, WBT and DPT are equal.
- When the **relative humidity of the air is 100%**, i.e. **the air is saturated**, the dew point temperature (DPT) equals the wet-bulb temperature (WBT), which is also equal to the dry-bulb temperature.

So $DBT = WBT = DPT$



P. E.
FACULTY

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Semester End Examination, February, 2022

Degree	B. Tech. (U. G.)	Program	Mechanical Engineering	Academic Year	2021 - 2022
Course Code	20ME303	Test Duration	3 Hrs. Max. Marks 70	Semester	III
Course	Material Science & Metallurgy				

Part A (Short Answer Questions 5 x 2 = 10 Marks)

No.	Questions (1 through 5)	Learning Outcome (s)	DoK
1	What is electron compound? Give examples	20ME303.1	L2
2	Compare spheroidal graphite cast Iron and gray cast Iron	20ME303.2	L2
3	What is meant by hardenability?	20ME303.3	L1
4	List any two applications of powder metallurgy	20ME303.4	L1
5	What are crystallized ceramics?	20ME303.5	L1

Part B (Long Answer Questions 5 x 12 = 60 Marks)

No.	Questions (6 through 15)	Marks	Learning Outcome (s)	DoK
6 (a)	What is a crystalline material? Distinguish between single crystal material and polycrystalline material?	6M	20ME303.1	L2
6 (b)	What are intermediate phases? What are the various types of intermediate phases?	6M	20ME303.1	L2
OR				
7 (a)	State and explain Gibbs phase rule.	6M	20ME303.1	L2
7 (b)	Draw a neat labeled Iron-Iron Carbide diagram and explain eutectic and eutectoid reaction in it.	6M	20ME303.1	L2
8 (a)	Explain structure and properties of Gray cast iron.	6M	20ME303.2	L2
8 (b)	Differentiate between white cast iron and malleable cast iron.	6M	20ME303.2	L2
OR				
9 (a)	Describe the composition, heat treatment and applications of the Titanium and its alloys	6M	20ME303.2	L2
9 (b)	Discuss briefly about super alloys and mention their applications.	6M	20ME303.2	L2
10 (a)	Differentiate between annealing and normalizing.	6M	20ME303.3	L2
10 (b)	Define hardenability of a material and explain the factors affecting hardenability.	6M	20ME303.3	L2
OR				
11 (a)	Draw a TTT diagram for 0.4%C steel and identify the micro structural phases. Explain its features.	6M	20ME303.3	L2
11 (b)	Describe briefly about induction hardening method.	6M	20ME303.3	L2
12 (a)	Explain powder extrusion process.	6M	20ME303.4	L2
12 (b)	Describe rapid sintering methods.	6M	20ME303.4	L2
OR				
13 (a)	Explain different stages of manufacturing of powder metallurgy components.	6M	20ME303.4	L2
13 (b)	What is sintering in powder metallurgy? Explain.	6M	20ME303.4	L2
14 (a)	Write the classification of ceramics? Explain with examples.	6M	20ME303.5	L2
14 (b)	Explain the importance and applications of composites.	6M	20ME303.5	L2
OR				
15 (a)	Describe various methods of the manufacturing composites.	6M	20ME303.5	L2
15 (b)	How do you manufacture the composites powder metallurgy components?	6M	20ME303.5	L1

ANSWER KEY AND SCHEME OF EVALUATION

Material Science and Metallurgy (20ME303)

18-02-2022

Academic Year: 2021-2022, Max Marks: 70, Semester - III, B.Tech (U-G)

PART - A (Short Answer Questions 5X2 = 10 Marks)

1. Definition - Electron Compound $\rightarrow$ 1M, Example $\rightarrow$ 1M
 2. 2 points comparison $\rightarrow$ 2M
 3. Definition - Hardenability $\rightarrow$ 2M
 4. Two applications of powder metallurgy $\rightarrow$ 2M
 5. Crystallized ceramics Definition $\rightarrow$ 2M
- } 10M

PART-B (Long Answer questions 5X12 = 60 Marks)

- 6a) Definition - crystalline material $\rightarrow$ 1M } 6M
 Differences, Diagram $\rightarrow$ 5M
- 6b) Definition - Intermediate phases $\rightarrow$ 1M } 6M
 Types of Intermediate phases $\rightarrow$ 3M
 Diagram, Example $\rightarrow$ 2M
 or
- 7a) Definition/Explanation Gibbs phase rule $\rightarrow$ 1M } 6M
 Formula $\rightarrow$ 2M, Diagram $\rightarrow$ 1M
 Notations with Example $\rightarrow$ 2M
- 7(b) Iron-Iron carbide diagram $\rightarrow$ 3M } 6M
 Reactions $\rightarrow$ 3M
- 8a) Structure $\rightarrow$ 2M } 6M
 Gray cast Iron properties $\rightarrow$ 4M
- b) Differences $\rightarrow$ 4M } 6M
 Structure $\rightarrow$ 2M
 or
- 9a) Titanium and its alloys - Explanation $\rightarrow$ 2M } 6M
 Composition $\rightarrow$ 1M, Applications $\rightarrow$ 3M
- b) Super alloys properties $\rightarrow$ 3M } 6M
 Applications $\rightarrow$ 3M

- 10 a) Differences $\rightarrow 4M$, Diagram $\rightarrow 2M$
 b) Definition Hardenability $\rightarrow 2M$
 Factors affecting hardenability $\rightarrow 4M$ } $6M$ } $12M$

or

- 11 a) T.T.T. diagram $\rightarrow 4M$
 Identification $\rightarrow 1M$
 features explanation $\rightarrow 1M$ } $6M$
 b) Induction hardening diagram $\rightarrow 2M$
 Explanation $\rightarrow 4M$ } $6M$

- 12 a) Powder extrusion process explanation $\rightarrow 3M$
 Diagram $\rightarrow 3M$ } $6M$

- b) Rapid Sintering Explanation $\rightarrow 2M$
 methods $\rightarrow 4M$ } $6M$

- 13 a) Different Stages of manufacturing of Powder metallurgy components $\rightarrow 6M$

- b) Sintering definition $\rightarrow 3M$
 process $\rightarrow 3M$ } $6M$

- 14 a) Classification of Ceramics $\rightarrow 4M$
 Examples $\rightarrow 2M$ } $6M$

- b) Importance of composites $\rightarrow 3M$
 Applications of composites $\rightarrow 3M$ } $6M$

or

- 15 a) Explanation of various methods of the manufacturing of composites $\rightarrow 4M$
 Diagrams $\rightarrow 2M$ } $6M$

- b) Process for manufacture the composite powder metallurgy components $\rightarrow 4M$
~~Diagram~~ flow chart $\rightarrow 2M$ } $6M$

Semester End Examination, February, 2022

Degree: B.Tech. (U.G.) Program: Mechanical Engineering, Academic Year 2021-2022

Course Code: 20ME303, Test Duration: 3 Hrs, Max Marks: 70, Semester III

Course: Material Science and Metallurgy, Date: 18-02-2022

Part A: (Short Answer Questions $5 \times 2 = 10$ Marks)

1. What is electron compound? Give examples

Electron compounds: They do not obey the valency laws and are known as Electron compounds. Many of these ~~electro~~ compounds fall into three classes according to the ratio of Valence electrons to the number of atoms

1. Ratio $\frac{3}{2}$ - beta (β), e.g., CuZn, Cu₃Al (B.C.C. Structure)
2. Ratio $\frac{21}{13}$ - gamma (γ), e.g., Cu₅Zn₈, Cu₉Al₄ (Complex Cubic)
3. Ratio $\frac{7}{4}$ - epsilon (ε), e.g., CuZn₃, Cu₃Sn (C.C.P.H. Structure)

2. Compare Spheroidal graphite cast iron and gray cast iron

- (i) unlike long flakes as in gray cast iron, graphite appears as rounded particles or nodules or spheroids in nodular cast iron.
- (ii) Spheroidal graphite cast iron has more impact resistant when compared to gray cast iron
- (iii) Ductile iron has lower thermal conductivity.
- (iv) Ductile iron is stronger and contains higher ductility.
- (v) nodular or ductile cast iron contains 3.2-4.2% C, 1.1-3.5% Si, 0.3-0.8% Mn, 0.08% P and 0.2% S
- (vi) Gray cast iron contains 2.5-3.8% C, 1.1-2.8% Si, 0.4-1% Mn, 0.15% P and 0.10% S.
- (vii) nodular cast iron uses Earth moving components, pipes, Gray cast iron uses Manhole covers, piston rings

3. What is meant by hardenability?

Hardness is a measure of resistance to plastic deformation (by indentation), Hardenability is the ease with which hardness may be attained in the depth direction of an object. Hardenability may also be remembered as the ability of a steel to become uniformly hard or to harden in depth direction.

4. List any two applications of powder metallurgy

- (i) Porous products, e.g. bearings and filters
- (ii) Use as parts in military and defense systems, e.g. in military arms

5. What are crystallized ceramics?

The crystalline ceramics are not eligible for a wide range of processing. They can be either be shaped in situ or formed with powders and sintered to form a solid body. Ceramic forming techniques include shaping by hand (sometimes including a rotation process called "throwing"), slip casting, tape casting (used for making very thin capacitors, e.g.), injection molding, dry pressing and other variations.

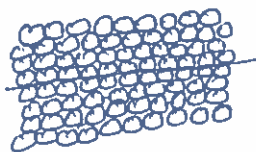
Part B. (Long Answer questions $5 \times 12 = 60$ Marks).

6(a) what is a crystalline material? Distinguish between single crystal material and polycrystalline material?

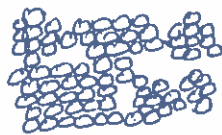
A crystalline material is defined as a material in which atoms are arranged in a periodically repeating geometric array. ~~these long atoms~~
~~crystalline~~ Crystalline materials have highly defined and repeatable arrangements of molecular chains. These materials tend to have sharp melting points. The examples of crystalline materials are sugar, diamonds,

A crystalline material with a single grain is called a single crystal. A crystalline material consisting of many grains of different orientation (like a ceramic) is called polycrystalline.

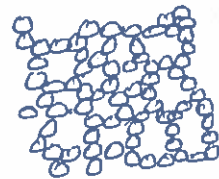
Single crystals have infinite periodicity. Polycrystals have local periodicity and ~~are~~ amorphous solids (and liquids) have no long-range order. A polycrystalline solid or polycrystal is comprised of many individual grains or crystallites.



Single crystal
Periodic across the whole volume

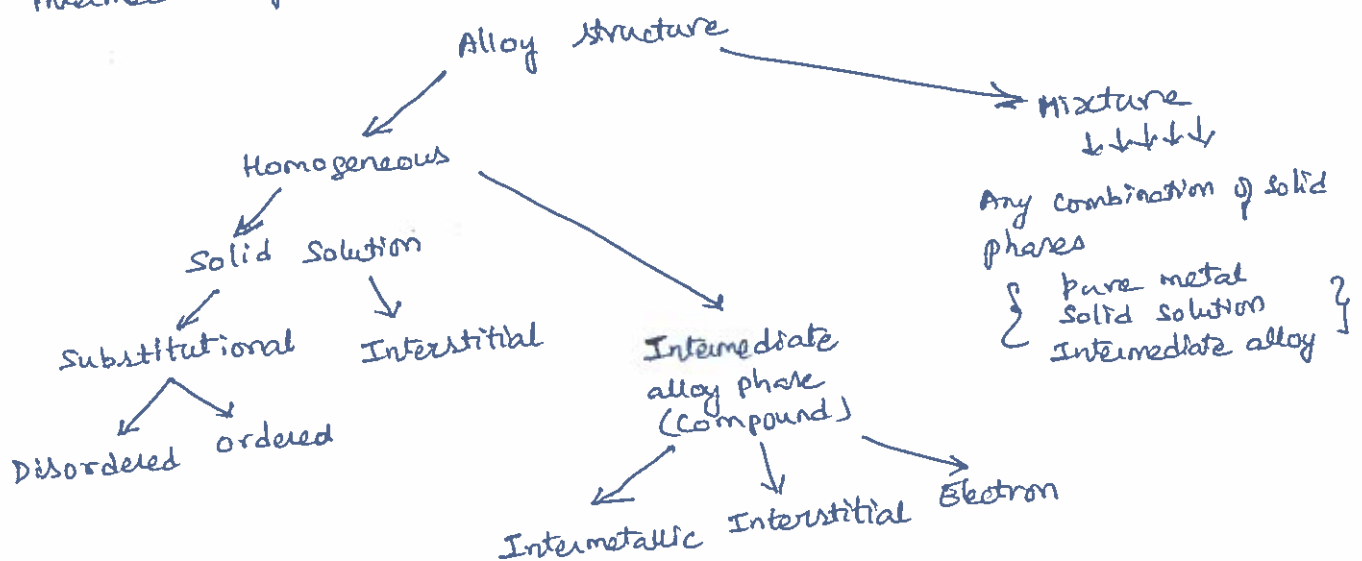


Polycrystal
Periodic across each grain



Amorphous solid
not periodic

6(b) what are intermediate phases? what are the various types of intermediate phases?



Intermetallic compounds or valency compounds: - These are generally formed between chemically dissimilar metals and are combined by following the rules of chemical valence. Since they generally have strong bonding (ionic or covalent) their properties are essentially nonmetallic. They usually show poor ductility and poor electrical conductivity and may have a complex crystal structure.

Examples of valency compounds are CaSe , Mg_2Pb , Mg_2Sn and Cu_2Se .

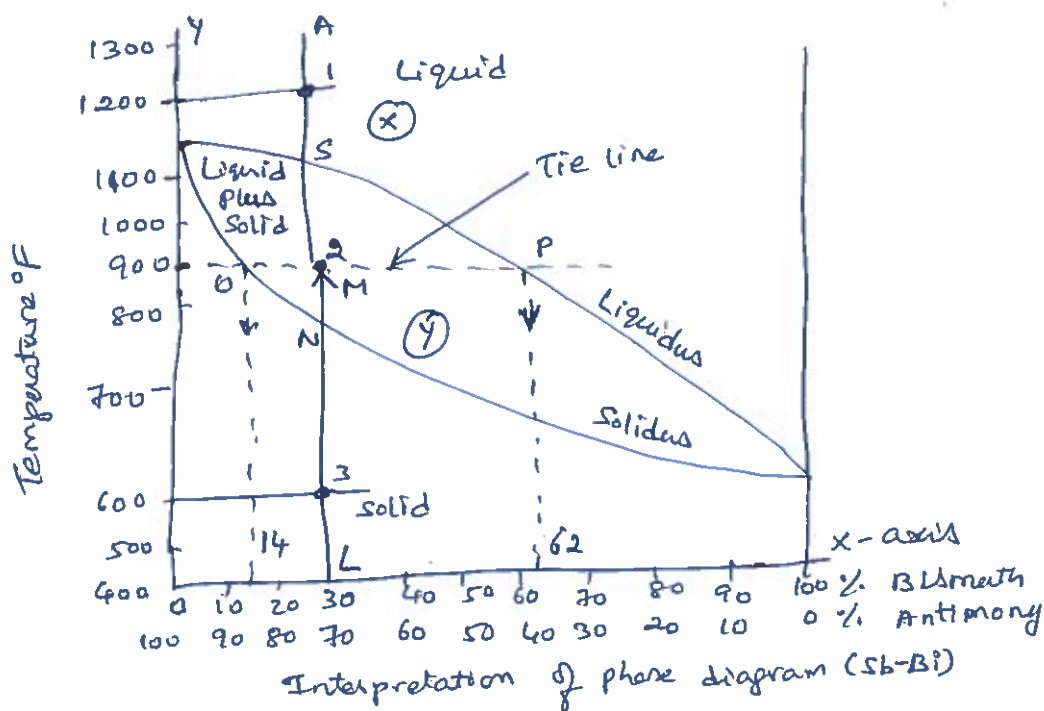
Interstitial compounds: - These compounds formed between the transition metals such as Scandium (Sc), titanium (Ti), tantalum (Ta), Tungsten (W) and iron (Fe) with hydrogen, oxygen, carbon, boron and nitrogen.

The interstitial compounds are metallic may have a narrow range of composition, high melting points and are extremely hard. Examples are TiC , TaC , Fe_4N , Fe_3C , W_2C , CrN and TiH many of these compounds are useful in hardening steel and in cemented carbide tools.

Electron compounds: - They do not obey the valency laws and are known as electron compounds. Many of these compounds fall into three classes according to the ratio of valence electrons to the number of atoms.

1. Ratio $\frac{3}{2}$ - beta (β), e.g. CuZn , Cu_3Al (B.C.C. structure)
2. Ratio $\frac{21}{13}$ - gamma (γ), e.g. Cu_5Zn_8 , Cu_9Al_4 (complex cubic)
3. Ratio $\frac{7}{4}$ - epsilon (ε) e.g. CuZn_3 , Cu_3Sn (C.C.P.H. structure)

7(a) State and explain Gibbs phase rule.



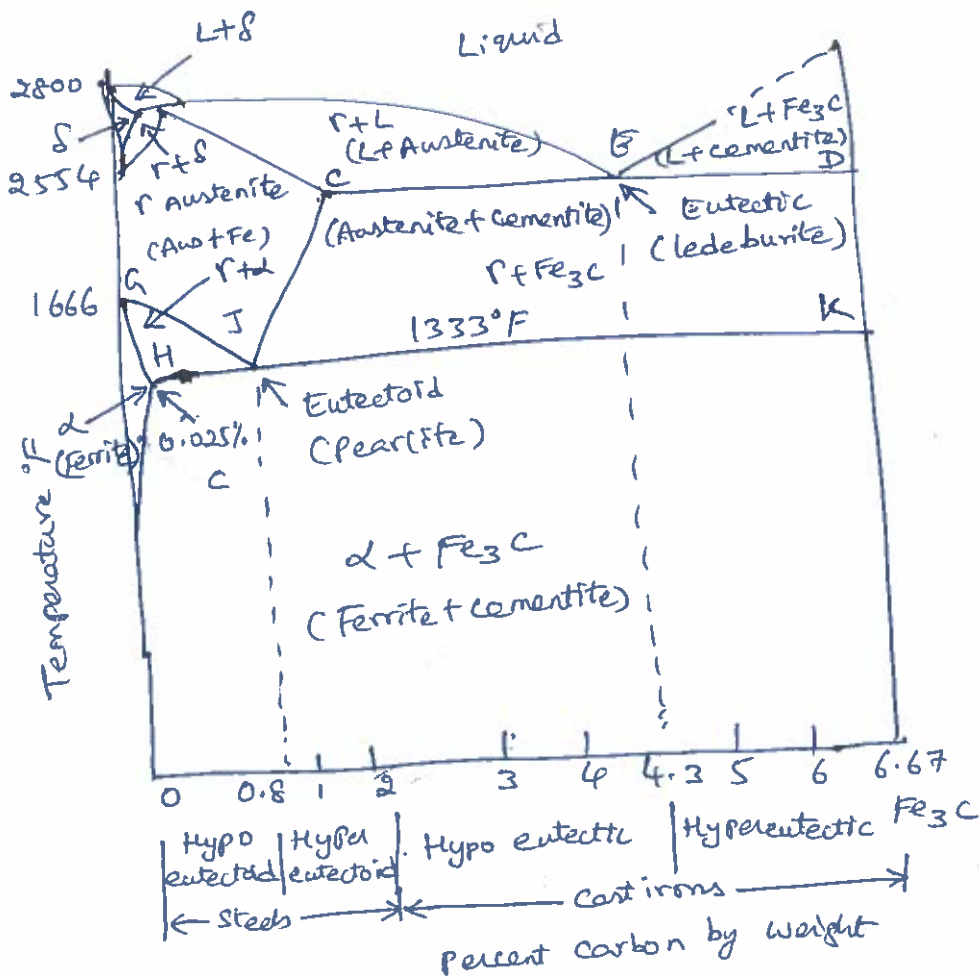
Considering points X and Y in Fig. it is required to determine the number of degrees of freedom using the phase rule.

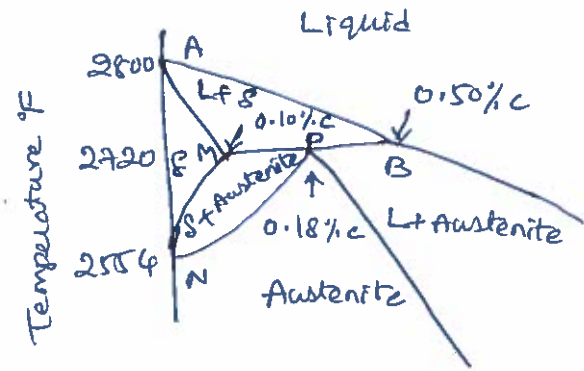
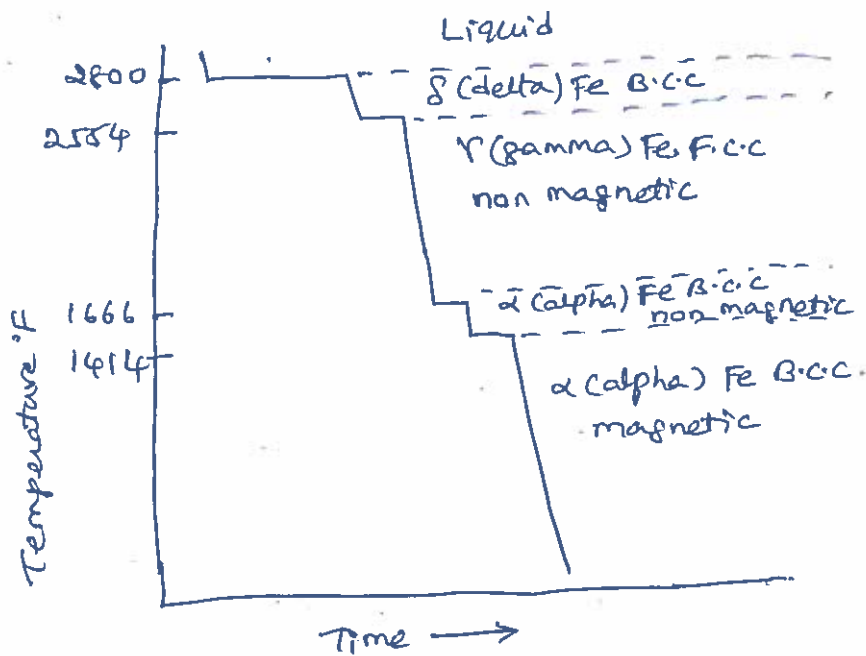
1. Point X in the region above the liquidus
 number of components $C=2$, Since it is a binary system of Si and Sb.
 number of phases (P) (liquid) = 1, Applying the rule $F = C + 2 - P = 2 + 2 - 1 = 3$ degrees of freedom
2. Point Y between the liquidus and solidus
 number of components (C) = 2
 number of phases (P) (liquid and solid) = 2
 Applying the rule $F = C + 2 - P = 2 + 2 - 2 = 2$ degrees of freedom.

The phase rule known as Gibbs phase rule establishes the relationship between the number of phases (P). It is expressed mathematically as follows $P + F = C + 2$

where P is the no. of phases (e.g. solid, liquid etc)
 F is the no. of degrees of freedom or the no. of physical variables (pressure, temperature and concentration) that can be independently changed without altering the equilibrium i.e. without causing disappearance of a phase or the formation of a new phase in the system.
 C is the number of components in the system for example Pb and Sn are the components of Pb-Sn equilibrium diagram.

7C6) Draw a neat labelled Iron-Iron Carbide diagram and explain eutectic and eutectoid reaction in it.





Cooling curve for pure iron

The equation of the peritectic reaction may be written as
 Liquid + δ $\xrightleftharpoons[\text{heating}]{\text{cooling}}$ Austenite

Liquid $\xrightleftharpoons[\text{heating}]{\text{cooling}}$ Austenite + Cementite
 eutectic mixture - Ledeburite

Liquid $\xrightleftharpoons[\text{heating}]{\text{cooling}}$ Ferrite + Cementite
 eutectoid mixture - pearlite.

8a) Explain structure and properties of Gray cast iron



Gray iron showing graphite flakes (black) and ferrite (white portion)

Applications

- 1) Machine tool structures (bed, frame and details).
- 2) Manhole covers
- 3) Gas & water pipes for underground purposes
- 4) Cylinder blocks and heads for I.C. Engines
- 5) Tunnel segment
- 6) Frames for electric motors
- 7) Ingot moulds
- 8) Sanitary wares
- 9) Piston rings
- 10) Rolling mills and general machinery parts.
- 11) Household appliances etc.

Gray cast Iron

A low-cost material that can be used for many purposes. Available to the foundry as virgin ingots and selected scrap

Mechanical properties:

1. High Compressive strength
2. Low tensile strength
3. High rigidity
4. High stability after 'weathering'

Castig properties:

1. High fluidity and ability to make sand castings
2. Relatively low melting temperature $1130^{\circ} - 1250^{\circ}\text{C}$

Machinability:

Easily machined to a good finish once the 'skin' has been removed. Forms a discontinuous chip.

Special properties:

'Self damping' does not vibrate or 'rig' Prevents chatter when used for machine frames. Self-lubricating. Good anti-friction properties.

A 'natural' for machine beds, frames and details. General castings

Characteristics

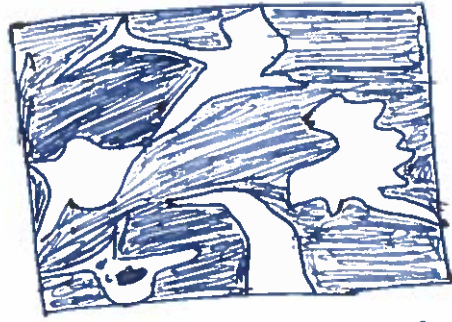
- (i) Gray Iron basically is an alloy of carbon and silicon with iron.
- (ii) It is readily cast into a desired shape in a sand mould.
- (iii) It contains 2.5-3.8% C, 1.1-2.8% Si, 0.4-1% Mn, 0.15% P and 0.10% S.
- (iv) Graphite flakes occupy about 10% of the metal volume.
- (v) It possesses machinability better than steel.
- (vi) It possesses high vibration damping capacity.
- (vii) It is marked by the presence of flakes of graphite in a matrix of ferrite, pearlite or austenite.
- (viii) It has high resistance to wear (including sliding wear).
- (ix) Gray cast iron has a solidification range of $2400 - 2000^{\circ}\text{F}$.
- (x) Gray cast iron possesses high compressive strength.

8(b) Differentiate between white cast iron and malleable cast iron.

Malleable iron starts out as white iron casting but is then treated at a heat of roughly 1650°F , 900°C . Malleable iron contains more blunt boundaries opposed to flakes, which eliminates the stress concentration problems that grey cast iron cause. Malleable iron has properties that are very similar to mild steel.



Ferritic malleable iron showing ferrite matrix and temper carbon (dark particles)



white cast iron showing massive carbide areas (white) in fine pearlite (dark) matrix.

~~Uses~~ - Malleable cast Iron - uses

- (1) Automotive industry
- (2) Rail road
- (3) Agricultural implements
- (4) Electrical line hardware
- (5) Conveyor chain links
- (6) Gear case
- (7) Universal Joint Yoke
- (8) Rear axle banjo housing
- (9) Truck tandem axle assembly parts.
- (10) Automotive crankshaft
- (11) Crankshaft sprocket etc.

Characteristics

- (1) It can be used from -60 to 1200°F
- (2) It has a solidification range of $2550 - 2065^{\circ}\text{F}$
- (3) It has shrinkage of $\frac{3}{16}$ inch per foot ($1.5\text{ mm}/100\text{ mm}$)
- (4) It has low to moderate cost
- (5) Malleable cast iron possesses high yield strength
- (6) Malleable cast iron is one which can be hammered and rolled to obtain different shapes
- (7) Malleable cast iron is obtained from hard and brittle white iron through a controlled heat conversion process.

White cast Iron (Mottled Iron)

- (1) White cast iron derives its name from the fact that its freshly broken surface shows a bright white fracture
- (2) Unlike gray iron, white cast iron has almost all its carbon, chemically bonded with the iron - as iron carbide, Fe_3C . Iron carbide is a very hard and brittle constituent.
- (3) Thus white iron possesses excellent abrasive wear resistance.
- (4) White iron under normal circumstances is brittle and not machinable.
- (5) By using a fairly low silicon content, cast iron may be made to solidify as white iron.
- (6) White cast iron castings can be made in sand moulds.
- (7) If iron of proper composition is cooled rapidly, the free carbon will go in the combined form and give rise to white iron casting.
- (8) The solidification range of white iron is $2550 - 2065^{\circ}\text{F}$
- (9) The white iron contains $1.8 - 3.6\% \text{C}$, $0.5 - 2.0\% \text{Si}$, $0.2 - 0.8\% \text{Mn}$, $0.15\% \text{P}$ and $0.10\% \text{S}$
- (10) White iron (of a particular, chemical composition) is the first step in the production of malleable iron castings.

uses

- (1) For producing malleable iron castings
- (2) For manufacturing those component parts which require a hard and abrasion resistant material.

9(c) Describe the composition, heat treatment and applications of the Titanium and its alloys.

Titanium and its alloys have attractive engineering properties. They are about 40% lighter than steel and 60% heavier than aluminium.

A second outstanding property of titanium materials is corrosion resistance. Other notable properties are a higher melting point than iron, low thermal conductivity, low coefficient of expansion and high electrical resistivity.

Titanium Alloys

Alpha Alloys:- These alloys contain such alloying elements as aluminium, tin, columbium, zirconium, vanadium and molybdenum in amounts varying from about 1 to 10 percent. They have a good combination of weldability, strength and toughness.

The 5% aluminium and 2.5% tin alloy ~~is~~ perhaps the most widely used alpha alloy has been employed in numerous space and aircraft applications. In addition it has good oxidation resistance and good weldability and formability.

Alpha-Beta Alloys:- This is the largest and most widely used group of titanium alloys.

The most popular alloy in this group is the 6% aluminium and 4% vanadium grade has good impact and fatigue strength and unlike other alpha-~~high~~ beta alloys is weldable.

Beta Alloys:- Although the beta alloys have exceptionally high strengths, their lack of toughness and low fatigue strength limits their use. They become brittle at temperature below -40°C .

9(c) Discuss briefly about super alloys and mention their applications.

1. The super alloys have superlative combinations of properties.
2. These materials are classified according to the predominant metal in the alloy which may be cobalt, nickel or iron other alloying elements include the refractory metals (Nb, Mo, W, Ta) chromium and titanium.
3. Super alloys are at least five times as strong as steels routinely used for making bridges and large buildings. They can withstand enormous strains and exhibit remarkable resistance to metal fatigue. They possess high impact strength and superior strength-to-weight ratio. They are probably the toughest materials ever produced.
4. In addition to turbine applications, these alloys are utilized in nuclear reactors and petrochemical equipment.
5. Superalloys are used in aircraft turbine components, which must withstand exposure to severely oxidizing environments and high temperatures for reasonable time periods.

11 a) Draw a TTT diagram for 0.4% C steel and identify the microstructural phases. Explain its features.

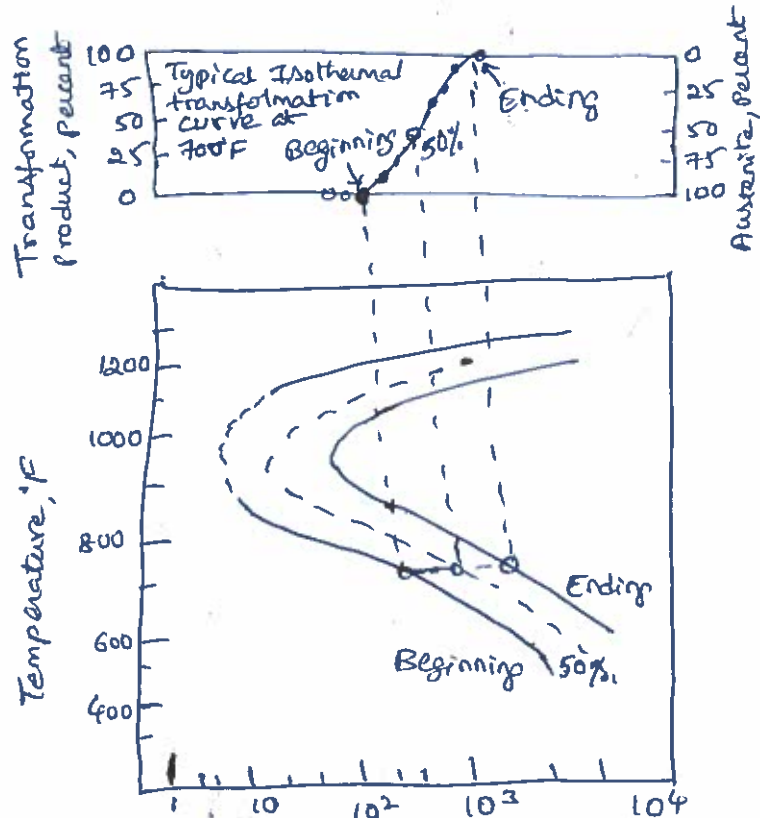
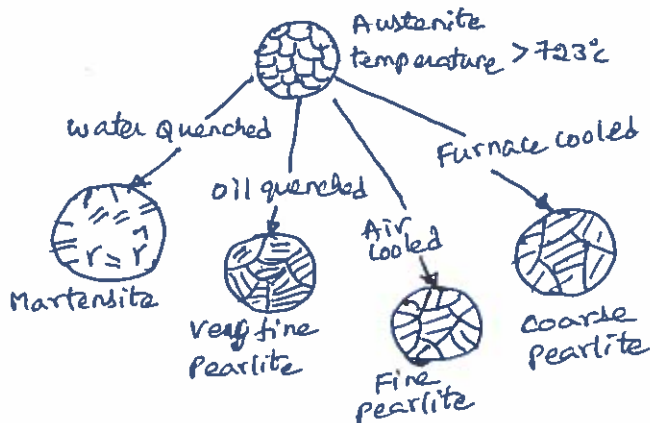
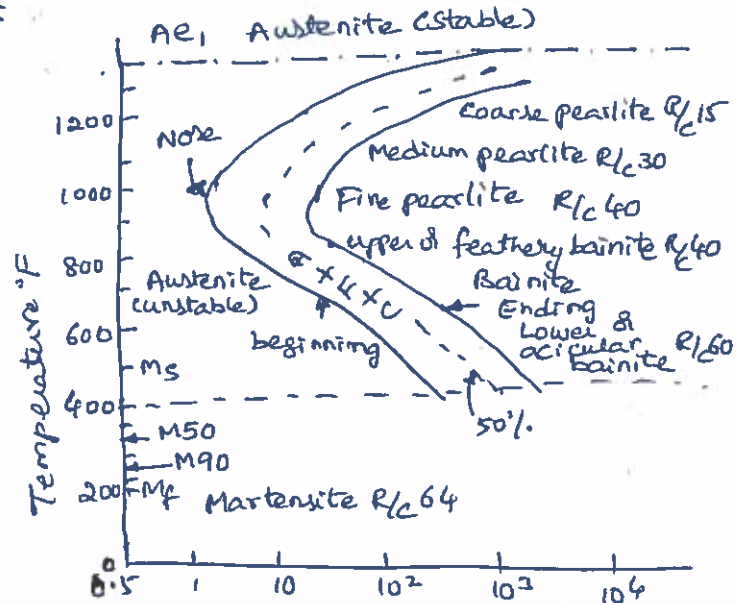


Diagram showing how measurements of isothermal transformation are summarized by the I-T diagram



T.T.T. (Time-Temperature-Transformation)

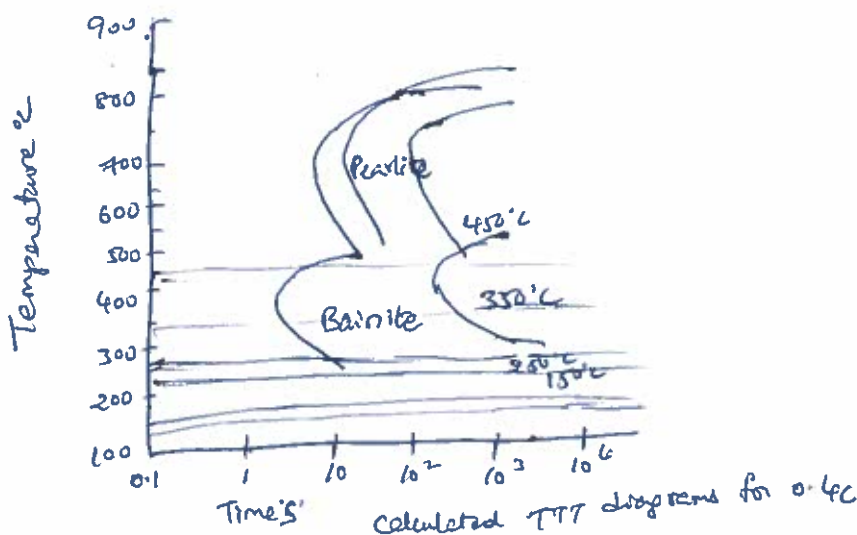
Diagram is also known as S-curve, C-curve, Bain's curve or Isothermal Transformation diagram.



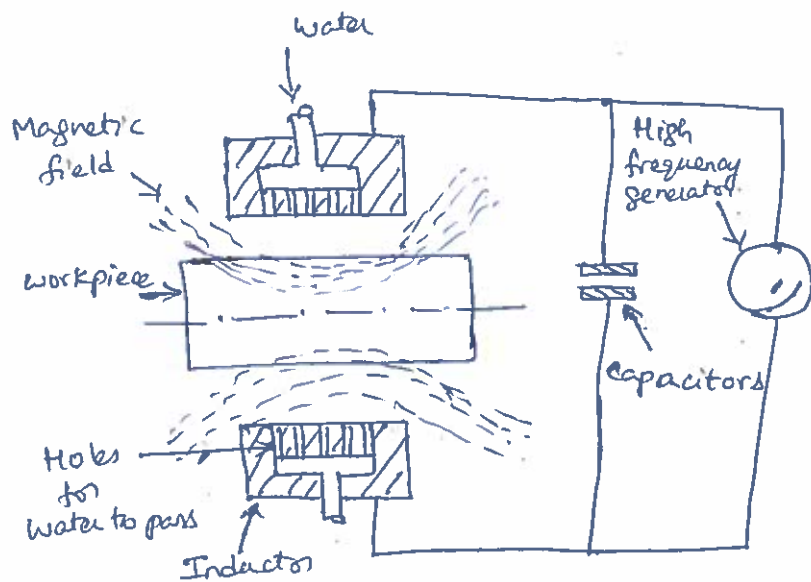
Isothermal-transformation diagram for a 1080 (eutectoid) steel.

M_s - start of martensite formation
 M_f - end of martensite formation

A T.T.T. diagram shows the relationship between temperature and time taken for a decomposition transformation to take place in a metal when the transformation is isothermal i.e. transformation is allowed to occur at constant temperature. T.T.T. diagram is used more particularly in the assessment of decomposition of austenite in a heat treatable steel.



11 b) Describe briefly about induction hardening method.



operation of induction hardening

The principle of induction hardening is similar to that employed in the induction melting of steels by the high frequency induction process in which heat is produced by currents induced in the metal charge itself. The higher the frequency of current, the closer the heat is to the surface of the component. The high frequency induced currents tend to travel at the surface of the metal. This is known as skin effect.

Applications :- 1) piston rods 2) Pump shafts 3) spur gears 4) Cam 5) Crankshaft 6) Automobile parts

Advantages :- 1) Time required for hardening a component is sharply reduced 2) It can be applied to both external and internal surfaces. 3) Hard case and tough core is obtained.

Disadvantages :- 1) cost of equipment is high 2) It is beneficial in mass production only.

10 b) Define hardenability of a material and explain the factors affecting hardenability.

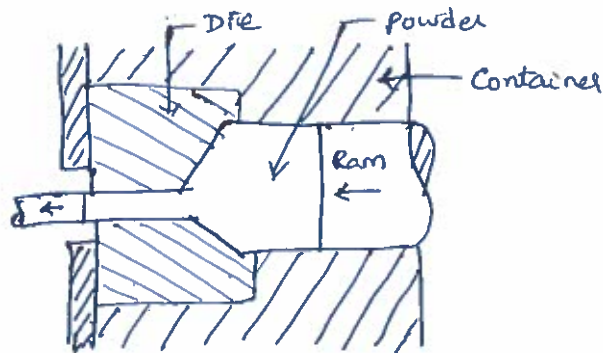
Hardenability :- Hardness is a measure of resistance to plastic deformation (by indentation). Hardenability is the ease with which hardness may be attained in the depth direction of an object. Hardenability may also be remembered as the ability of a steel to become uniformly hard or to be hardened in depth direction. In a ferrous alloy, hardenability is the property that determines the depth and distribution of hardness induced by quenching.

Methods to determine hardenability 1. The Grossmann method 2. Jominy Test

Factors affecting hardenability.

1. In general the composition of the steel and method of manufacture.
2. The quenching media and the method of quenching
3. Section of the steel.
4. The size of austenite grains before quenching. The larger the austenite grain size prior to quenching, the greater the degree of hardenability.
5. The homogeneity of the austenite before quenching
6. The presence of undissolved carbides & non-metallic inclusions before quenching (tend to reduce hardenability).

12 a) Explain powder extrusion process



After blending and mixing, the next step is that of compacting or pressing the powders into their semi-finished form preparatory to sintering.

The purpose of compacting is to consolidate the powder into the desired shape and as closely as possible to final dimensions, taking into account any dimensional changes that result from sintering. Compacting also imparts

(i) the desired level and type of porosity and (ii) adequate strength for handling.

Powders are compacted by using high pressures. The degree of pressure required depends upon

(i) The required density of the final product and (ii) The ease with which the powder particles will weld together.

Compacting pressures may be applied in the following ways

1. Die pressing 2. Roll pressing 3. Extrusion

The extrusion method of compacting does not give such efficient control as that given by pressing or even by rolling. It is difficult to obtain high densities and some porosity is always left.

12 (b) Describe rapid sintering methods
Secondary operations 1) Sizing 2) Coining 3) Joining of sintered parts 4. Heat treatment 5. Machining, 6. Infiltration 7) Plating 8) Impregnation (SPS) Spark plasma sintering:- external pressure and an electric field are applied.

Simultaneously to enhance the densification of the metallic/ceramic powder compacts.

Field assisted sintering technique (FAST), Electric field assisted sintering (EFAS) and direct current sintering (DCS).

Electro sinter forging:- is an electric ~~as~~ current assisted sintering (ECAS) technology originated from capacitor discharge sintering. It is used for production of diamond metal matrix composites and is under evaluation for the production of hard metals.

Microwave sintering:- Heat is sometimes generated internally within the material, rather than via surface radiative heat transfer from an external heat source. Some materials fall to couple and often exhibit run-away behavior. A benefit of microwave sintering is faster heating for small loads, meaning less time is needed to reach the sintering temperature. Less heating energy is required and there are improvements in product properties.

Pressureless sintering:- is the sintering of a powder compact without applied pressure. This avoids density variations in the final component, which occur with more traditional hot pressing methods. Plastic sintering:- are formed by sintering for applications that ~~material~~ require material of specific porosity, to sinter.

13a) Explain different stages of manufacturing of powder metallurgy components.

- The principal steps in a powder - metallurgy process include
- obtaining / producing metal powders in a suitable degree of fineness and purity
 - weighing and mixing of the necessary powders (and lubricants) to arrive at a composition which processes satisfactorily and which produces desired properties in the fabricated part.
 - Pressing the powder (mixture) in a suitable mold (of required size and shape) to cause cohesion to occur between the powder particles.
 - pressing the powder compact by heating and holding it at a moderate temperature. Pressing develops additional green strength and drives off mixing lubricants and/ or mixture.
 - sintering the compacted mass at a temperature high enough to cause diffusion and intergranular crystal growth to occur.
 - Finishing and sizing the final product.

In addition to above, one or more of the following treatments may also be required for metal components.

- Annealing
- Repressing for greater density or closer dimensional control
- Machining
- Polishing
- Rolling, forging or drawing
- Surface treatments to protect against corrosion

Production of metal powders is by atomization, Reduction, electrolysis, crushing, milling, condensation of metal, hydride and carbonyl processes.

13 b) What is sintering in powder metallurgy? Explain

- After being compressed into a briquette of the shape required in the finished component, the agglomerated metals are sintered
- Sintering is done to achieve all possible final strength and hardness needed in the finished product.
- Sintering consists of heating pressed metal or cement compacts in batch or continuous furnaces to a temperature below the melting point of the major constituent in an inert or reducing atmosphere (of hydrogen, dissociated ammonia or cracked hydrocarbon). Where time, temperature, heating rate and cooling rate are automatically controlled.
- Most metals are sintered at 70 to 80% of the melting temperature. Certain refractory materials may be sintered at 90% of the melting point.
- The sintering time varies from thirty minutes up to several hours.

The relationship between time t , and temperature T is given by

$$\log \left(\frac{t_1}{t_2} \right) = \frac{Q_s}{R} \left(\frac{1}{T_1} - \frac{1}{T_2} \right)$$

$Q_s \rightarrow$ is the activation energy of sintering
 $R \rightarrow$ is the gas constant.

- Sintering is essentially a process of bonding solid bodies (particles) by atomic forces. Bonding of powder particles during sintering can take place in any of the three ways
 - melting of a minor constituent
 - volume diffusion, although surface diffusion, evaporation and condensation also contribute to bonding
 - Mechanical bonding or entrapment of non-metallic particles in a metal matrix
 - Sintering temperatures and times vary considerably with different materials.
- Secondary operations:- 1) Sizing 2) Corning 3) Joining of sintered parts
 4) Heat treatment 5) Machining 6) Infiltration 7) Impregnation 8) Plating

10 a) Differentiate between annealing and normalizing.

Annealing :- is the process of heating a metal which is in a metastable or distorted structural state, to a temperature which will remove the instability or distortion and then cooling is (usually at a slow rate) so that the room temperature structure is stable and / or strain free purpose

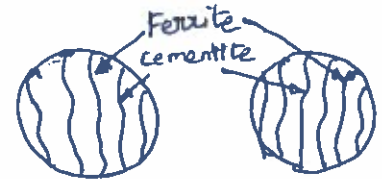
- (1) Inducing a completely stable structure (full annealing)
- (2) Refining and homogenizing the structure
- (3) Reducing hardness
- (4) Improving machinability
- (5) Removing pores
- (6) Improving cold working characteristics (process annealing) for facilitating further cold work.
- (7) producing desired microstructure
- (8) Removing residual stresses
- (9) Improving mechanical, physical, electrical and magnetic properties.

1) Full annealing 2) stress-Relief annealing 3) Process-Annealing 4) Spheroidizing

Normalizing :- Normalizing or air quenching consists in heating steel to about $40-50^{\circ}\text{C}$ (100°F) above its upper critical temperature (i.e. A_3 and A_{cm} line) and if necessary holding it at that temperature for a short time and then cooling in still air at room temperature

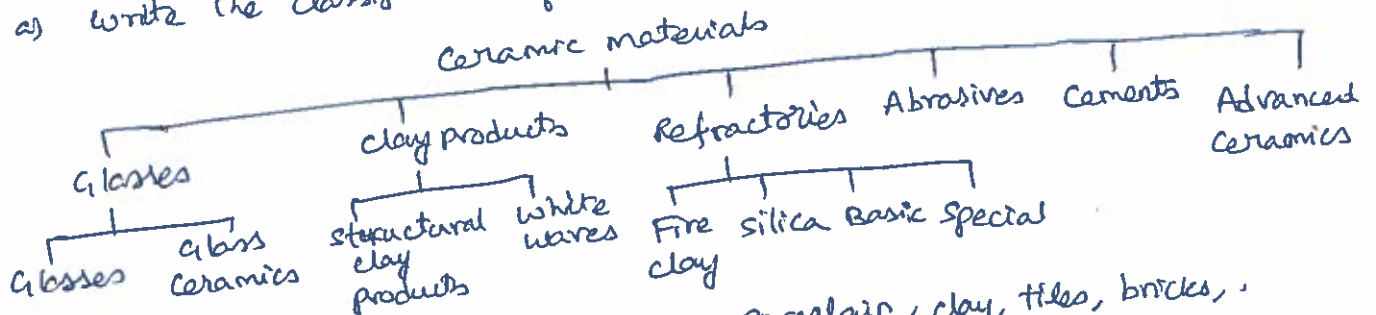
Normalizing differs from full annealing in that the rate of cooling is more rapid and there is no extended soaking period. The type of structure obtained by normalizing will depend largely on the thickness of cross-section as this will affect the rate of cooling. Thin sections will give a much finer grain than thick sections. The purpose of normalizing is to produce a harder and stronger steel than full annealing, so that for some applications normalizing may be a final heat treatment.

- i) produces a uniform structure
- ii) Reduces internal stresses
- iii) improves structure in welds
- iv) resulting in reduced hardness and increased ductility
- v) normalizing may also be used to improve machinability.



Schematic picture of the difference in pearlitic structure due to annealing and normalizing.

11 a) Write the classification of Ceramics? Explain with Examples.



Examples :- Earthenware, stoneware, Porcelain, clay, tiles, bricks, Silicon carbide, Aluminium oxide, Carbides of W and B, Gmel, diamond, cermets, sand stone, quartz, Tripoli, Rottenstone and pumice, garnet and flint, Walnut shells, glass.

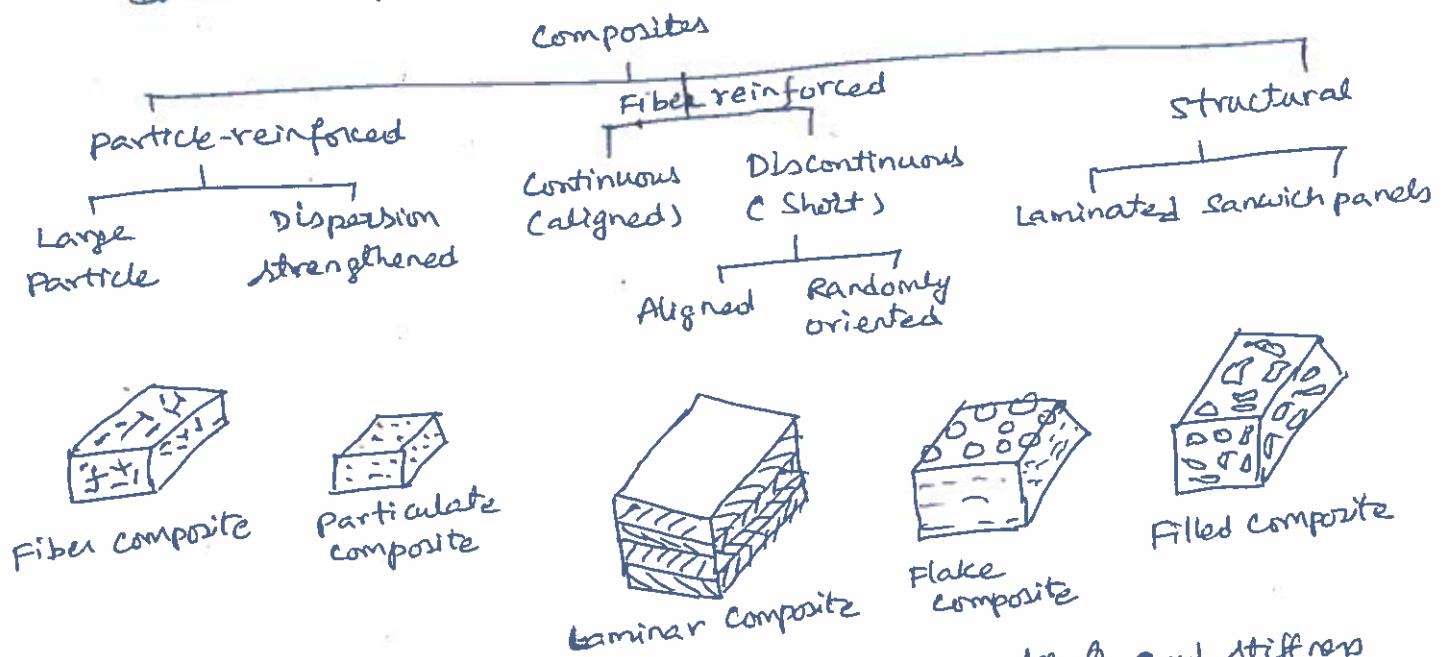
14(b) Explain the importance and applications of composites.

Composite materials are produced by combining two dissimilar materials into a new material that may be better suited for a particular application than either of the original materials alone.

The most common example of a composite material is the fiber-glass-reinforced plastic commonly used in household goods and in many industrial applications.

Metals, ceramics, glasses, polymers and cement can be obtained in composite materials to produce unique characteristics such as stiffness, toughness and high temperature strength, cost efficient alternative to metals.

Many composite materials are composed of just two phases one is termed the matrix, which is continuous and surrounds the other phase, often called the dispersed phase.



The greatest advantage of composite materials is strength and stiffness combined with lightness. Currently composites are used extensively in consumer products and building materials. Other advantages are durable, corrosion resistance, design flexibility, high performance at elevated temperature, High impact strength.

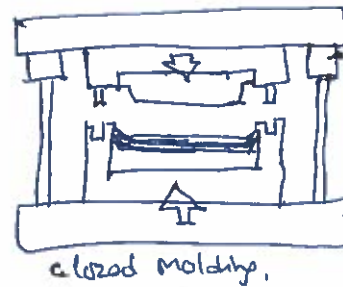
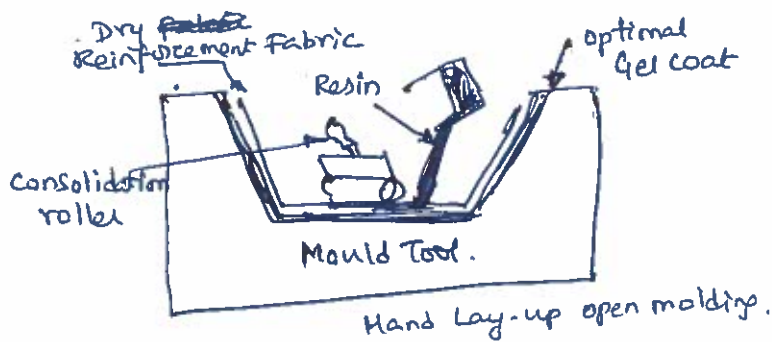
15a) Describe various methods of the manufacturing composites.

Open molding :- composite materials (resin and fibers) are placed in an open mold, where they cure or harden while exposed to the air. Tooling cost for open molds is often inexpensive, making it possible to use this technique for prototype and short production runs.

Closed molding :- composite materials are processed and cured inside a vacuum bag of a two-sided mold, closed to the atmosphere. closed molding may be considered for two cases first if a two-sided finish is needed and second if high production volumes are required.

Cast polymer molding :- A mixture of resin and fillers are poured into a mold (typically without reinforcements) and left to cure or harden. These molding methods sometimes use open molding and sometimes use closed molding.

1. Hand Lay-up
2. Spray up :- process chopped fibers and resins are sprayed simultaneously into or onto the mold.
- Hand Lay up is the most common and least expensive open molding method because it requires the least amount of equipment. Fiber reinforcements are placed by hand in a mold and resin is applied with a brush or roller. This process is used to make both large and small items, including boats, storage tanks, tubs and showers.
3. Vacuum Bagging :- uses hand lay-up uses atmospheric pressure to ~~compact~~ Compact laminate
4. Automated tape - Laying machines :- The differences are in the methods of applying the laminations to the mold, alternative curing techniques, and other differences.



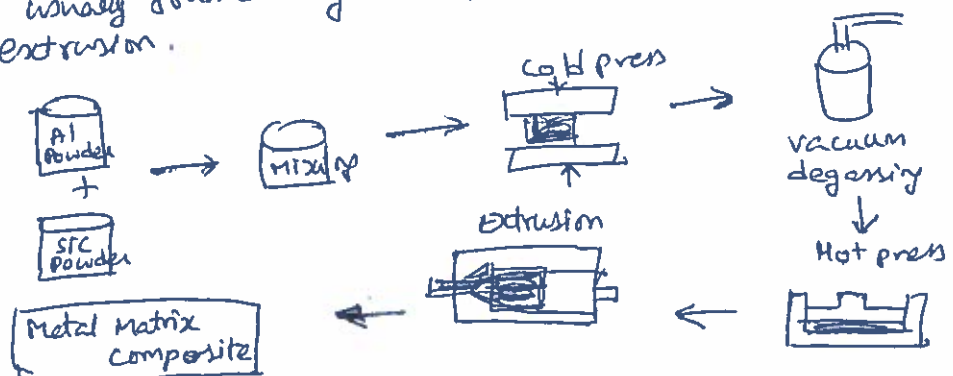
15b) How do you manufacture the composites powder metallurgy components.

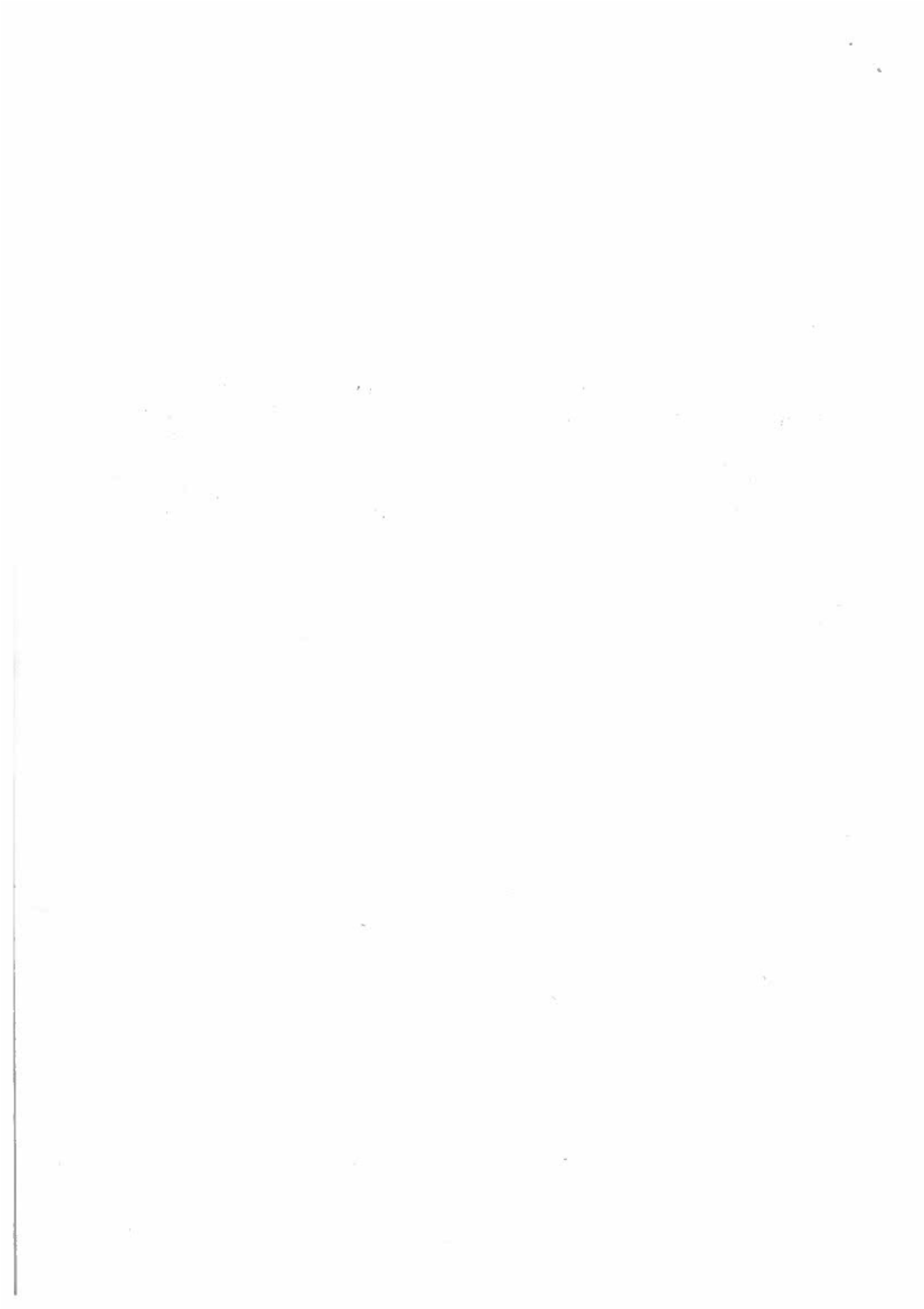
Powder production ↓	Chemical reduction Atomization Electrolytic Deposition Milling
Compaction ↓	Cold compaction Hot compaction
Sintering ↓	Solid state sintering Liquid phase sintering Activated sintering Microwave sintering Spark plasma sintering
Finishing operation	Forging Extrusion

production of Metal Matrix Composites.

Powder metallurgy consists of three distinct steps

1. mixing metal and reinforcement powders.
2. powder ~~compaction~~ compaction to produce green materials
3. Sintering usually followed by a deformation process such as hot extrusion.





Semester End Examination, February, 2022

Degree	B. Tech. (U. G.)	Program	ME	Academic Year	2021 - 2022
Course Code	20ME303	Test Duration	3 Hrs. Max. Marks 70	Semester	III
Course	Mechanics of Solids				

Part A (Short Answer Questions 5 x 2 = 10 Marks)

No.	Questions (1 through 5)	Learning Outcome (s)	DoK
1	Define Poisson's ratio.	20ME303.1	L1
2	Write the relationship between Shear force and Bending moment.	20ME303.2	L1
3	$M/I = f/y = E/R$ – justify.	20ME303.3	L2
4	What are the methods for finding out the slope and deflection at a section?	20ME303.4	L2
5	Define thin cylinder.	20ME303.5	L1

Part B (Long Answer Questions 5 x 12 = 60 Marks)

No.	Questions (6 through 15)	Marks	Learning Outcome (s)	DoK
6 (a)	The stresses on two perpendicular planes through a point in a body are 90 MPa (Tensile) and 70 MPa (Compression). Determine the normal and tangential stress on a plane at an angle 25° with the vertical. Draw configuration and Mohr's circle.	6M	20ME303.1	L2
6 (b)	A steel rod of length 5 m and 30 mm diameter is subjected to load of 50 kN. Determine change in length, change in diameter and change in volume. Take $E = 2 \times 10^5 \text{ N/mm}^2$ and Poisson's ratio = 0.25.	6M	20ME303.1	L2
OR				
7 (a)	A steel rod of length 60 cm and two copper rods of length 30 cm each together support a load of 450 kN. C/S area of steel rod is 2800 mm^2 and each copper rod is 1200 mm^2 . Solve the stresses in the rods. Take $E_s = 2 \times 10^5 \text{ N/mm}^2$ and $E_c = 1 \times 10^5 \text{ N/mm}^2$.	6M	20ME303.1	L2
7 (b)	Draw a neat stress- strain curve diagram of stainless steel and explain.	6M	20ME303.1	L2
8	A cantilever beam 2 m long fixed at A is carrying point load of 10 kN at B, C and D each and at a distance of 0.5 m, 1.0 m, and 1.5 m from the fixed end. Calculate the Shear force and bending moment at salient points.	12M	20ME303.2	L2
OR				
9 (a)	A beam of length 7 m is simply supported at its ends. It is loaded with a Uniformly Distributed load of 10 kN/m throughout and 20 kN concentrated load acting at centre. Draw the SF and BM diagrams.	7M	20ME303.2	L2
9 (b)	Draw the Shear force and bending moment diagrams for a simply supported beam of length L carrying a point load W at its middle point.	5M	20ME303.2	L2
10 (a)	Discuss the assumptions involved in the theory of simple bending.	6M	20ME303.3	L2
10 (b)	Derive the section modulus of rectangular and solid circular.	6M	20ME303.3	L2
OR				
11	Derive the bending equation.	12M	20ME303.3	L3
12 (a)	Derive the equation for deflection of a cantilever beam with a point load acting at free end by double integration method.	6M	20ME303.4	L2
12 (b)	A beam of length 7m is simply supported at its ends. It is loaded with a Uniformly Distributed load of 10 kN/m throughout and 20 kN concentrated load acting at centre. Find the central deflection by moment area method.	6M	20ME303.4	L2
OR				
13	Derive the torsion equation for circular shafts.	12M	20ME303.4	L2
14 (a)	Determine the change in dimensions of a thin cylinder due to internal pressure.	6M	20ME303.5	L3
14 (b)	Explain about circumferential and longitudinal stress.	6M	20ME303.5	L2
OR				
15	Derive Euler's crippling formula for columns.	12M	20ME303.5	L2



N S RAJU INSTITUTE OF TECHNOLOGY
(AUTONOMOUS)
SONTYAM, ANANDAPURAM, VISAKHAPATNAM – 531 173

ANSWER KEY AND SCHEME OF EVALUATION

ME-III sem

Mechanics of Solids (20ME 303)

Max Marks:70

1. Poisson's Ratio

- a. Poisson's ratio is the negative of the ratio of transverse strain to lateral or axial strain. 2M

2. Relation between Shear force and bending force

The bending moment is maximum where shear force is zero or changes its direction. The point where bending moment changes its sign is known as point of contraflexure or point of inflection. 2M

3. $M/I = f/y = E/R$ – justify.

Bending Equation. The axial deformation of the beam due to external load that is applied perpendicularly to a longitudinal axis is called the Bending Theory. The bending equation stands as $f/y = E/R = M/I$. 2M

4. Methods for finding slope and deflection at a section.

Double integration method.

Moment-area method.

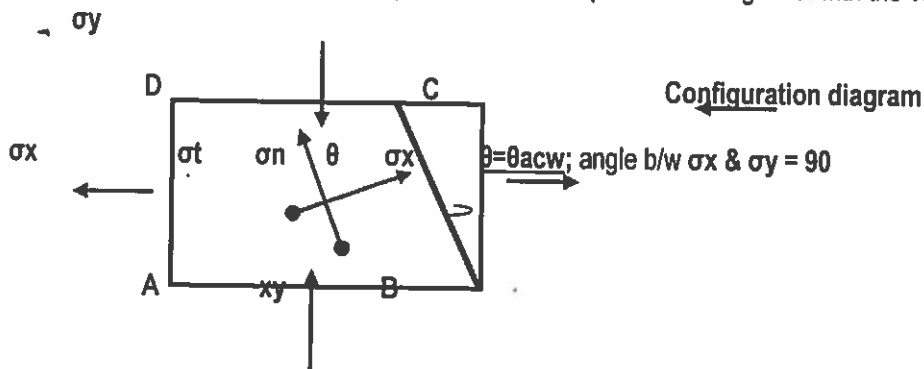
Mecaulay's method.

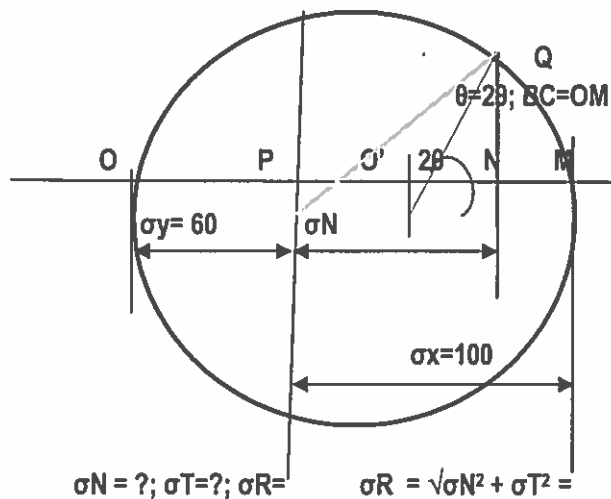
Conjugate beam method. 2M

5. Define thin cylinder

The cylinder which has a thickness is less than $1/10$ to $1/20$ of its Diameter, that cylinder is called a thin cylinder. The cylinder which has Thickness is more than $1/20$ of its diameter, that Cylinder is called a thick Cylinder. 2M

6A. The stresses on two perpendicular planes through a point in a body are 90 MPa (Tensile) and 70 MPa (Compression). Determine the normal and tangential stress on a plane at an angle 25° with the vertical (ACW).





Mohr's circle

$\sigma_x > \sigma_y$; $(\sigma_x - \sigma_y)/2$; $\theta=180$, $\Sigma r, \sigma_T$

6B.

A steel rod of 5 m long and 20mm diameter is subjected to load 50 kN. Find the elongation when young's modulus is 2×10^6 N/mm². Poisson's ratio is 0.25

Sol: Length, $l = 5000 \text{ mm}$; cross sectional area, $A = 22/7 \times 20 \times 20/4 = 400 \text{ mm}^2$; Tensile force, $P = 50 \text{ KN} = 50 \times 10^3 \text{ N}$; Young's Modulus, $E = 200 \times 10^6 \text{ N/mm}^2$.

As per the young's modulus, $E = [(P/A) / (\delta l / l)]$

Therefore, change in length or elongation, $\delta l = (Pl / AE)$

$$\delta I = (50 \times 10^3 \times 10^3) / (400 \times 200 \times 10^3)$$

$$= \frac{1}{2} = 0.6 \text{ mm}$$

7A. Area of steel bar = 2800mm²

Data: Area of steel, $A_s = 16 \text{ cm}^2$

Area of brass, $A_b = 10 \text{ cm}^2$

Weight, $W = 500 \text{ kg}$

Δl is same for steel and brass, so

$$A_s = YI \Delta l$$

The sum of forces of steel and brass is, $F_s + 2F_b = 5000$ ———(1)

$$S_s = (F_s) / (A_s), S_b = (F_b) / (A_b)$$

$$F_s = S_s A_s, F_b = S_b A_b$$

$$S_s = (Y_s)(I_s)(\Delta l)$$

S_s is directly proportional to $I_s Y_s$

Similarly, S_b is directly proportional to $I_b Y_b$

Where I_s and I_b are the moment of inertia of steel and brass

S is the stress

2M

$$S_b S_s = Y_b I_s Y_s I_b = 2 \times 32 = 34$$

$$S_s = 34 S_b$$

$$S_s A_s + 2(S_s A_b) = 5000 \text{ ———from eq. (1)}$$

$$(S_s)(A_s + 2 \times 43)(S_s A_b) = 5000$$

$$S_s (16 + 23 \times 10) = 5000$$

$$S_s (31) = 5000$$

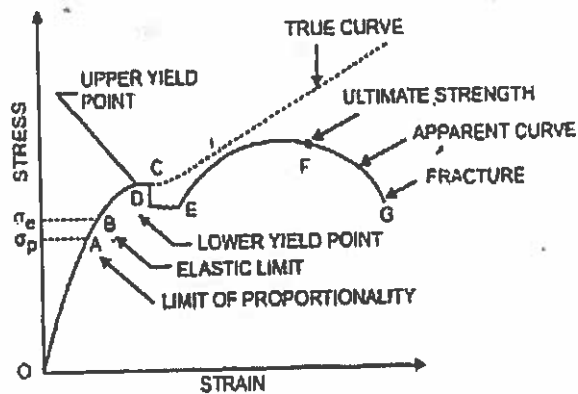
$$S_s = 315000 \cong 161 \text{ kg/cm}^2$$

$$S_b = 43 \times 115000$$

$$S_b \cong 121 \text{ kg/cm}^2$$

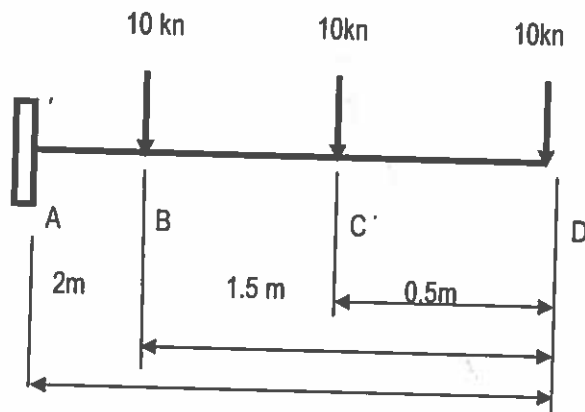
2M

7B.

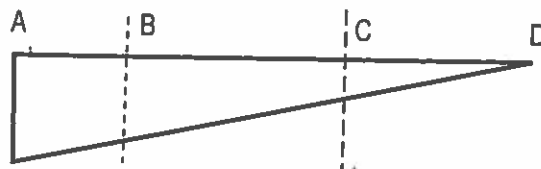
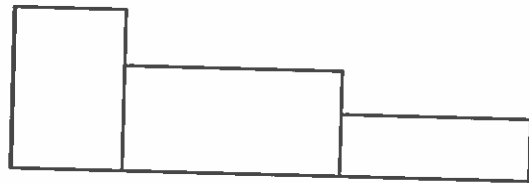


6M

8.



Shear Force Dia:



Bending Moment Dis:

Concentrated loads at B, C and D are 10 Kg each

Shear force at A = 0

Shear force at B = 30

Shear force at C = 20

Shear force at D = 10

Bending Force at D = $-10 \times 0 = 0$ kN

Bending Force at C = $-10 \times 0.5 = -5$ kN

Bending Force at B = $-10 \times 1.5 = -15$ kN

Bending Force at A = $-10 \times 2 = -20$ kN

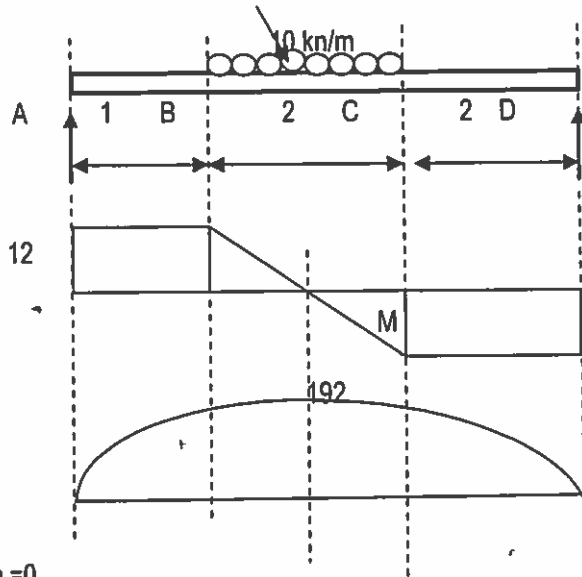
3M

3M

3M

3M

9A.



$$M_a = 0$$

$$M_c = 12 \times 1 = 12 \text{ kN-m}$$

$$M_d = 0 = 8 \times 2 = 16 \text{ kN-m}$$

$$x/12 = (2-x)/8$$

$$x = 1.2 \text{ kN-m}$$

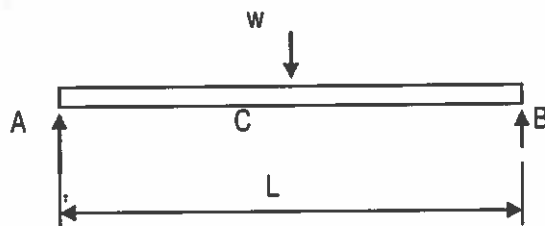
$$M_m = 12(1+1.2) - 10 \times 1.2 \times 2/2 = 19.2 \text{ kN-m}$$

8

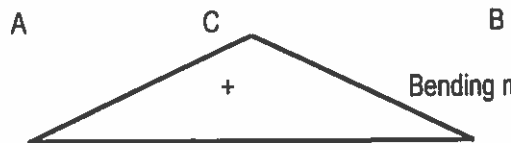
3M

3M

9B.



Force diagram



Bending moment diagram

2M

2M

2M

10A. Assumptions

1. The material of the beam is homogeneous¹ and isotropic².
2. The value of Young's Modulus of Elasticity is same in tension and compression.
3. The transverse sections which were plane before bending, remain plane after bending also.
4. The beam is initially straight and all longitudinal filaments bend into circular arcs with a common centre of curvature.

5. The radius of curvature is large as compared to the dimensions of the cross-section.
 6. Each layer of the beam is free to expand or contract, independently of the layer, above or below it.

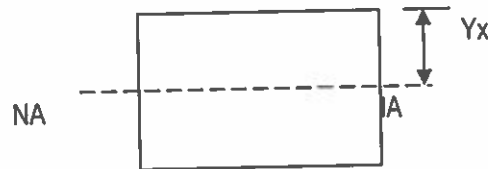
6M

10B.

The modulus of section may be defined as the ratio of moment of inertia to the distance to the extreme fibre. It is denoted by Z . $Z = I/y$; For rectangular section, $I = bd^3/12$ & $y = d/2$. $Z = bd^2/6$.

It is the ratio of moment of inertia (I) of the beam cross-section about the neutral axis to the distance (y_{max}) of extreme fiber from the neutral axis. y = distance from the centroid to top or bottom edge i.e. $y = d/2$.

2M



2M

It is the ratio of moment of inertia (I) of the beam cross-section about the neutral axis to the distance (y_{max}) of extreme fiber from the neutral axis. y = distance from the centroid to top or bottom edge i.e. $y = d/2$.

2M

11. Bending equation:

$$M I = \sigma y = E R$$

Where, E = Modulus of Elasticity of beam material

R = Radius of Curvature of the beam

M = Moment of resistance

I = Moment of Inertia of beam c/s about N.A

σ = Bending stress at the layer situated at a distance y from N.

3M

y = Distance of layer from N.A.

$$\sigma/y = E/R \text{ --- 1}$$

At the distance ' y ', let us consider an elementary strip of very small thickness dy .

' σ ' is the bending stress in this strip.

Let dA = area of this elementary strip.

The force developed in this strip = $\sigma \cdot dA$.

The elementary moment of resistance due to this elementary force is given by $dM = \sigma \cdot dA \cdot y$

3M

Total moment of resistance due to all such elementary forces is given by

$$\int dM = \int \sigma \times dA \times y \text{ or } M = \int \sigma \times dA \times y \text{ --- 2}$$

From Eq. 1, $\sigma = y \times (E/R)$ Putting this value of σ in Eq. 2

we get where I = Moment of inertia of the whole area about the neutral axis N-A.

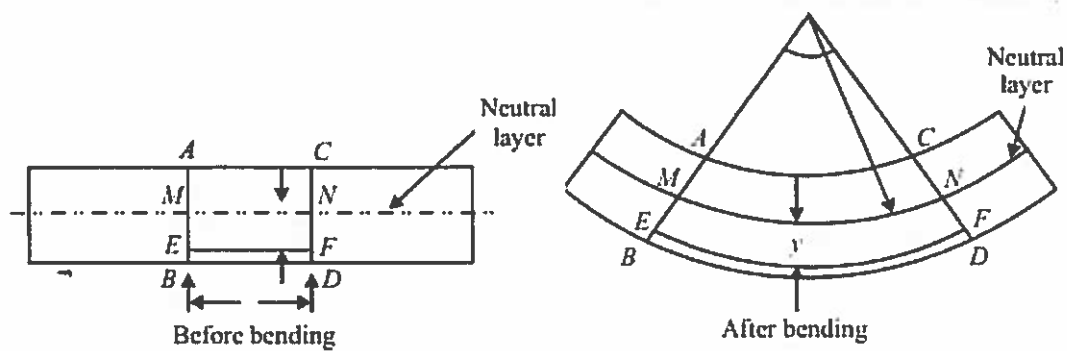
Where; M = Bending moment I = Moment of Inertia about the axis of bending i.e; I_{xx} y = Distance of the layer

The extreme distance of extreme fibre from N.A.)

E = Modulus of elasticity of the beam material.

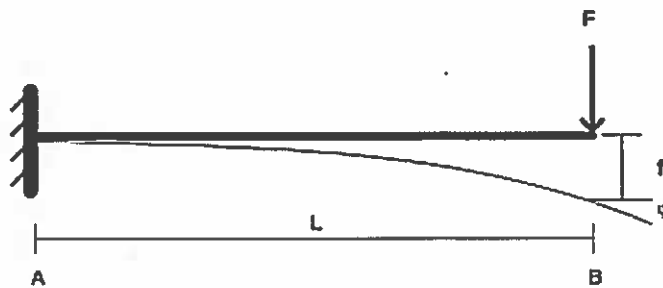
R = Radius of curvature

3M



3M

12.



4M

$$EI \frac{d^2y}{dx^2} = -Px^2 + c_1EIy = -Px^3 + c_1x + c_2EI \frac{dy}{dx}$$

$$= -Px^2 + c_1EIy$$

$$= -Px^3 + c_1x + c_2$$

$$\text{at } x=0, y=0, \text{ so } c_2=0$$

$$\text{at } x=0, \frac{dy}{dx}=0, \text{ so } c_1=0$$

4M

$$\text{So, } EIy = -Px^3/6$$

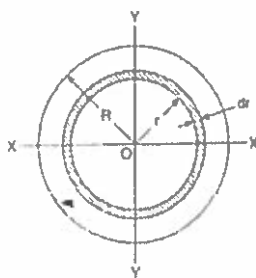
$$EIy_{\max} \text{ occurs at } L = 3L/3,$$

$$EIy_{\max} = -10 \cdot 336 = -45$$

$$EIy_{\max} = -46.67$$

4M

13.



Total torque could be easily determined by integrating the above equation between limits 0 and R.

Therefore, total torque transmitted by a circular solid shaft is, $T = \int_0^R \tau r^2 dA$ ----- 1

4M

Polar moment of inertia is ----- 2

$$J = \int r^2 dA$$

Therefore total torque transmitted by a circular solid shaft is,

$$T = tJ/R$$

$$T/J = t/R$$

4M

The expression for shear stress produced in a circular shaft subjected to torsion and therefore we have following result from that expression

$$T/J = C\theta/L$$

Considering above two equations, the torsion equation for circular shaft is, $T/J = t/R = C\theta/L$ -----3m

4M

Where,

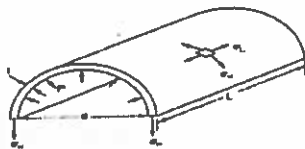
C = Modulus of rigidity

L = Length of shaft

θ = Angle of twist

14. A.

The hoop stress or circumferential stress. The bursting effect of the applied pressure is considered the equilibrium of the half of the cylinder, and



Total force on half of owing to internal pressure = $P \times \text{projected area} = pxdl$ -----1

Total resisting force owing to hoop stress σ_h in the cylinder walls = $2 \sigma_h x Lt$ -----2

2M

Equate both equations,

2M

$$pxdl = 2 \sigma_h x Lt$$
 -----3

$$\text{Circumferential or hoop stress} = pd/2t$$

Longitudinal stress

The force on the end of cylinder owing to internal pressure = $p \times \pi d^2/4$ -----4

2M

14 B. Explain about circumferential and longitudinal stresses, f_1 and f_2

Define circumferential stresses, f_1

The hoop stress is the force over area exerted circumferentially (perpendicular to the axis and the radius of the object) in both directions on every particle in the cylinder wall.

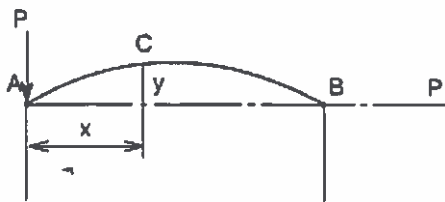
3M

Define longitudinal stresses, f_2

Longitudinal stress is defined as the stress produced when a pipe is subjected to internal pressure. Longitudinal stress is also known as axial stress.

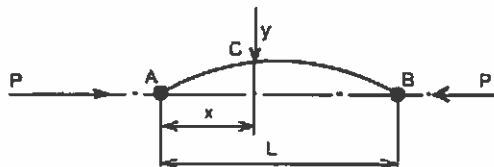
3M

15.



According to sign convention

4M



4M

Consider an axially loaded strut, shown below, and is subjected to an axial load 'P' this load 'P' produces a deflection 'y' at a distance 'x' from one end.

Assume that the ends are either pin jointed or rounded so that there is no moment at either end.

$$B \sin \{L\sqrt{p/EI}\} = 0$$

$$B=0 \text{ or } \sin \{L\sqrt{p/EI}\} = 0$$

$$\sin \{L\sqrt{p/EI}\} = 0 \text{ or } \{L\sqrt{p/EI}\} = \pi \text{ or } nL = \pi$$

$$\text{or } \{\sqrt{p/EI}\} = \pi/L \text{ or } P = \pi^2 EI / L^2$$

4M

Reply

✍

Semester End Examination, February, 2022

Degree	B. Tech. (U. G.)	Program	MECH	Academic Year	2021 - 2022
Course Code	20ME305	Test Duration	3 Hrs. Max. Marks 70	Semester	III
Course	MANUFACTURING PROCESS				

Part A (Short Answer Questions 5 x 2 = 10 Marks)

No.	Questions (1 through 5)	Learning Outcome (s)	DoK
1	List any four advantages of casting.	20ME304.1	L1
2	Illustrate the steel making processes	20ME304.2	L2
3	List any four differences between the welding and soldering	20ME304.3	L2
4	Define forge welding	20ME304.4	L1
5	Illustrate thread rolling process.	20ME304.5	L1

Part B (Long Answer Questions 5 x 12 = 60 Marks)

No.	Questions (6 through 15)	Marks	Learning Outcome (s)	DoK
6 (a)	Explain steps involved in a casting process with a neat sketch.	6M	20ME304.1	L2
6 (b)	Explain the various types of pattern and also explain any three patterns with a neat sketch.	6M	20ME304.1	L1
OR				
7 (a)	Explain injection molding and Blow molding.	6M	20ME304.1	L2
7 (b)	What are the steps involved in the preparation of a Casting? Explain briefly the die casting process.	6M	20ME304.1	L1
8 (a)	Explain the construction and working principle of Cupola Furnace with a neat sketch.	6M	20ME304.2	L2
8 (b)	How the upsetting is differ from fullering in forging process?	6M	20ME304.2	L2
OR				
9 (a)	How do you compare forged components with cast components?	6M	20ME304.2	L2
9 (b)	Explain the two types of crucible furnaces with diagrams	6M	20ME304.2	L2
10 (a)	Explain the advantages and applications of oxy-acetylene welding.	7M	20ME304.3	L2
10 (b)	What are the parameters that control the weld quality in manual metal-arc welding?	5M	20ME304.3	L2
OR				
11 (a)	Explain the TIG systems of arc-welding give the applications of each.	6M	20ME304.3	L2
11 (b)	Explain the MIG systems of arc-welding give the applications of each.	6M	20ME304.3	L2
12 (a)	Describe the electro slag welding process	5M	20ME304.4	L2
12 (b)	Describe the electron beam welding process	7M	20ME304.4	L2

OR



13 (a)	What applications would require diffusion welding?	6M	20ME304.4	L2
13 (b)	What is the term HAZ in welding process explain briefly?	6M	20ME304.4	L2
14 (a)	Explain hot rolling operations through (i) two high (ii) three high and (iii) four high rolling mill	7M	20ME304.5	L1
14 (b)	Write a note on thread rolling process.	5M	20ME304.5	L2
OR				
15 (a)	Enumerate the typical applications of cold working	4M	20ME304.5	L1
15 (b)	Explain the various methods available for blow moulding of thermoplastics giving their relative applications	8M	20ME304.5	L2

ANSWER KEY AND SCHEME OF EVALUATION

①

PART-A

1. list any four advantages of casting (2M)

Ans: 1. casting can produce very complex geometry parts
2. It can be used to make small to very large size.
3. Any intricate shape can be produced.
4. Any material can be cast ferrous & non ferrous.

2. illustrate the steel making processes. (2m)

Ans:- There are two main processes for producing steels. by means of blast furnace in combination with a converter (or) by means of an electric furnace. in the former process, iron ore is the main raw material in electric furnace scrap iron is used and occasionally also sponge iron.

3. list any four differences between the welding and soldering. (2m)

s. no	welding	soldering
1.	welding joints are the strongest	1. soldering joints are brazed joints
2.	welding requires about 6,500 °F	2. soldering requires about 840 °F.
3.	In welding the metal are heated and melted	3. It requires no heating of the workpiece

S.NO	welding	Soldering
4	metal may change at the joint due to heating and cooling during weld	4. mechanical properties don't change at all.

4. Define forge welding. (2m)

Ans: forge welding, also called fire welding is a solid state welding process that joins two pieces of metal by heating them to a high temperature and then hammering them together.

5. illustrate thread rolling process. (2m)

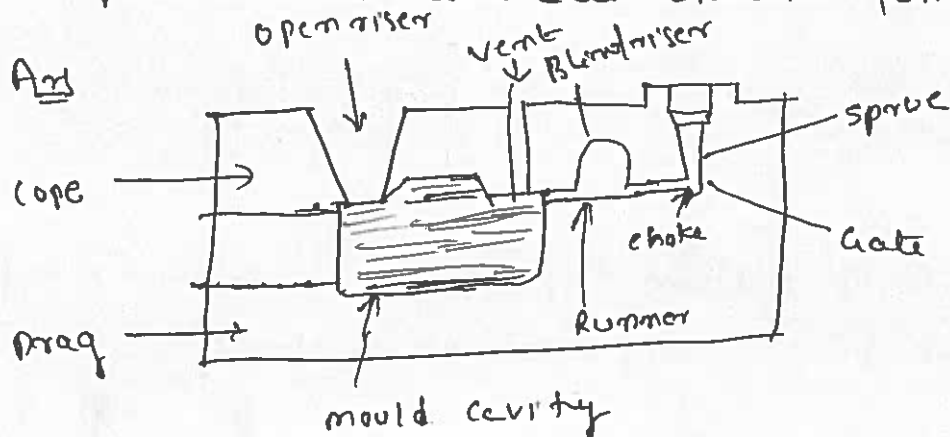
Ans:- Thread rolling is a metal forming process that forms threads in to the mirror image of a roller die. the process is different from metal cutting grinding and chaming because it does not remove any metal from the workpiece, instead, thread rolling use hardened steel dies to displace and mould ductile metals.

ANSWER KEY AND SCHEME OF EVALUATION

2

PART-B

6. a. Explain steps involved in a casting process with a neat sketch (6m)



- Process
1. place mould pattern in sand
 2. set up the gating system
 3. Remove the mould pattern
 4. pour molten metal into mould cavity
 5. wait for metal to cool.
 6. Break open mould to remove the metal casting.

6. b. Ans Explain various types of pattern and also explain any three pattern with a neat sketch. (6m)

Ans:- Types of patterns

1. solid or single piece pattern
2. split pattern
3. multi-piece patterns

4. Cope and Drag pattern

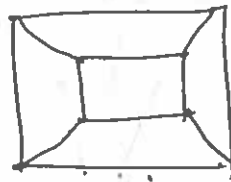
5. match plate pattern.

6. Coated pattern

7. skeleton pattern

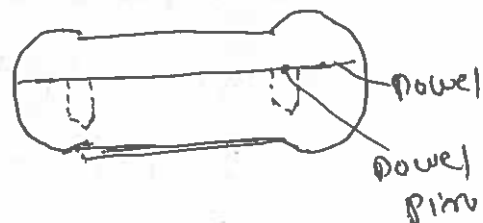
8. Sweep pattern

① Solid pattern



The solid pattern type is a most and simple method for simple shape casting.

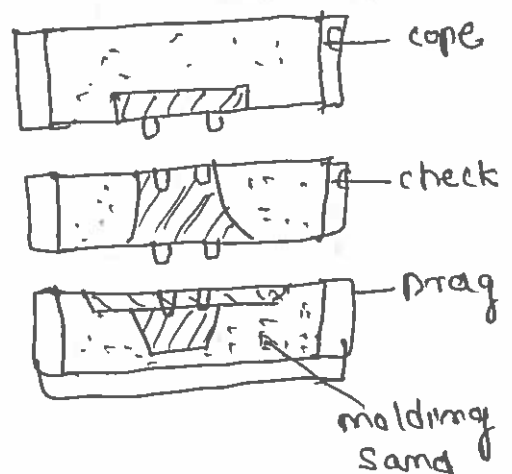
② Split pattern:-



When contour of casting manufacture, the sand casting pattern making withdraw from the mold is difficult or when the part depth too high in casting.

③ Multi-piece pattern:-

To measure a sand casting pattern making in more than two parts in order to casting piece with easily withdrawl.



ANSWER KEY AND SCHEME OF EVALUATION

3

7. a. Explain injection molding and Blow molding
Injection molding:- (2m) Blow molding
Total (6m)

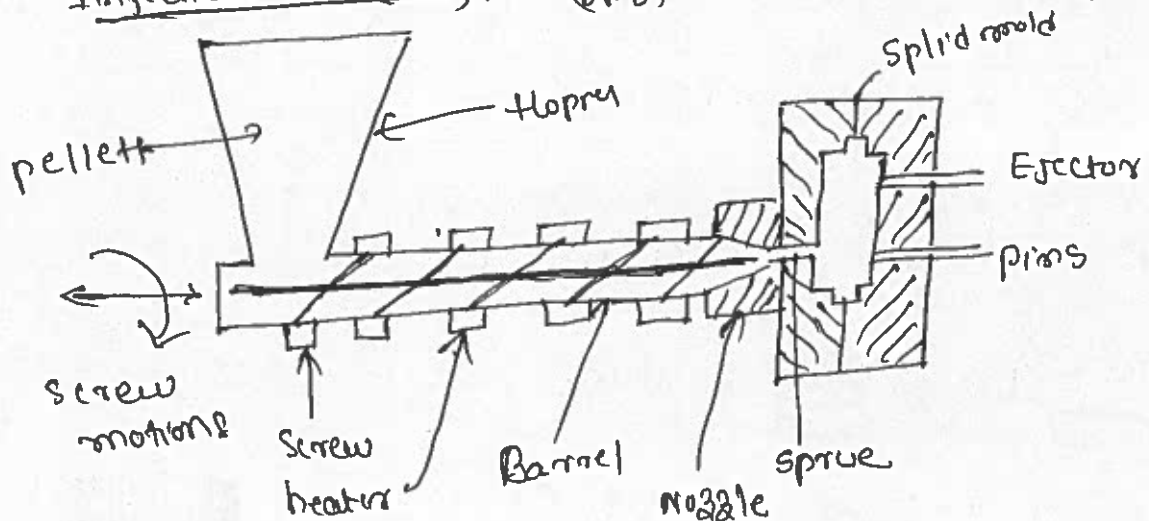
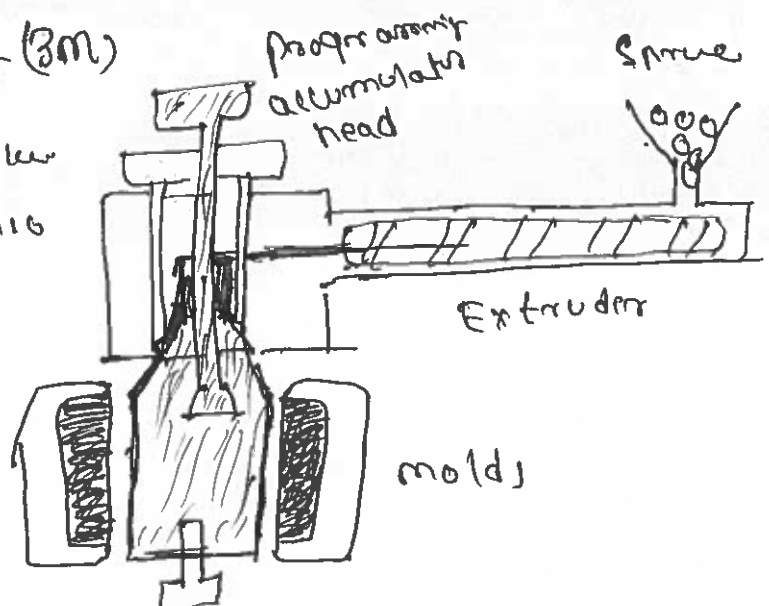


Fig: injection molding

Process:- It is the most commonly used manufacturing process for the fabrication of plastic parts. A wide variety of products are manufactured using injection molding.

Blow molding:- (3m)

Joining together hollow plastic parts. it is also used for forming it is also used for forming glass bottles and other hollow shapes



7.b. what are the steps involved in the preparation of a casting? Explain briefly the die casting process.
—(6m)

Steps:-

- ① including mold preparation
- ② filling
- ③ Injection
- ④ shake out

Die casting process:-

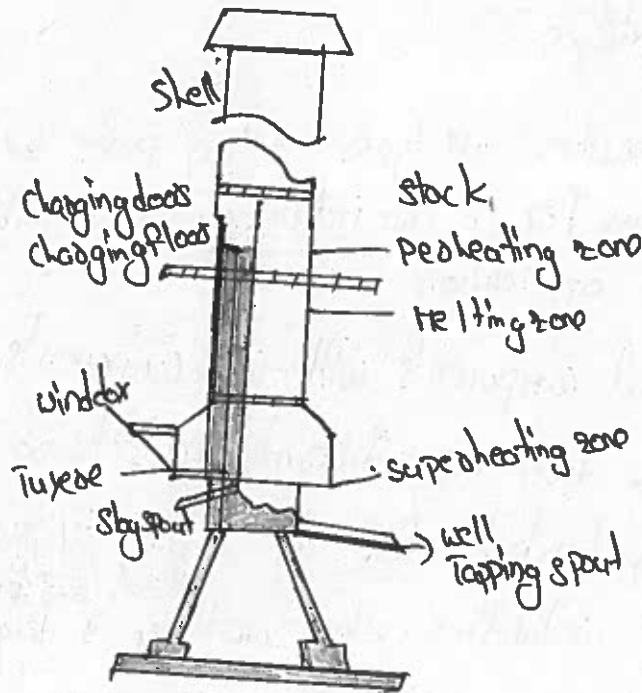
It is a metal casting process that is characterized by forcing the molten metal into a mold cavity under the application of high pressure or under gravity. Because of the metal mold, the same mold can be used for producing the infinite number of casting, hence the process is also called a permanent mold casting process.

ANSWER KEY AND SCHEME OF EVALUATION

(4)

8(a) Explain the construction and working principle of cupola furnace with a neat sketch. (6M)

Ans

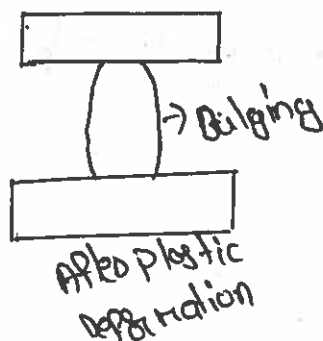
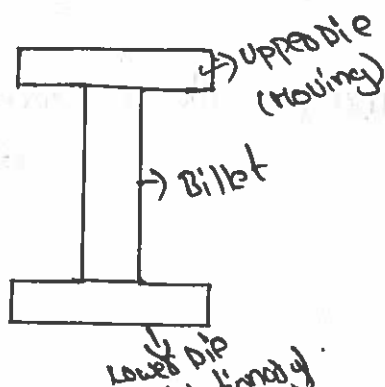


It is a very old device we used in manufacturing for melting because this system produces good cast iron from pig iron. The shape of this device is cylindrical but other size is also available.

While choosing a cupola furnace we keep some valid factors that are melting point, temperature, shape, and many more.

8(b) How the upsetting is differ from fulling in forging process? (6M)

Ans



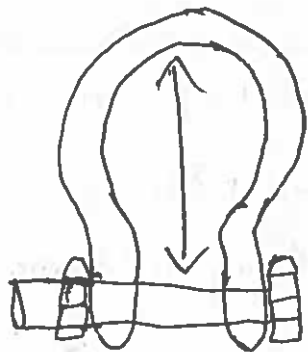
upset forging involves locally heating a metal bar and then, while holding it firmly with special tooling, applying pressure to the end of the bar in the direction of its axis to deform it.

The process allows the production of long parts with a forged end. upset forging does not allow for as large a variation between the initial bar diameter and the forged head diameter as with the electro-upsetting process. It does, however, allow for such more complex geometries to be obtained as the deformation is carried out in several stages.

upset forging is used to produce all types of long parts such as shafts, screws, piston rods and others for the automotive industry, public works, aircraft and many other fields of application.

Q(a) How do you compare forged components with cast components? (6m)

(Ans) Forging and casting are two very different manufacturing methods. When something is cast, the material is heated above its melting temperature and poured into a mold where it solidifies. When something is forged it is physically forced into shape while remaining in a solid state - although it is frequently heated.



Qb Explain the two types of crucible furnaces with diagrams. (6m)

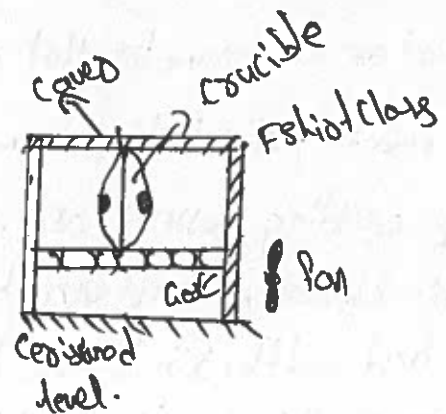
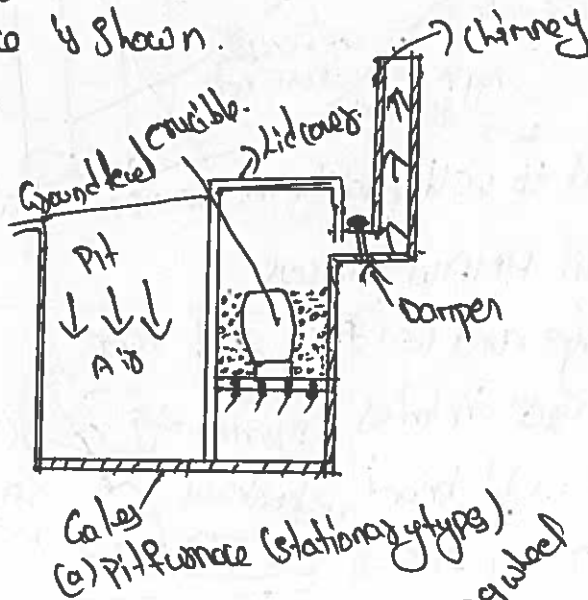
ANSWER KEY AND SCHEME OF EVALUATION

5

Ans) As the name indicates, the pit furnace is made below ground level. So the crucible can easily be lifted, operating from of the shop. The metal to be melted is put into the crucibles. The crucibles are placed in the heating chamber and sufficient coke being packed around and above the crucible.

A tall chimney attached, serves as a natural draught. The chimney draught is controlled by means of a damper.

A removable fire gate is provided at the bottom of the furnace. After sufficient melting, the crucibles are lifted by means of long handle tongs. A pit type furnace is shown.

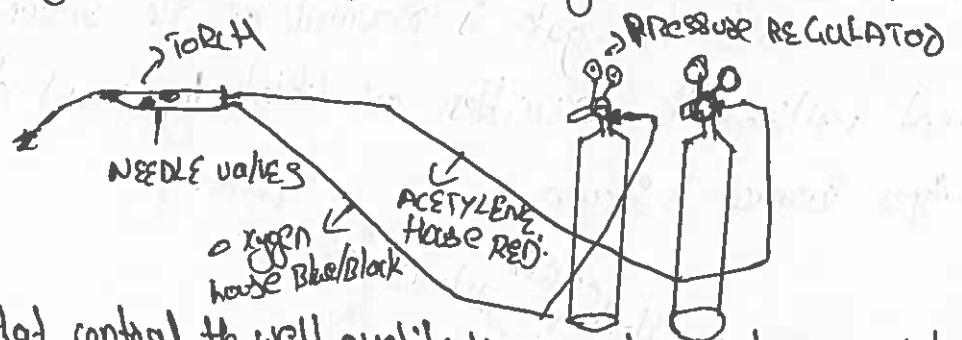


The shape of the furnace is cylindrical and it is built above the ground level. The cylindrical shape helps flames to sweep uniformly around the crucible. A fan is provided to forced draught and waste gases are taken out from the opening at the top. Back-out types furnaces are generally oil or gases fired, and are used for melting non-ferrous alloys like aluminum, magnesium, zinc, lead etc. A back out type furnace is shown.

10a) Explain advantages and applications of oxy-acetylene welding. (7m)

Ans) One of the most commonly used applications of acetylene gas is oxy-acetylene welding. The oxy-acetylene welding process uses a combination of oxygen and acetylene gases in the approved solution to produce a high temperature flame. In fact, the process uses fuel gases and oxygen to weld and cut metals respectively.

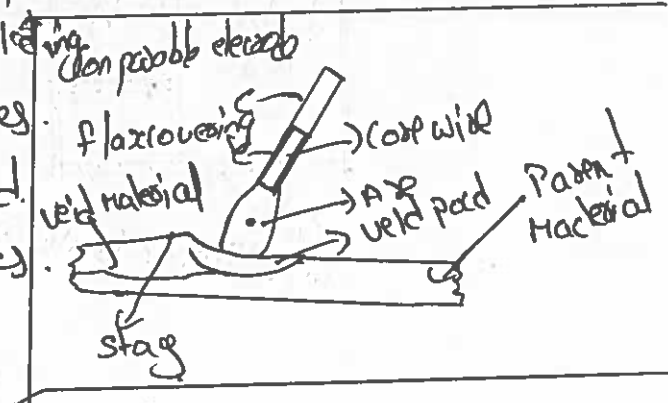
The oxygen-acetylene welding was developed by French engineers Edmond Fouche and Charles 1903.



10b) What are the parameters that control the weld quality in manual metal-arc welding? (5m)

Ans) The most prominent parameters in MMAW process

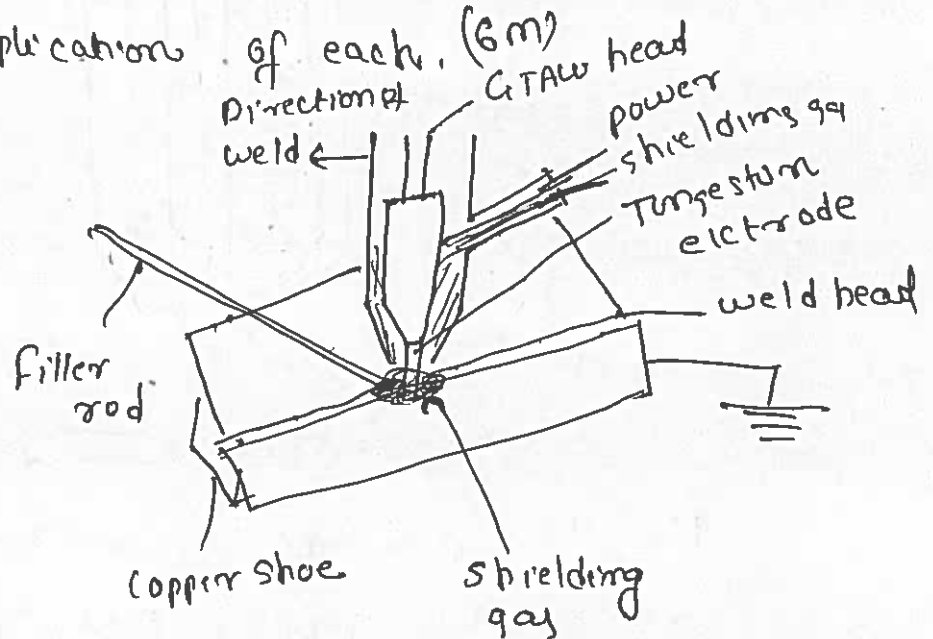
include welding current, arc voltage and welding speed. Like this, process quality measure includes weld bead width, reinforcement and weld bead hardness. To determine the feasible working ranges of each input variable, several preliminary test were conducted.



ANSWER KEY AND SCHEME OF EVALUATION

⑥

11. a Explain the TIG systems of arc welding
give the applications of each. (6m)



Process:-

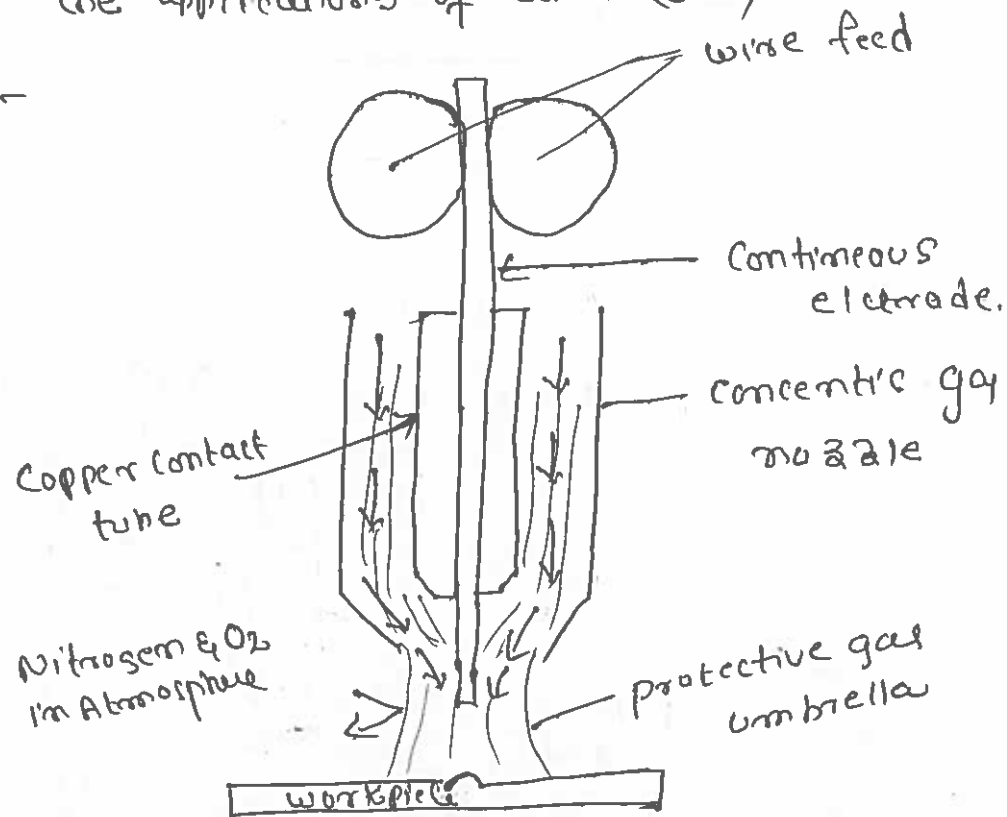
Tig welding another name gas tungsten arc welding. It can be used for more metal than any other type of process. For this reason a variety of industry rely on TIG welding.

Applications:-

1. Aerospace
2. Automotive
3. Repair
4. Art

11. b. Explain MIG system of arc-welding give the applications of each. (6m)

Ans:-



Process:- This type of welding is basically an arc welding process that joins two metals. This is done by heating the metals with the help of an arc.

Applications:-

1. Used in automotive industry
2. it is used in sheet metal welding,
3. it is used in pipe welding
4. It is also save on time and cost.

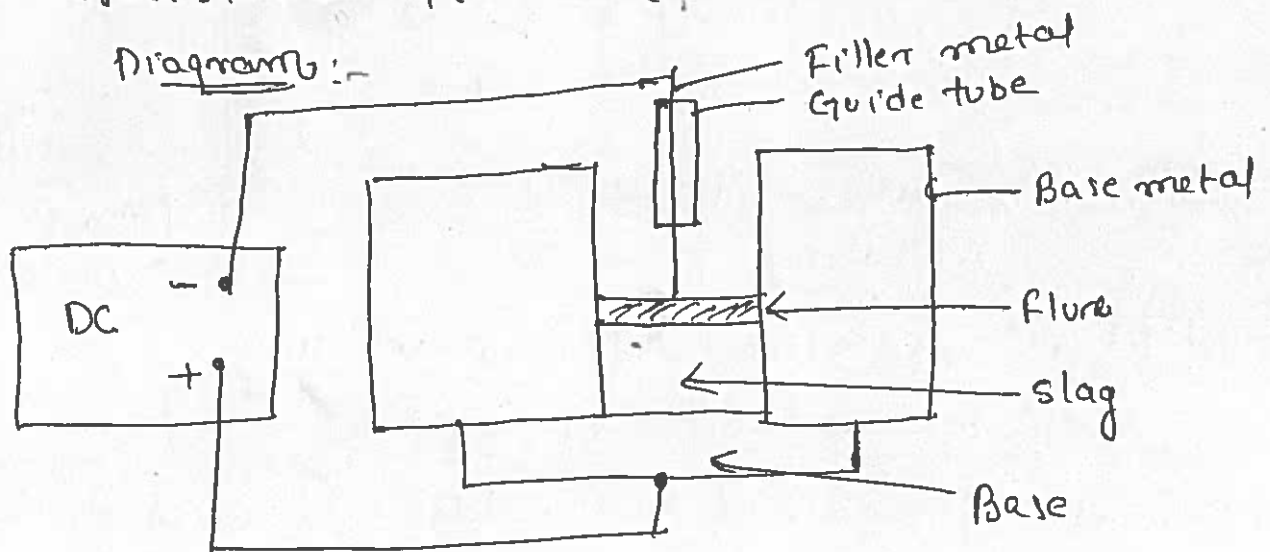
ANSWER KEY AND SCHEME OF EVALUATION

⑦

12. a. Describe the electro slag welding. (5m)

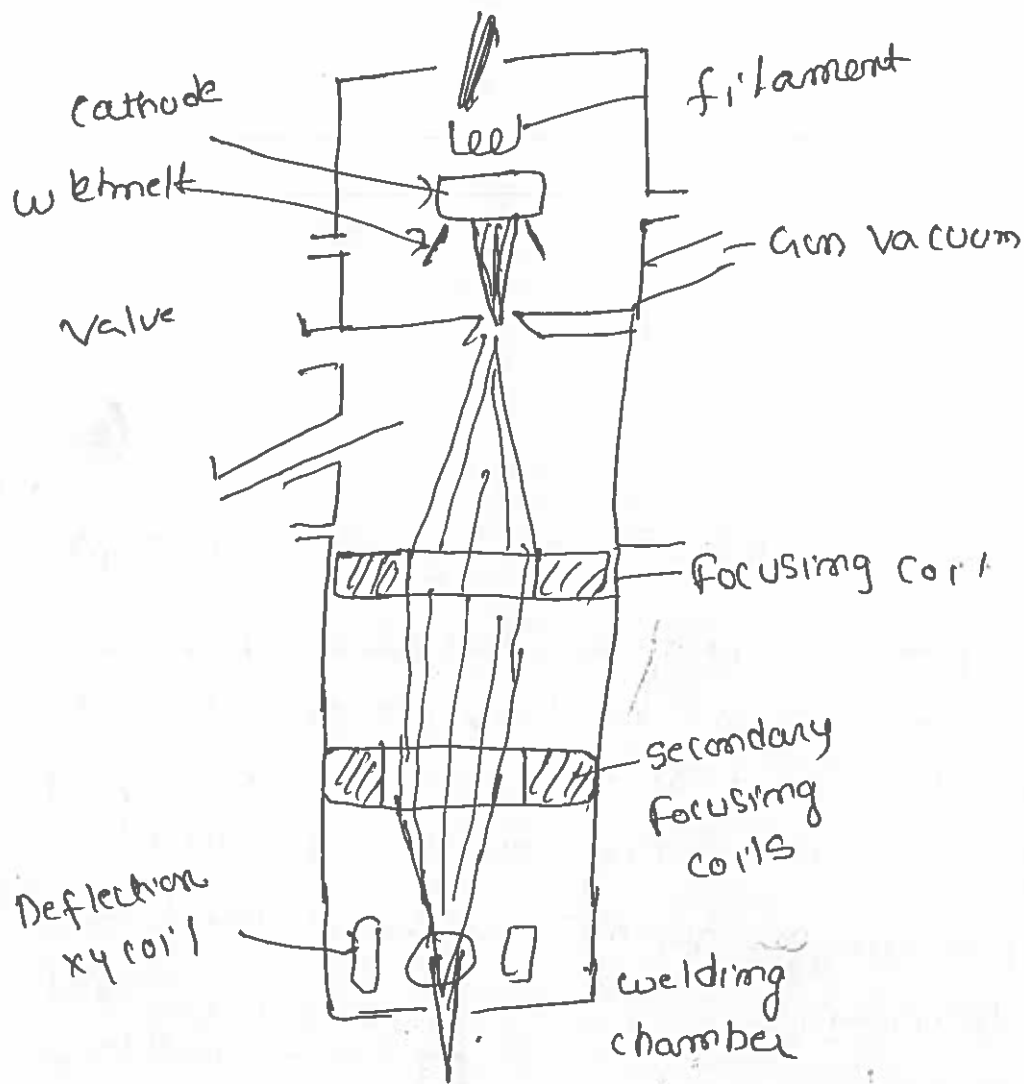
Ans:- prereq: It's a combination of Arc welding and Resistance welding. This is because an arc is created at the very beginning of the process and then weld is produced by the resistance offered by the flux used.

Diagram:-



12. b. Describe the electron beam welding process. (7m)

Prereq:- It's a fusion welding process whereby electrons are generated by an electron gun and accelerated to high speeds using electrical fields. The high speed stream of electrons is tightly ~~focus~~ focused using magnetic fields and applied to the materials to be joined.

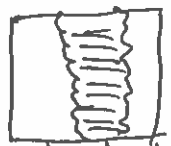


13. a. what applications would require diffusion welding. (6m)

- Ans
- 1) Solid state diffusion welding.
 - 2) Activated diffusion welding
 - 3) Liquid interface diffusion welding
 - 4) Transient liquid phase diffusion welding

13. b. what is the term HAZ in welding process explain briefly. (6m)

Ans: The heat affected zone is more melted area of metal that has undergone change in material properties or a result being exposed to high temperature. these change in material property are usually a result of welding.



ANSWER KEY AND SCHEME OF EVALUATION

(8)

14. a. Explain hot rolling operations through

- (i) two high (ii) three high (iii) four high rolling mill. (7m)

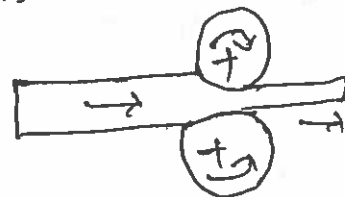
Ans :- The process is vastly used in respect of any other rolling process. In this process the metal is heated above recrystallization temperature. In the hot working process the metal is changing its grain structure because of the heat. Some types of rolling mills are listed below

- (i) Two high Rolling mill
(ii) Three high Rolling mill
(iii) Four high Rolling mill

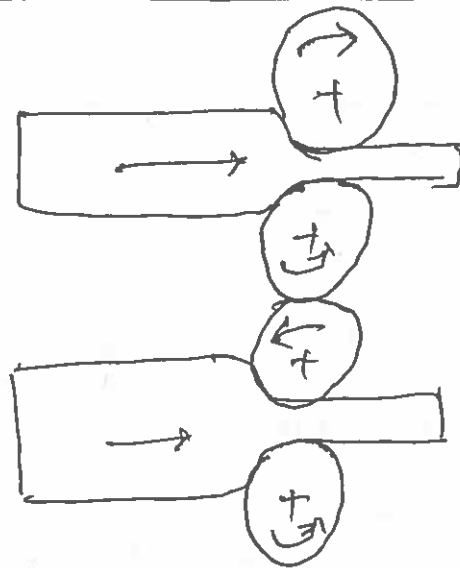
(i) Two high rolling mill

① Reversing mill

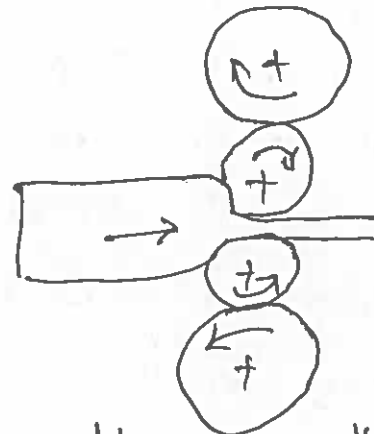
② Forward rolling mill.



(2) Three - high Rolling mills.



(3) Four high Rolling mills



14. b write a note on thread rolling process.

Ans:-

Thread rolling produces rolled ^(5m) thread on a wide variety of parts. often rolled threads are required by design because of their superior tensile, shear and fatigue strength. other process remove material to produce the thread form but thread rolling displaces the material with hardened steel dies.

ANSWER KEY AND SCHEME OF EVALUATION

9

15/a) Enumerate the typical applications of cold working.

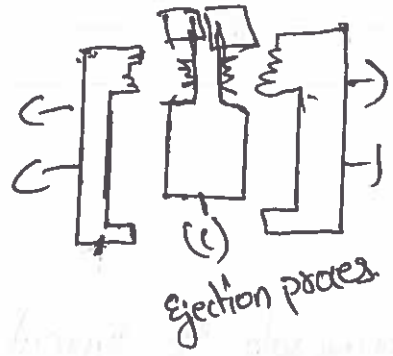
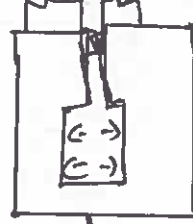
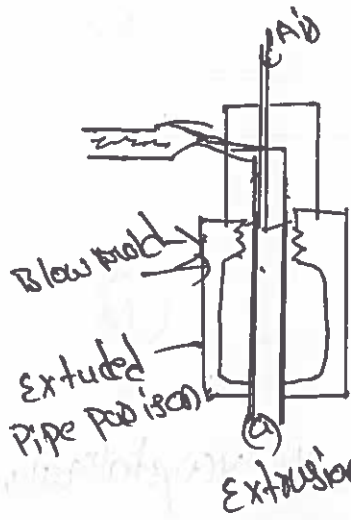
Ans cold working is the plastic deformation of metals below the recrystallization temperature. In most cases, such cold forming is done at room temperature. The major cold working operations can be classified basically as: squeezing, bending, shearing and drawing.

Cold working is the plastic deformation of metals below the recrystallization temperature. In most cases of manufacturing, such cold forming is done at room temperature. Sometime however, the working may be done at room temperature. Sometime, however the working may be done at elevated temperature that will provide increased ductility and reduced strength but will be below the recrystallization temperature.

15/b) Explain the various methods available for blow molding of thermoplastic giving their relative applications.

Ans There are two processes used to manufacture high quality plastic moldings: injection molding and blow molding. With injection molding, the thermoplastic resin molds are injected with liquid polymers at high temperature under external pressure.

Blow molding process



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HOD-ME

Semester End Examination, January/February, 2022

Degree	B. Tech. (U. G.)	Program	ECE	Academic Year	2021 - 2022
Course Code	20BSX14	Test Duration	3 Hrs.	Max. Marks	70
Course	Complex Variable & Transforms				
Semester	III				

Part A (Short Answers Questions 5 X 2 =10 Marks)

No	Questions (1 through 5)	Learning Outcome(s)	DoK
1	Prove that u, v are harmonic functions if $f(z) = u+iv$ is analytic function.	20BSX14.1	L1
2	Define Residue of a complex function.	20BSX14.2	L1
3	Find bn of the Fourier series to represent the function $f(x) = \sin x , -\pi < x < \pi$	20BSX14.3	L3
4	Find L { $\cos^2 x$ }	20BSX14.4	L2
5	Express as Fourier sine and cosine transform of $f(x) = \frac{e^{-ax}}{x}$	20BSX14.5	L2

Part B (Long Answers Questions 5 X 12 =60 Marks)

No	Questions (1 through 5)	Marks	Learning Outcome(s)	DoK
6(a)	Find the analytic function, whose real part is $\frac{\sin 2x}{\cosh 2y - \cos 2x}$	6M	20BSX14.1	L3
6(b)	Evaluate $\int_{-i}^{2+i} (2x+1+iy)dz$ along the straight line joining (1, -i) and (2, i)	6M	20BSX14.1	L2
OR				
7(a)	Prove that $f(z) = \begin{cases} \frac{xy^2(x+iy)}{x^2+y^4}, z \neq 0 \\ 0, z = 0 \end{cases}$ is not analytic at $z = 0$, although CR equations are satisfied at the origin.	6M	20BSX14.1	L2
7(b)	Using Cauchy's Integral formula, evaluate $\int_C \frac{z^2 - z + 1}{z - 1} dz$, where C is $ z = \frac{1}{2}$.	6M	20BSX14.1	L3
8(a)	Obtain the Taylor's series of $f(z) = \frac{2z^3 + 1}{z^2 + z}$ valid about the point $z = i$.	6M	20BSX14.2	L2
8(b)	Evaluate $\int_C \frac{z-3}{z^2+2z+5} dz$, where C is $ z+1+i = 2$.	6M	20BSX14.2	L3
OR				
9(a)	Explain various types of singular points and find the nature of singularity of the function $f(z) = \sin \frac{1}{1-z}$.	6M	20BSX14.2	L2

9(b)	By Cauchy's residue theorem, evaluate $\int_C \frac{dz}{(z^2 + 4)^2}$, where C is $ z - i = 2$.	6M	20BSX14.2	L3
10(a)	Find cosine series for $f(x) = (\pi - x)^2$ in $[0, \pi]$.	6M	20BSX14.3	L3
10(b)	Find the Fourier series expansion for $f(x)$, if $f(x) = \begin{cases} 2, & \text{if } -2 \leq x \leq 0 \\ x, & \text{if } 0 < x < 2 \end{cases}$	6M	20BSX14.3	L3
OR				
11(a)	Expand $f(x) = \begin{cases} 1, & 0 < x < \pi \\ 0, & \pi < x < 2\pi \end{cases}$ as Fourier series	6M	20BSX14.3	L3
11(b)	Find half range sine series for $f(x) = x + x^2$ in $[0, 1]$.	6M	20BSX14.3	L3
12		12M	20BSX14.4	L2
Find (i) $L\{t^2 + 1\}$ (ii) $L\{\sin t + \cos t\}$				
OR				
13(a)	$f(t) = \begin{cases} 1, & 0 < t < 1 \\ -1, & 1 \leq t < 2 \end{cases}$ is a periodic function with period 2. Then find its Laplace transform.	6M	20BSX14.4	L3
13(b)	Solve $y'' + y = \sin 3t$, $y(0) = y'(0) = 0$	6M	20BSX14.4	L3
14(a)	Find the Fourier sine and cosine transform of $f(x) = \frac{e^{-2x}}{x}$ and deduce that $\int_0^\infty \frac{e^{-2x} - e^{-3x}}{x} \sin sx = \tan^{-1}\left(\frac{s}{2}\right) - \tan^{-1}\left(\frac{s}{3}\right)$	6M	20BSX14.5	L3
14(b)	Find the Fourier transform of $f(x) = \begin{cases} a^2 - x^2, & \text{if } x < a \\ 0, & \text{if } x > a \end{cases}$ $a > 0$. Hence Show that $\int_0^\infty \frac{\sin x - x \cos x}{x^3} dx = \frac{\pi}{4}$	6M	20BSX14.5	L3
OR				
15(a)	Using Fourier integral show that $\int_0^\infty \frac{1 - \cos \pi \lambda}{\lambda} \sin x \lambda d\lambda = \begin{cases} \pi/2, & \text{if } 0 < x < \pi \\ 0, & \text{if } x > \pi \end{cases}$	6M	20BSX14.5	L3
15(b)	Find the inverse Fourier cosine transform of $\frac{\sin ap}{p}$.	6M	20BSX14.5	L3

I. ANSWER ALL THE QUESTIONS: $5 \times 2 = 10$

① Prove that u, v are harmonic functions if $f(z) = u + iv$ is analytic function.

Ans:- Given $f(z) = u + iv$.

Harmonic function Means. $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

and $\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0 \rightarrow \text{① Mark}$

For writing formulae $\rightarrow \text{① Mark}$

Total : 2 Marks

② Define residue of a complex Function.

Ans: The co-efficient of $\frac{1}{z-a}$ in the Laurent series expansion of $f(z)$ about the isolated singularity $z=a$ is called the residue of $f(z)$ at the point

$$\int_c f(z) dz = 2\pi i \times [R_1 + R_2 + \dots + R_n] \rightarrow \text{1 Mark}$$

$\rightarrow \text{1 Mark}$

- ③ Find b_n of the Fourier series to represent the function $f(x) = |\sin x|$, $-\pi < x < \pi$.

Sol:- Given that $f(x) = |\sin x|$
the Limits are $-\pi < x < \pi$

Since $|\sin x|$ is an even function

$\therefore b_n = 0$ for all $n \longrightarrow 1 \text{ Mark}$

Hence the solution.

- ④ Find $L\{\cos^2 3t\}$.

Sol:- Let $f(t) = \cos^2 3t$

$$\therefore \cos^2 t = \frac{1 + \cos 2t}{2}$$

$$\cos^2 t = \frac{1 + \cos 6t}{2}$$

$$\cos^2 3t =$$

$$L\{\cos^2 3t\} = L\left\{\frac{1 + \cos 6t}{2}\right\}$$

$$= \frac{1}{2} L\{1\} + \frac{1}{2} L\{\cos 6t\}$$

$$= \frac{1}{2} \left(\frac{1}{s}\right) + \frac{1}{2} \left(\frac{6}{s^2 + 6^2}\right)$$

$$= \frac{1}{2s} + \frac{3}{s^2 + 36}$$

5) Find Fourier sine and cosine transform of $f(x) = \frac{e^{-ax}}{x}$ (3)

Sol:- Fourier Sine Transform

We have

$$F_s \{ f(x) \} = \int_0^{\infty} f(x) \sin px \, dx$$

$$F_c \{ f(x) \} = \int_0^{\infty} f(x) \cos px \, dx$$

$$\therefore F_s \{ f(x) \} = \int_0^{\infty} \frac{e^{-ax}}{x} \sin px \, dx = \frac{a}{a^2 + b^2}$$

Similarly cosine $\rightarrow 1 \text{ Mark}$
Total $\rightarrow 2 \text{ Marks}$

$$\therefore \frac{a}{a^2 + b^2} \text{ and } -\frac{b}{a^2 + b^2}$$

II. LONG ANSWER QUESTIONS: 5x12=60 Marks.

6) (a) Find the analytic function, whose real part is $\frac{\sinh 2y - \cos 2x}{\cosh 2y - \cos 2x}$

Sol:- Given that $u = \frac{\sinh 2y - \cos 2x}{\cosh 2y - \cos 2x}$

$$\text{Let } f(z) = u + iv$$

$$f'(z) = \frac{\partial u}{\partial x} + i \frac{\partial v}{\partial x} = \frac{\partial u}{\partial x} - i \frac{\partial u}{\partial y}$$

$$= \frac{[\cosh 2y - \cos 2x] 2 \cos 2x - \sinh 2x [2 \sinh 2x]}{(\cosh 2y - \cos 2x)^2} \quad (4)$$

$$- i \frac{[\sinh 2x (-2 \sinh 2y)]}{(\cosh 2y - \cos 2x)^2} \longrightarrow 2 \text{ Marks.}$$

Now using Milne-Thomson Method

put $x = z$ and $y = 0$. $\longrightarrow 2 \text{ Marks}$

$$\therefore f'(z) = \frac{-2}{1 - \cos 2z} = -\cot^2 z$$

$$\therefore f(z) = \cot z + ic \longrightarrow 2 \text{ Marks.}$$

Total = 6 Marks.

⑥ ⑥ Evaluate

$\int_{1-i}^{2+i} (2x+1+i y) dz$ along the A. line
joining $(1, -i)$ and $(2, i)$.

Sol:- Given that $\int_{1-i}^{2+i} (2x+1+i y) dz$

The given points are $(1, -1)$, $(2, 1)$

$$\therefore m = \frac{1+1}{2-1} = 2$$

$$y - y_1 = m(x - x_1)$$

$$y + 1 = 2(x - 1)$$

$$y + 1 = 2x - 2$$

$$\Rightarrow 2x - y - 2 + 1 = 0$$

$$2x - y - 1 = 0$$

$$2x = y + 1 \text{ or } y = 2x - 1$$

$$\int_{(1,-1)}^{(2,1)} [2x+1 + i(2x-3)] [dx + i2dy] \rightarrow 4 \text{ Marks} \quad (5)$$

$$\int_{(1,-1)}^{(2,1)} [2x+1 + i(2x-3)] (1+2i) dx$$

$$= (1+2i)(4) = 4 + 8i \rightarrow 2 \text{ Marks}$$

$$\text{Ans} = 4 + 8i \rightarrow 6 \text{ Marks.}$$

7(a) Prove that

$$f(z) = \begin{cases} \frac{xy^2(x+iy)}{x^2+y^4}, & z \neq 0 \\ 0, & z = 0 \end{cases}$$

is not analytic at $z=0$ although CR conditions are satisfied at the origin

Sol:- $\frac{f(z) - f(0)}{z - 0} = \frac{f(z)}{z}$

$$= \frac{xy^2(x+iy)}{(x^2+y^4)z} = \frac{xy^2}{x^2+y^4}$$

clearly $\lim_{\substack{x \rightarrow 0 \\ y \rightarrow 0}} \frac{xy^2}{x^2+y^4} = 0 = \lim_{\substack{y \rightarrow 0 \\ x \rightarrow 0}} \frac{xy^2}{x^2+y^4}$

Along the path $y = mx \rightarrow 1 \text{ Mark}$

we get $= 0 \rightarrow 1 \text{ Mark}$

Along the path $x = my^2 \neq 0$.

To prove C-R conditions are satisfied at (0)
the origin

$$\text{Let } f(z) = u + iv = \frac{xy^2(x+iy)}{x^4+y^4}$$

$$\therefore u = \frac{x^2 y^2}{x^4 + y^4} \text{ and } v = \frac{xy^3}{x^4 + y^4}$$

Also $u(0,0) = 0$ and $v(0,0) = 0$

$$\text{Now } \frac{\partial u}{\partial x} = \lim_{h \rightarrow 0} \frac{u(x,0) - u(0,0)}{h} = 0$$

$$\frac{\partial u}{\partial y} = 0$$

$$\frac{\partial u}{\partial x} = 0 \text{ and } \frac{\partial v}{\partial y} = 0 \rightarrow 2 \text{ Marks}$$

Thus C-R conditions are satisfied
at the origin

Hence $f(z)$ is not analytic at $z = 0$
even though C-R conditions are
satisfied at the origin $\rightarrow 2 \text{ Marks}$

Hence the solution

Total Marks = 6 Marks

⑦ ⑥ Using Cauchy's integral formula ⑦

$$\int_c \frac{z^2 - z + 1}{z - 1} dz, \text{ where } c \text{ is } |z| = \frac{1}{2}$$

Sol:- Given that $\int_c \frac{z^2 - z + 1}{z - 1} dz$
and $|z| = \frac{1}{2} \rightarrow 2 \text{ Marks}$

$\therefore$ The singular point $z = 1$

$\therefore$ By Cauchy integral formula

$$f(a) = \frac{1}{2\pi i} \int \frac{f(z)}{z - a} dz \rightarrow 2 \text{ Marks}$$

$$2\pi i f(a) = \int \frac{z^2 - z + 1}{z - 1} dz$$

put $a = \frac{1}{2}$ and solving

we get $\text{Ans} = 0 \rightarrow 2 \text{ Marks}$

$\text{Ans} = 0$ Total $\rightarrow 6 \text{ Marks}$

⑧ (a) Obtain Taylor's series of

⑧

$$f(z) = \frac{2z^3+1}{z^2+z} \text{ Valid about the point } z=i$$

Sol:- $f(z) = \frac{2z^3+1}{z^2+z}$ at $z=i$

$$f(z) = \frac{2z^3+1}{z^2+z} = (2z-2) + \frac{2z+1}{z(z+1)}$$
$$= 2(z-1) + \frac{1}{z} + \frac{1}{z+1}$$

Differentiate 'n' times we get $\rightarrow$ 2 Marks

$$f^{(n)}(z) = (-1)^n n! \left[\frac{1}{z^{n+1}} + \frac{1}{(z+1)^{n+1}} \right]$$

$$f^{(n)}(i) = (-1)^n n! \left[\frac{1}{i^{n+1}} + \frac{1}{(1+i)^{n+1}} \right]$$

$$\therefore f(z) = f(a) + \sum_{n=1}^{\infty} \frac{(z-a)^n}{n!} f^{(n)}(a)$$

put $a=i$

$$f(z) = f(i) + \sum_{n=1}^{\infty} \frac{(z-i)^n}{n!} f^{(n)}(i) \rightarrow 2 \text{ Marks}$$

$$= \frac{1}{2} - \frac{3}{2} + (z-i) \left(3 + \frac{i}{z} \right) + \dots$$

$\rightarrow$ 2 Marks.

Total: 6 Marks.

⑧ (b) Evaluate $\int_c \left(\frac{z-3}{z^2+2z+5} \right) dz$ (9)

where c is the $|z+1+i|=2$

Sol:- Here $f(z) = \frac{z-3}{z^2+2z+5}$

Here only the pole $z = -1-2i$ lies in the circle $|z+1+i|=2 \rightarrow 2 \text{ Marks}$

$\therefore f(z)$ is analytic everywhere except at the point $z = -1-2i$

$$\therefore (\text{Res } f(z))_{z=-1-2i} = \lim_{z \rightarrow -1-2i} \frac{(z+1+2i)(z-3)}{z^2+2z+5}$$

$$= \frac{1}{2} - i \rightarrow 2 \text{ Marks}$$

$\therefore$ By Residue theorem

$$\int_c \left(\frac{z-3}{z^2+2z+5} \right) dz$$

$$= 2\pi i (\text{Res } f(z))_{z=-1-2i}$$

$$= 2\pi i \left(\frac{1}{2} - i \right)$$

$$= \pi (2+i) \rightarrow 2 \text{ Marks}$$

Total $= \underline{6 \text{ Marks}}$

9) a) Explain Various types of Singular points and find the nature of $f(z) = \sin\left(\frac{1}{1-z}\right)$ (10)

Ans: - (1) Isolated Singularity: A point $z = a$ is called an isolated singularity of an analytic function $f(z)$, if

$f(z)$ is not analytic at $z = a$

(2) $f(z) = \frac{e^z}{z^2 + 1}$ then $z = \pm i$ are two isolated points } 2 Marks

(3) Removable Singularity

$f(z) = \sin\left(\frac{1}{1-z}\right)$ then $z = 1$ is a

removable singularity
→ 2 Marks

(4) $f(z) = \frac{\sin z}{z}$ so $z = 0$ is called

removable singularity
→ 2 Marks

Total → 6 Marks

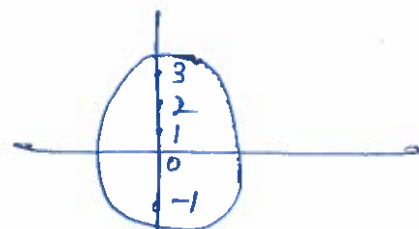
Hence the problem

Q) By Cauchy's residue theorem

(11)

$$\int_C \frac{dz}{(z^2+4)^2} \text{ where } C \text{ is } |z-i|=2$$

Sol:- The integrand $\frac{1}{(z^2+4)^2}$ has double poles at $z = \pm 2i$ of these poles only $z = 2i$ lies inside C



→ 1 Mark

Again $\int_C \frac{dz}{(z^2+4)^2} = \int_C \frac{1}{(z+2i)^2} \cdot \frac{dz}{(z-2i)^2} \rightarrow 1 \text{ Mark}$

Since $\frac{1}{(z+2i)^2}$ is analytic in C , apply Cauchy's integral formula for derivatives

$$\begin{aligned} \therefore \int_C \frac{1}{(z^2+4)^2} dz &= \frac{2\pi i}{1!} \frac{d}{dz} \left[\frac{1}{(z+2i)^2} \right]_{z=2i} \rightarrow 2 \text{ Mark} \\ &= 2\pi i \left[\frac{-2}{(z+2i)^3} \right]_{z=2i} = \frac{\pi}{16} \end{aligned}$$

Total = 6 Marks

→ 2 Mark

10Q) Find the cosine series for

$$f(x) = (\pi-x)^2 \text{ in } [0, \pi]$$

Sol:- Given that $f(x) = (\pi-x)^2 \rightarrow$

$$\therefore f(x) = (\pi-x)^2 = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx \rightarrow 1 \text{ Mark}$$

$$\therefore a_0 = \frac{2}{\pi} \int_0^{\pi} f(x) dx$$

$$\begin{aligned}
 &= \frac{2}{\pi} \int_0^{\pi} (\pi - x)^2 dx = \frac{2}{\pi} \int_0^{\pi} [\pi^2 + x^2 - 2\pi x] dx \quad (12) \\
 &= \frac{2}{\pi} \left[\pi^2 x + \frac{x^3}{3} - 2\pi \frac{x^2}{2} \right]_0^{\pi} \\
 &= \frac{2}{\pi} \left[\pi^3 + \frac{\pi^3}{3} - \pi^3 \right] = \frac{2\pi^2}{3} \rightarrow 1 \text{ Mark}
 \end{aligned}$$

$$\therefore a_n = \frac{2}{\pi} \int_0^{\pi} f(x) \cos nx dx$$

$$= \frac{2}{\pi} \int_0^{\pi} (\pi - x)^2 \cos nx dx$$

$$= \frac{2}{\pi} \left[(\pi - x)^2 \cdot \frac{\sin nx}{n} - \int \frac{2(\pi - x)(-1) \sin nx}{n} dx \right]_0^{\pi}$$

$$= \frac{2}{\pi} \left[(\pi - x)^2 \sin nx + \frac{2}{n} \int (\pi - x) \sin nx dx \right]_0^{\pi}$$

$$= \frac{2}{\pi} \left[(\pi - x)^2 \sin nx + \frac{2}{n} \left\{ (\pi - x) \int \sin nx dx - \int (-1)(-\cos nx) dx \right\} \right]_0^{\pi}$$

$$= \frac{2}{\pi} \left[(\pi - x)^2 \sin nx + \frac{2}{n} \left\{ \frac{(\pi - x)(-\cos nx)}{n} - \frac{1}{n} \frac{\sin nx}{n} \right\} \right]_0^{\pi}$$

$$= \frac{2}{\pi} \left[(\pi - x)^2 \sin nx - \frac{2}{n^2} (\pi - x) \cos nx - \frac{2}{n^3} \sin nx \right]_0^{\pi}$$

$$= \frac{2}{\pi} \left[\left(0 - 0 - \frac{2}{n^3} \sin n\pi \right) - \left(0 - \frac{2\pi}{n^2} \right) \right]$$

$$= \frac{2}{\pi} \left[-\frac{2}{n^3} \sin n\pi + \frac{2\pi}{n^2} \right]$$

$$= \frac{-4}{\pi n^4} \sin n\pi + \frac{4\pi}{n^3 \pi} = \frac{4}{n^3} \rightarrow 2 \text{ Marks}$$

(13)

$$\therefore f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$$

$$= \frac{\frac{2\pi^2}{3 \times 2}}{3} + \sum_{n=1}^{\infty} \frac{4 \cos nx}{n^3}$$

$$= \frac{\pi^2}{3} + \sum_{n=1}^{\infty} \frac{4 \cos nx}{n^3} \rightarrow 3 \text{ Marks}$$

Hence the solution

10 (b) Find the Fourier series expansion for $f(x)$.

$$f(x) = \begin{cases} 2 & \text{for } -2 \leq x \leq 0 \\ x & \text{for } 0 < x < 2 \end{cases}$$

Sol:- The Fourier series of $f(x)$ in the interval $(0, 2\pi)$ is given by

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

$$\therefore a_0 = \frac{1}{\pi} \int_0^{2\pi} f(x) dx$$

but $(-2, 2)$

$$a_0 = \frac{1}{\pi} \int_{-2}^2 f(x) dx = \frac{1}{\pi} \left[\int_{-2}^0 f(x) dx + \int_0^2 f(x) dx \right]$$

$$= \frac{1}{\pi} \left[\int_{-2}^0 2 dx + \int_0^2 x dx \right]$$

$$= \frac{1}{\pi} \left[2(x)_{-2}^0 + \left(\frac{x^2}{2}\right)_0^2 \right]$$

$$= \frac{1}{\pi} [4 + 2] = \frac{6}{\pi} \text{ Here } \pi = 2$$

$$\Rightarrow \frac{6}{2} = 3 \rightarrow 1 \text{ Mark}$$

$$a_n = \frac{1}{2} \int_{-2}^2 f(x) \cos \frac{n\pi x}{2} dx \quad (14)$$

$$= \frac{1}{2} \left[\int_{-2}^0 2 \cos \frac{n\pi x}{2} dx + \int_0^2 x \cos \frac{n\pi x}{2} dx \right]$$

$$= \frac{1}{2} \left[2 \left[\frac{\sin \frac{n\pi x}{2}}{\frac{n\pi}{2}} \right]_{-2}^0 + \left\{ x \left(\frac{\sin \frac{n\pi x}{2}}{\frac{n\pi}{2}} \right) - \left(\frac{\cos \frac{n\pi x}{2}}{\frac{n^2 \pi^2}{4}} \right) \right\}_0^2 \right]$$

$$= \frac{2}{n^2 \pi^2} [(-1)^n - 1]$$

$\therefore a_n = 0$ when n is even

$$= \frac{4}{n^2 \pi^2} \text{ when } n \text{ is odd.} \quad \rightarrow 2 \text{ Marks}$$

$$\therefore b_n = \frac{1}{2} \int_{-2}^2 f(x) \sin \frac{n\pi x}{2} dx$$

$$= \frac{1}{2} \left(\frac{-4}{n\pi} \right) = \frac{-2}{n\pi} \rightarrow 1 \text{ Mark}$$

$$\therefore f(x) = \frac{3}{2} - \frac{4}{\pi^2} \sum_{n=1,3,5} \frac{1}{n^2} \cos \frac{n\pi x}{2}$$

$$- \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \sin \frac{n\pi x}{2} \quad \rightarrow 2 \text{ Marks}$$

Hence the solution

Total 6 Marks.

⑪ ② Expand $f(x) = \begin{cases} 1 & \text{for } 0 < x < \pi \\ 0 & \text{for } \pi < x < 2\pi \end{cases}$ (15)
as Fourier series

Sol:- We have $f(x) = \begin{cases} 1 & \text{for } 0 < x < \pi \\ 0 & \text{for } \pi < x < 2\pi \end{cases}$

$$\therefore f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

$$\therefore a_0 = \frac{1}{\pi} \int_0^{2\pi} f(x) dx = \frac{1}{\pi} \int_0^{\pi} f(x) dx$$

$$= 1 \rightarrow 1 \text{ Mark}$$

$$a_n = \frac{1}{\pi} \int_0^{2\pi} f(x) \cos nx dx$$

$$= \frac{1}{\pi} \int_0^{\pi} [1 \cdot \cos nx dx] + \int_{\pi}^{2\pi} 0 dx$$

$$= \frac{1}{\pi} \left(\frac{\sin nx}{n} \right)_0^{\pi} = 0 \rightarrow 2 \text{ Mark}$$

$$b_n = \frac{1}{\pi} \int_0^{2\pi} f(x) \sin nx dx$$

$$= \frac{1}{\pi} \int_0^{\pi} \sin nx dx = \frac{1}{\pi} \left[-\frac{\cos nx}{n} \right]_0^{\pi}$$

$$= -\frac{1}{n\pi} [(-1)^n - 1]$$

$$\therefore b_n = \begin{cases} 0 & \text{for } n \text{ is even} \\ \frac{2}{n\pi} & \text{for } n \text{ is odd} \end{cases} \rightarrow 2 \text{ Mark}$$

$$\therefore f(x) = \frac{1}{2} + \frac{2}{\pi} \sum \frac{1}{n} \sin nx = \frac{1}{2} + \frac{2}{\pi} \left(\sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \dots \right)$$

Hence the series $\rightarrow 1 \text{ Mark}$

Find the half range sine series (16)
U(b): for $f(x) = x + x^2$ in $[0, 1]$

Sol:- $\therefore f(x) = x + x^2$ in $[0, 1]$

$$\therefore f(x) = \sum_{n=1}^{\infty} b_n \sin n\pi x \rightarrow 1 \text{ Mark}$$

$$\therefore b_n = 2 \int_0^1 f(x) \sin n\pi x \, dx$$

$$= 2 \int_0^1 (x + x^2) \sin n\pi x \, dx$$

$$= 2 \int_0^1 x \sin n\pi x \, dx + 2 \int_0^1 x^2 \sin n\pi x \, dx$$

$$= 2 \left[x \left(\frac{-\cos n\pi x}{n} \right) - \int \left(\frac{-\cos n\pi x}{n} \right) dx \right]_0^1$$

$$= 2 \left[-\frac{x \cos n\pi x}{n} + \frac{1}{n} \frac{\sin n\pi x}{n} \right]_0^1$$

$$= 2 \left[-\frac{x \cos n\pi x}{n} + \frac{1}{n^2} \sin n\pi x \right]_0^1$$

$$= 2 \left[\left(-\frac{\cos n\pi}{n} + \frac{1}{n^2} \sin n\pi \right) - 0 - 0 \right]$$

$$= 2 \left[-\frac{\cos n\pi}{n} + 0 \right] = -\frac{2}{n} (-1)^n \rightarrow 2 \text{ Marks}$$

$$\therefore f(x) = \sum_{n=1}^{\infty} b_n \sin n\pi x$$

$$= \sum_{n=1}^{\infty} \frac{-2}{n} (-1)^n \sin n\pi x \rightarrow 3 \text{ Marks}$$

Hence the solution.

(12) i, Find $L\{t^2+1\}$

(17)

Sol:- Given $L\{t^2+1\} = L\{t^2\} + L\{1\}$

$$\Rightarrow L\{t^2\} = \frac{2!}{s^{2+1}} = \frac{2}{s^3} \rightarrow 2 \text{ Marks}$$

$$L\{1\} = \frac{1}{s} \rightarrow 2 \text{ Marks}$$

$$\therefore L\{t^2+1\} = \frac{2}{s^3} + \frac{1}{s} \rightarrow 2 \text{ Marks}$$

Total $\rightarrow 6 \text{ Marks}$.

ii, Find $L\{\sin t + \cos t\}$

$$= L\{\sin t\} = \frac{1}{s^2+1} = \frac{1}{s^2+1}$$

$\rightarrow 2 \text{ Marks}$

$$L\{\cos t\} = \frac{s}{s^2+1} = \frac{s}{s^2+1}$$

$\rightarrow 2 \text{ Marks}$

$$\therefore L\{\sin t + \cos t\} = \frac{1}{s^2+1} + \frac{s}{s^2+1}$$

$$= \frac{s+1}{s^2+1}$$

$\rightarrow 2 \text{ Marks}$

Total $\rightarrow 6 \text{ Marks}$.

$$(13) (a) \text{ If } f(t) = \begin{cases} 1 & \text{for } 0 < t < 1 \\ -1 & \text{for } 1 < t < 2 \end{cases} \quad (18)$$

is a periodic function with period 2.

Then find its Laplace Transform.

$$\text{Sol: } \therefore L\{f(t)\} = \frac{1}{1 - e^{-sT}} \int_0^T e^{-st} f(t) dt$$

$$\text{Given } f(t) = \begin{cases} 1 & \text{for } 0 \leq t < 1 \\ -1 & \text{for } 1 \leq t < 2 \end{cases}$$

$$\begin{aligned} L\{f(t)\} &= \frac{1}{1 - e^{-sT}} \int_0^2 e^{-st} f(t) dt \\ &= \frac{1}{1 - e^{-sT}} \left[\int_0^1 e^{-st} f(t) dt + \int_1^2 e^{-st} f(t) dt \right] \end{aligned}$$

→ 2 Marks

$$= \frac{1}{1 - e^{-2s}} \left[-\frac{1}{s} (e^{-s} - 1) + \frac{1}{s} (e^{-2s} - e^{-s}) \right]$$

→ 2 Marks

After simplification we get

$$= \frac{1 - e^{-s}}{s(1 + e^{-s})} \rightarrow 2 \text{ Marks}$$

Total → 6 Marks

(13) (b) Solve $y'' + y = \sin 3t$, $y(0) = y'(0) = 0$. (19)

Sol:- Given that $y'' + y = \sin 3t$

$$\Rightarrow (D^2 + 1)y = \sin 3t$$

$$L\{y''\} + L\{y\} = L\{\sin 3t\}$$

$$[s^2 L\{y\} - s y(0) - y'(0)] + L\{y\} = \frac{3}{s^2 + 9} \rightarrow 2 \text{ Marks}$$

$$s^2 L\{y\} - s(0) - (0) + L\{y\} = \frac{3}{s^2 + 9}$$

$$L\{y\} (s^2 + 1) = \frac{3}{s^2 + 9}$$

$$\therefore L\{y\} = \frac{3}{(s^2 + 1)(s^2 + 9)} \rightarrow 2 \text{ Marks}$$

$$\therefore y = L^{-1} \left\{ \frac{3}{(s^2 + 1)(s^2 + 9)} \right\}$$

$$y = 3L^{-1} \left\{ \frac{1}{s(s^2 + 1)} - \frac{1}{s(s^2 + 9)} \right\}$$

$$= \frac{3}{s} L^{-1} \left\{ \frac{1}{s^2 + 1} \right\} - \frac{3}{s} L^{-1} \left\{ \frac{1}{s^2 + 9} \right\}$$

$$= \frac{3}{s} \sin t - \frac{3}{s} \sin 3t \rightarrow 2 \text{ Marks}$$

Hence the solution

Total 6 Marks

(14) (a) Find the Fourier sine and cosine transforms of $f(x) = \frac{e^{-2x}}{x}$ and deduce that

$$\int_0^{\infty} \frac{e^{-2x} - e^{-3x}}{x} \sin px = \tan^{-1}\left(\frac{p}{2}\right) - \tan^{-1}\left(\frac{p}{3}\right)$$

Sol:- Fourier sine transform:-

$$\begin{aligned} F_s \{f(x)\} &= \int_0^{\infty} f(x) \sin px \, dx \\ &= \int_0^{\infty} \frac{e^{-ax}}{x} \sin px \, dx \end{aligned}$$

differentiate w.r.t 'p' we get-

$$\frac{d}{dp} [F_s \{f(x)\}] = \int_0^{\infty} e^{-ax} \cos px \, dx = \frac{a}{a^2 + p^2}$$

Integrating w.r.t "p" we get - → 2 Marks.

$$F_s \{f(x)\} = \int \frac{a}{a^2 + p^2} dp = \tan^{-1}\left(\frac{p}{a}\right) + C$$

put $p = 0$ we get $\tan^{-1}\left(\frac{p}{a}\right)$

$$\text{Let } f(x) = \frac{e^{-ax} - e^{-bx}}{x}$$

$$\int_0^{\infty} \frac{e^{-ax} - e^{-bx}}{x} dx = \tan^{-1}\left(\frac{p}{a}\right) - \tan^{-1}\left(\frac{p}{b}\right)$$

(21)

$$\therefore \int_0^{\infty} \frac{e^{-2x} - e^{-3x}}{x} \sin px \, dx = \tan^{-1}\left(\frac{p}{2}\right) - \tan^{-1}\left(\frac{p}{3}\right)$$

→ 2 Marks.

Similarly we can find

$$\begin{aligned} F_c\{f(x)\} &= \int_0^{\infty} f(x) \cos px \, dx \\ &= \int_0^{\infty} \frac{e^{-ax}}{x} \cos px \, dx \\ &= \log \left[\frac{1}{\sqrt{p^2 + a^2}} \right] \end{aligned}$$

→ 2 Marks.

Total → 6 Marks

Hence the solution.

(14) (b) Find the Fourier Transform of

$$f(x) = \begin{cases} a^2 - x^2 & \text{for } |x| < a \\ 0 & \text{for } |x| > a \end{cases}$$

Hence show that

$$\int_0^{\infty} \frac{\sin x - x \cos x}{x^3} \, dx = \frac{\pi}{4}$$

Sol:- We have $F\{f(x)\} = \int_{-\infty}^{\infty} e^{ipx} f(x) \, dx$

$$= \int_{-\infty}^{-1} e^{ipx} \cdot 0 \, dx + \int_{-1}^1 e^{ipx} (1-x^2) \, dx + \int_1^{\infty} e^{ipx} \cdot 0 \, dx$$

$$= \int_{-1}^1 (1-x^2) e^{ipx} dx$$

$$= \frac{-1}{2} \left[\frac{-2 \cos p}{p^2} + \frac{2i \sin p}{ip^3} \right]$$

$$= 4 \left[\frac{-p \cos p + \sin p}{p^3} \right] \rightarrow 2 \text{ Marks.}$$

Now by inversion formula, we have

$$f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(p) e^{-ipx} dp$$

$$\Rightarrow \left\{ \begin{array}{ll} 1-x^2 & \text{for } |x| \leq 1 \\ 0 & \text{for } |x| > 1 \end{array} \right\}$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{4}{p^3} (p \cos p - \sin p) e^{-ipx} dp$$

$$\text{put } x = \frac{1}{2}$$

$$\text{we get } \int_0^{\infty} \frac{x \cos x - \sin x}{x^3} \cos \frac{p}{2} dp = -\frac{3\pi}{8}$$

$$\text{put } x=0.$$

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{-4}{p^3} (p \cos p - \sin p) dp = 1$$

$\rightarrow 2 \text{ Marks}$

$$\therefore \int_0^{\infty} \frac{x \cos x - \sin x}{x^3} dx = -\frac{\pi}{4}$$

$$\Rightarrow \int_0^{\infty} \frac{\sin x - x \cos x}{x^3} = \frac{\pi}{4} [\because a=1] \rightarrow 2 \text{ Marks.}$$

15) a) Using Fourier integral

(23)

then show that

$$\int_0^{\infty} \frac{1 - \cos k\pi}{k} \sin xk \, dk = \begin{cases} \frac{\pi}{2} & \text{for } 0 < x < \pi \\ 0 & \text{for } x > \pi \end{cases}$$

Sol:- let $f(x) = \begin{cases} \frac{\pi}{2} & \text{for } 0 < x < \pi \\ 0 & \text{for } x > \pi \end{cases}$

put $x = t$ $\rightarrow$ 2 Marks

$$f(x) = \frac{2}{\pi} \int_0^{\infty} \sin kx \int_0^{\infty} f(t) \sin kt \, dt \, dk$$

$$\therefore f(x) = \frac{2}{\pi} \int_0^{\infty} \sin kx \left[\int_0^{\pi} f(t) \sin kt \, dt \right] dk$$

$$= \int_0^{\infty} \sin kx \left(\frac{1 - \cos k\pi}{k} \right) dk$$

$\rightarrow$ 4 Marks

$$\therefore \int_0^{\infty} \frac{1 - \cos k\pi}{k} \sin xk \, dk = \begin{cases} \frac{\pi}{2} & \text{for } 0 < x < \pi \\ 0 & \text{for } x > \pi \end{cases}$$

Hence the solution

Total $\rightarrow$ 6 Marks

(15) (b) Find the inverse Fourier cosine transform of $\frac{\sin ap}{p}$ (24)

Sol: — Let $F_c(p) = \frac{\sin ap}{p}$

$$\therefore f(x) = \frac{2}{\pi} \int_0^{\infty} F_c(p) \cos px \, dp \rightarrow 2 \text{ Marks}$$
$$= \frac{2}{\pi} \int_0^{\infty} \frac{\sin ap}{p} \cos px \, dp$$

= After simplification we get

$$= \frac{1}{\pi} \left[\int_0^{\infty} \frac{\sin(a+x)p}{p} \, dp + \int_0^{\infty} \frac{\sin(a-x)p}{p} \, dp \right]$$

$$f(x) = \frac{1}{\pi} \left[\frac{\pi}{2} + \frac{\pi}{2} \right] = \text{if } a-x > 0.$$

$$= \frac{1}{\pi} \left[\frac{\pi}{2} - \frac{\pi}{2} \right] = \text{if } x-a > 0.$$

————— $\rightarrow$ 2 Marks

$$\text{Thus } f(x) = \begin{cases} 1 & \text{for } x < a \\ 0 & \text{for } x > a \end{cases}$$

————— $\rightarrow$ 2 Marks

Hence the solution

————— x —————

Semester End Examination, January/February, 2022

Degree	B. Tech. (U. G.)	Program	EEE & ECE	Academic Year	2021 - 2022
Course Code	20EC302	Test Duration	3 Hrs. Max. Marks 70	Semester	III
Course	Electronic Devices and Circuits				

Part A (Short Answer Questions 5 x 2 = 10 Marks)

No.	Questions (1 through 5)	Learning Outcome (s)	DoK
1	What is meant by depletion region?	20EC302.1	L1
2	List any two applications of LED	20EC302.2	L1
3	Define TUF and what is the TUF of Bridge rectifier	20EC302.3	L1
4	Why we call BJT as a current Controlled Device?	20EC302.4	L1
5	List the advantage and disadvantages of fixed bias method	20EC302.5	L1

Part B (Long Answer Questions 5 x 12 = 60 Marks)

No.	Questions (6 through 15)	Marks	Learning Outcome (s)	DoK
6 (a)	With neat diagram explain the principle and operation of P-N junction diode in forward and reverse bias	8M	20EC302.1	L2
6 (b)	Calculate the forward bias current of a Si diode when forward bias voltage of 0.4V is applied, the reverse saturation current is 1.17×10^{-9} A and the thermal voltage is 25.2mV.	4M	20EC302.1	L2
OR				
7 (a)	Derive the expression for Diffusion capacitance of a diode	7M	20EC302.1	L2
7 (b)	What is the effect of temperature on P-N junction diode?	5M	20EC302.1	L2
8 (a)	Explain the working of Zener diode in reverse bias condition with neat diagram.	6M	20EC302.2	L2
8 (b)	With the help of V-I characteristics, explain SCR operation.	6M	20EC302.2	L2
OR				
9 (a)	A sinusoidal voltage whose $V_m=12$ V is applied to half-wave rectifier. The diode may be considered to be ideal and $R_L=1.5$ K Ω is connected as load. Find out peak value of current, RMS value of Current, DC value of current and Ripple factor.	6M	20EC302.2	L2
9 (b)	Derive the expression for Ripple factor for Full Wave Rectifier with L filter.	6M	20EC302.2	L3
10 (a)	Explain the operation of NPN Transistor with neat diagram	5M	20EC302.3	L2
10 (b)	Write the differences between CB, CE, and CC Configurations	7M	20EC302.3	L2
OR				
11 (a)	Derive the relation between α and β	5M	20EC302.3	L2
11 (b)	Sketch a family of CE output characteristics for a transistor. Explain cutoff, active, saturation region	7M	20EC302.3	L2
12 (a)	Obtain an expression of stability factor for fixed bias	5M	20EC302.4	L2
12 (b)	Distinguish Thermal runaway and Thermal stability	7M	20EC302.4	L2
OR				
13 (a)	With suitable expressions explain self bias of BJT	6M	20EC302.4	L2
13 (b)	Explain about Thermistor and Sensistor bias compensation techniques	6M	20EC302.4	L2
14 (a)	Draw the hybrid parameter equivalent circuit for an NPN common emitter transistor and explain.	8M	20EC302.5	L3
14 (b)	Explain the construction and working of Enhancement MOSFET.	4M	20EC302.5	L2
OR				
15 (a)	With neat sketch, discuss about common source FET amplifier.	4M	20EC302.5	L3
15 (b)	Draw the hybrid parameter equivalent circuit for PNP common base transistor and explain.	8M	20EC302.5	L3

ANSWER KEY AND SCHEME OF EVALUATION

II - B TECH SEMESTER SOE
ELECTRONICS COMMUNICATION & ENGINEERING
EDC
Common For ECE & EEE

PART-A

- ① depletion region meaning - 2marks.
- ② Application of LED - 2mark
- ③ define TUF - 1mark
TUF of bridge rectifier - 1mark } 2mark
- ④ BJT as a current controlled device why - 2mark
- ⑤ advantage of Fixed bias - 1m
Disadvantage - 1m } 2marks

PART-B

- 6(a) Diagram - 2m
principle & operation of Forward & Reverse - 6m } 8m
- 6(b) Problem - 4m
- 7(a) meaning of diffusion capacitance - 2m
Expression - 5m
- 7(b) Diagram - 2m
equation - 3m } 5m
- 8(a) Diagram - 2m
working of Zener diode - 4m } 6m
- 8(b) SCR operation - 4m
V-I characteristics - 2m } 6m

- 9(a) - problem - 6m
- 9(b) Expression of FULL wave rectifier with L Filter - 6m
- 10(a) diagram - 1m
operation NPN - 4m } 5m
- 10(b) difference of CB, CE, CC - 7m
- 11(a) relation b/w α & β - 5m
- 11(b) CE o/p character. Diagram - 4m } 7m
cutoff - 1m
active - 1m
saturation - 1m
- 12(a) Expression for stability factor - 5m
- 12(b) thermal runaway - 3.5 } 7m
thermal stability - 3.5
- 13(a) suitable expression of BJT in Fixed bias - 3m } 6m
Explanation - 3m
- 13(b) thermistor. Explanation - 3m } 6m
sensistor explanation - 3m
- 14(a) hybrid parameter ckt - 3m } 8m
Explanation - 4m
Diagram - $\frac{1m}{8m}$
- 14(b) construction - 2m } 4m
working of enhancement - 2m
- 15(a) diagram - 2m } 4m
Common source amplifier - 2m
- 15(b) circuit diagram - 2m } 8m
PNP explanation - 6m

total - 70 marks

S. Jayalakshmi
16/12/22

Group
16/12/22

NSRIT COLLEGE
EDC SEMESTER KEY
ELECTRONICS & COMMUNICATION ENGINEERING.
COMMON FOR ECE & EEE

1. What is meant by depletion region?

The depletion region is so named because it is formed because it is formed from a conducting region by removal of all free charge carriers, leaving none to carry a current.

2. list any two applications of LED?

- 1) Picture phones and digital watches.
- 2) Used as flash light to travel light for more distance.
- 3) Microprocessors and multiplexers.

Q3: Define TUF and what is the TUF of Bridge rectifier?

A: TUF (Transformer utilization factor): It is defined as the ratio of dc power output to the Transformer Volt-Ampere (VA) rating required by the secondary winding.

TUF of Bridge rectifier is 0.812.

4. Why we call BJT as a Current Controlled device?

A: BJT is a current controlled device because the output characteristics of BJT are determined by the input current.

5. list the advantages and disadvantages of Fixed bias Method.

A: Advantages of Fixed bias

1. This is a simple circuit which uses very few components.
2. It provides maximum flexibility in the design.

Disadvantages:-

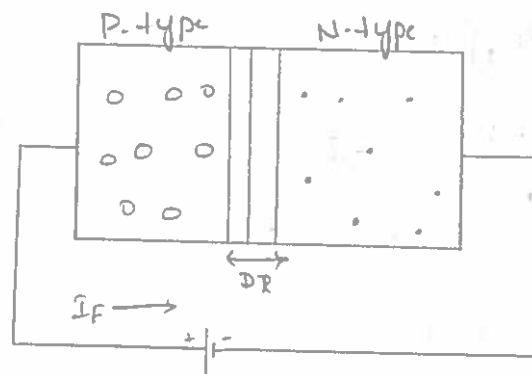
- 1) Thermal stability is not provided in the circuit. Thus Q-Point is not Maintained.

6a). With neat diagram explain the principle and operation of PN junction diode in forward bias and reverse bias.

Forward bias.

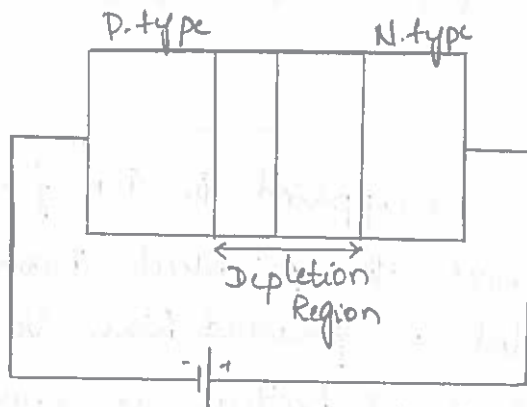
When an external voltage is applied to PN junction in such a direction that it cancels the potential barrier and permits the current flow is called as forward bias. To apply a forward bias the positive terminal of battery is connected to p-type and negative terminal to n-type of the PN junction diode, then the bias applied is forward bias.

In forward bias condition, the width of the depletion region decreases on increasing applied voltage and potential barrier becomes less. When the voltage is increased the depletion region breaks from and holes from p-type is attracted to n-type and electrons from n-type is attracted to p-type. forward current I_F moves from anode to cathode.



Reverse bias condition.

The p-type is connected to negative terminal and N-type is connected to positive terminal of the battery. The holes in P-type is attracted by negative terminal electrons in N-type moves towards to positive terminal so the width of the depletion region increases and potential barrier increases. There is slight current flow in reverse bias PN junction diode.



6b) Calculate the forward bias current of a Si diode when forward bias voltage of 0.4V is applied the reverse saturation current is $1.17 \times 10^{-9} \text{ A}$. and the thermal voltage is 25.2mV

Given data

Silicon $\eta = 2$

forward bias voltage $V = 0.4 \text{ V}$

$I_0 = 1.17 \times 10^{-9} \text{ A}$

Thermal voltage $V_T = 25.2 \times 10^{-3} \text{ V}$

Current $I = I_0 (e^{V/\eta V_T} - 1)$

$$I = (1.17 \times 10^{-9}) \times \left(e^{\frac{0.4}{2 \times 25.2 \times 10^{-3}}} - 1 \right)$$

$$I = (1.17 \times 10^{-9}) (e^{7.9365} - 1)$$

$$I = (1.17 \times 10^{-9}) (2796.55)$$

$$I = 3.27 \times 10^{-6} \text{ A}$$

$$I = 3.27 \mu\text{A}$$

7a) Derive the expression for Diffusion capacitance of a diode.

Diffusion capacitance or Storage Capacitance (C_D).

Let I be the diode current

$$I = \frac{Q}{\tau}$$

$$Q = I\tau$$

where τ = Mean life time of a charge carrier.

$$C_D = \frac{dQ}{dV}$$

$$I = I_0 [e^{V/\eta V_T} - 1]$$

$$\text{eff } V \gg V_T \\ e^{V/\eta V_T} \gg 1$$

$$I = I_0 e^{V/\eta V_T}$$

$$Q = I_0 \cdot e^{V/\eta V_T}$$

$$\frac{dQ}{dV} = \frac{\tau I e^{V/\eta V_T}}{\eta V_T}$$

$$\boxed{\frac{dQ}{dV} = \frac{I\tau}{\eta V_T} = C_D}$$

7b) What is the effect of temperature on PN-junction diode?

The Boltzmann's equation of diode current is expressed as

$$I = I_0 \exp\left\{\left(\frac{V}{\eta V_T}\right) - 1\right\} \rightarrow (1)$$

where

I = diode current (positive when forward bias and negative when reverse bias).

I_0 = diode reverse current at temperature T .

V = diode voltage

$\eta = 1$ for Germanium, 2 for Silicon

$V_T = (T/11600)$ a quantity in volts dependent upon temperature

T = temperature of diode junction

At room temperature say 22°C , $T = 295\text{K}$

$$V_T = \frac{T}{11600}$$

$$= 0.025 \text{ volts}$$

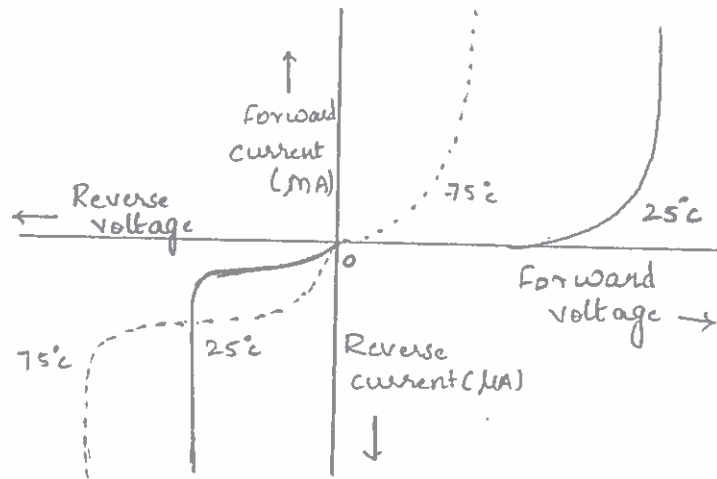
$$I = I_0 (e^{40V} - 1) \text{ for Germanium}$$

$$I = I_0 (e^{20V} - 1) \text{ for Silicon.}$$

I_0 = Reverse saturation current

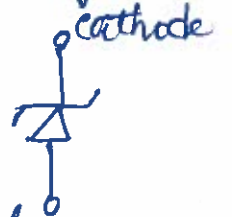
From equation (1) with the increase in temperature, the exponential term will reduce and hence the diode current should also reduce. The reverse saturation current is also temperature dependent. It can be shown that it increases 7% per $^\circ\text{C}$ for both Germanium and Silicon. This represents a doubling for every 10°C rise. Germanium is more temperature dependent than Silicon because its saturation current is much greater than the exponential term.

The effect of temperature on the diode characteristics.



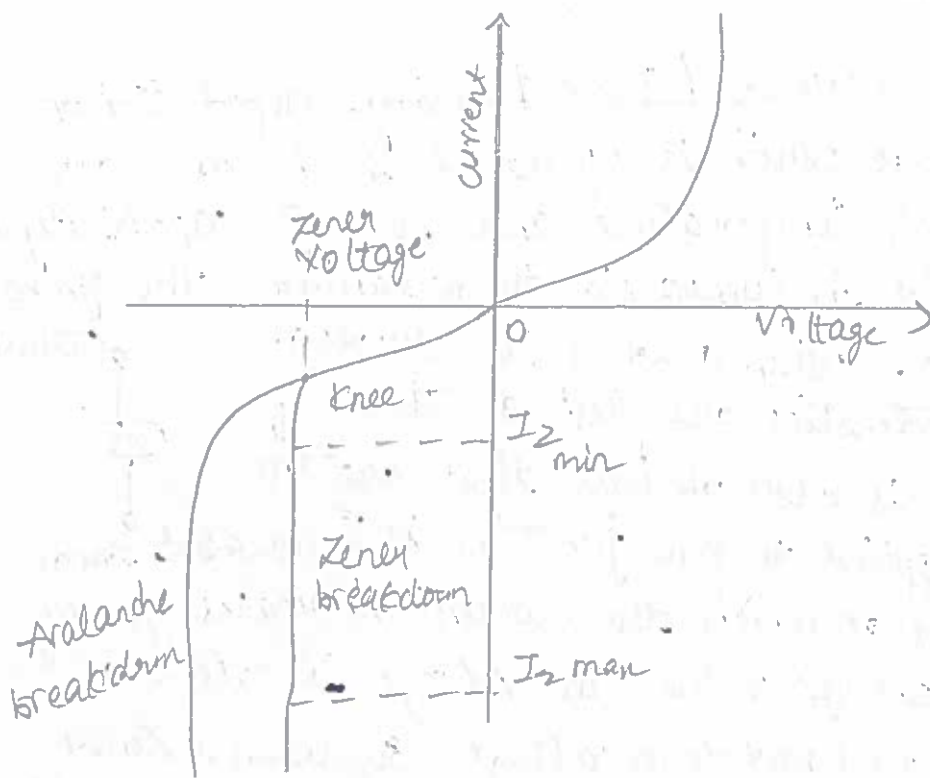
8(a) Explain the working of Zener diode in reverse bias condition with neat diagram

Ans Zener diode is a reverse biased heavily-doped Silicon P-N Junction diode which is operated in the breakdown region. Due to high temperature and current capability, Silicon is preferred in comparison to germanium. The symbol of a Zener diode is. When a Zener is forward-biased its characteristics are just as those of ordinary diode. It is also obvious that as the reverse voltage applied to P-N junction is increased, a value is reached at which the current increases greatly from its normal cut off value. The voltage is called a Zener voltage V_Z or breakdown voltage. So, when a Zener diode is reverse-biased, it has sharp breakdown voltage called Zener voltage V_Z .



(i) Zener breakdown :- Zener breakdown takes place in very thin junction when both sides of junction are very heavily doped and consequently the depletion layer. When a small reverse bias voltage is applied, a very strong field (about 10^7 V/m) is set up across the thin depletion layer. Zener current is independent of the applied voltage. depends only on the external resistance.

(ii) Avalanche breakdown :- This type of breakdown takes place when the both sides of junction are lightly doped and consequently the depletion layer is large. In this electric field across the depletion layer is not so strong to produce Zener breakdown. This breakdown is called avalanche breakdown.

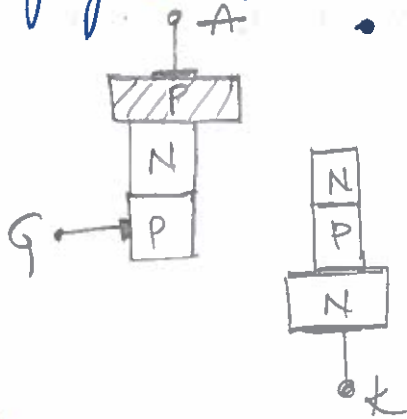


8(b) With the help of V-I characteristics, Explain SCR operation

Ans SCR: Silicon Controlled Rectifier:-

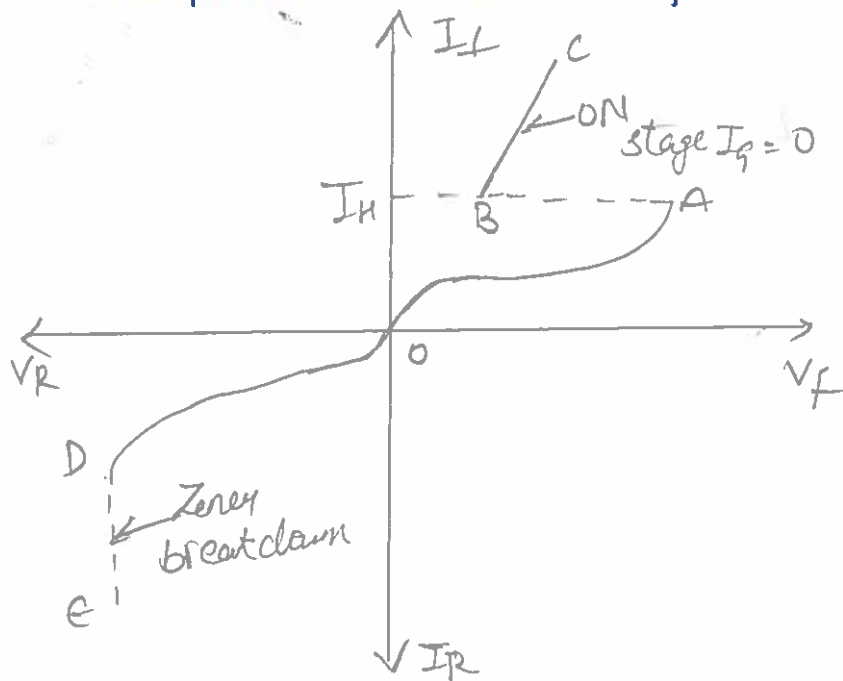
1. If the anode is made negative wrt Cathode, SCR is blocked due to two reverse-biased junctions J_1 and J_3 . If the voltage is increased, a stage is reached. When Zener breakdown occurs and the SCR is destroyed. This shows that SCR
2. Either SCR does not conduct or conducts heavily. There is no state in between. Thus SCR behaves like a switch.
3. To make the SCR non-Conducting from conducting state, the supply voltage is reduced to zero.
4. To close the SCR, normally, a small positive voltage; the supply voltage is applied at the gate because the breakover voltage is usually much greater than supply voltage.

5. The SCR can be turn on either by keeping the gate open and making the supply voltage equal to break over voltage or making the supply voltage less than break over voltage and applying a small voltage to the gate.



V-I characteristics of SCR:-

The volt-ampere characteristics of a SCR



At this voltage junction J_2 breaks down and SCR switches suddenly to a high conducting state. When gate is open, the break over voltage.

The voltage across SCR suddenly drops as shown by dotted curve AB.

The current corresponding to point B is denoted by I_H and is known as holding current.

- 3 Above holding current, SCR is in ON state and the current keep flowing until the current is opened.
- 4 Reverse break-over voltage - The reverse voltage at which avalanche breakdown takes place is known as reverse break-over voltage.

Q(a) A sinusoidal voltage whose $V_m = 12V$ is applied to half-wave rectifier the diode may be considered to be ideal and $R_L = 1.5k\Omega$ is connected to be ideal and $R_L = 1.5k\Omega$ is connected as load. find out peak value of current, RMS value of current, DC value of a current and Ripple factor.

Sol Given data,

Voltage $V_m = 12V$

load resistance, $R_L = 1.5 \times 10^3 \Omega$

Peak value of current = $I_m = ?$

RMS value of current $I_{rms} = ?$

$I_{DC} = ?$

Ripple factor, $\gamma = ?$

Peak value of a current, $I_m = \frac{V_m}{R_L}$

$$I_m = \frac{12}{1.5 \times 10^3} = 8 \times 10^{-3} A \quad \boxed{I_m = 8mA}$$

RMS value of current, $I_{rms} = \frac{I_m}{2}$

$$I_{rms} = \frac{8 \times 10^{-3}}{2} = 4 \times 10^{-3} \quad \boxed{I_{rms} = 4mA}$$

$$I_{DC} = \frac{I_m}{\pi} = \frac{8 \times 10^{-3}}{\pi} = 2.546 \times 10^{-3} A \Rightarrow \boxed{I_{DC} = 2.546mA}$$

Ripple factor $\gamma = \sqrt{\left(\frac{I_{rms}}{I_{DC}}\right)^2 - 1}$

$$\gamma = \sqrt{\left(\frac{4 \times 10^{-3}}{2.546 \times 10^{-3}}\right)^2 - 1}$$

$$= \sqrt{2.468 - 1} = 1.0211 \quad \boxed{\gamma = 1.0211}$$

Q(15) Derive the Expression for Ripple factor for full wave Rectifier with L filter.

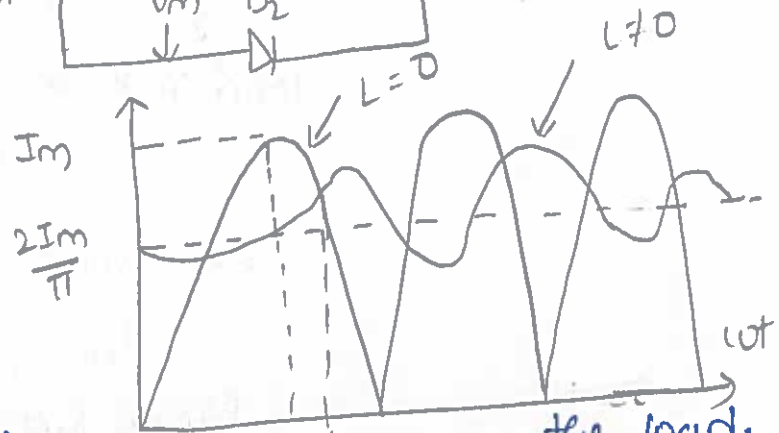
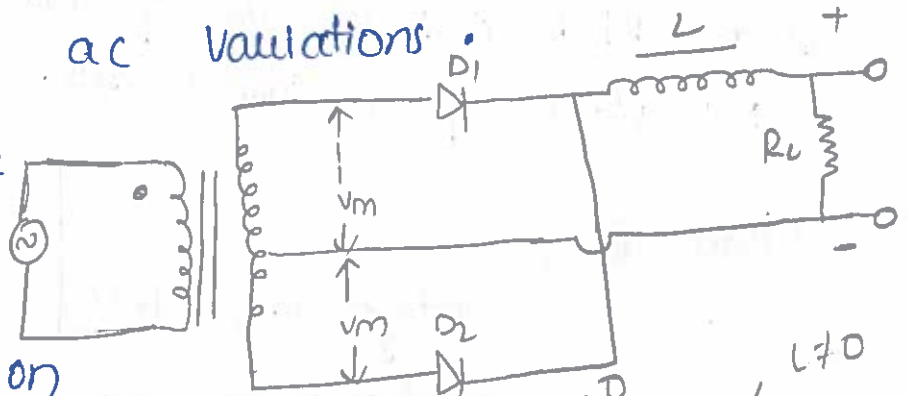
Rectifier with L filter.

→ A full wave rectifier with series Inductor. The inductor in series with load serves as a filter. The inductor offers high impedance to ac variations.

→ DC resistance of large inductance L is very small.

The action is based on property of inductance to oppose any change of current that may flow through it.

∴ AC Component of rectified output is blocked while the dc component reaches the load. Theoretically output should contain dc voltage. It contains small ac components as well.



Ripple factor ∴ The ripple factor γ is defined as the ratio of rms value of ac component to dc value of the wave.

$$\gamma = \frac{(I_r)_{rms}}{I_{dc}}$$

$$\gamma = \frac{\frac{4V_m}{3\pi R_L} \cdot \frac{1}{\sqrt{R_L^2 + 4\omega^2 L^2}}}{\frac{2V_m}{\pi R_L}}$$

$$= \frac{2}{3\sqrt{2}} \cdot \frac{1}{\sqrt{1 + (4\omega^2 L^2 R_L^2)}}$$

If $\frac{4\omega^2 L^2}{R_L^2} \gg 1$, $\rho = \frac{1}{3\sqrt{2}} \cdot \frac{R_L}{\omega L} = 0.236 \frac{R_L}{\omega L}$

$$\rho = \frac{2}{3\sqrt{2}} = 0.471$$

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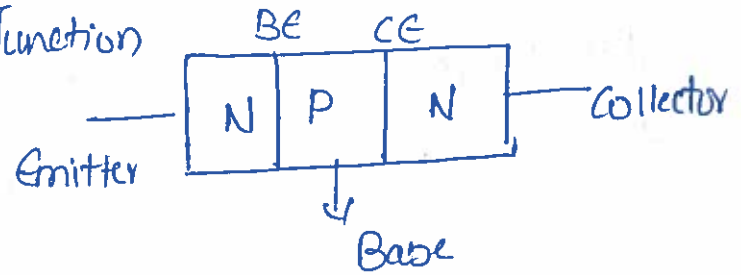
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10(a) Explain the operation of NPN Transistor with neat diagram.

→ A NPN Transistor is the most commonly used bipolar Junction transistor, and it is constructed by sandwiching a p-type Semiconductor between two N-type Semiconductors. A NPN transistor has three terminals a Collector, Emitter and base. The NPN Transistor behaves like two PN Junction diodes are connected back to back.

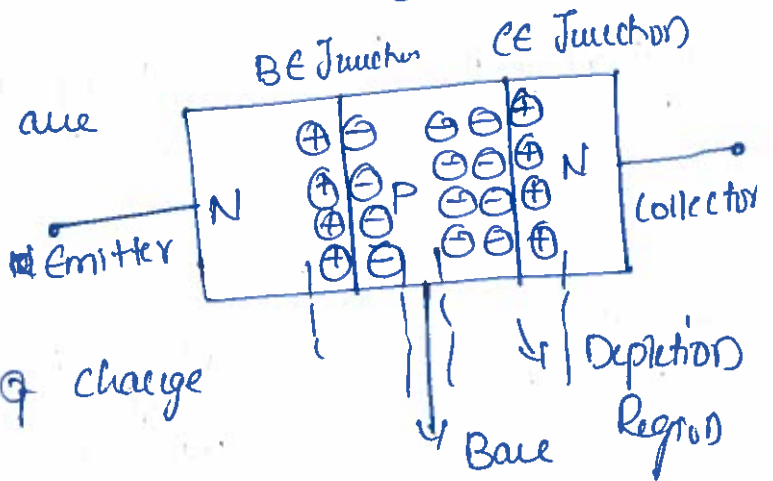
→ These back to back PN Junction diodes are known as the Collector Base Junction and base Emitter Junction.



→ The Emitter & Collector layers are wider compared to the base.

The Emitter is heavily doped.

It can emit a large number of charge carriers to the base.



→ The base is lightly doped & very thin construction of NPN Transistor compared to other two regions. It passes most of all charge carriers to the collector which is emitted by emitter.

→ The collector is moderately doped and collects charge carriers from the base layer.

10(15) Write the differences between CB, CE, CC Configurations.

<u>Characteristics</u>	<u>CB</u>	<u>CE</u>	<u>CC</u>
Common terminal for Input & Output	Base Terminal	Emitter Terminal	Collector Terminal
Input voltage applied between	Emitter & Base terminal	Base & Emitter Terminal	Base & Collector terminal
Output voltage taken across	Collector & Base Terminal	Collector & Emitter Terminal	Emitter & Collector Terminal
Input Impedance	Very low (only 50 to 500 ohm)	Medium (500 to 5000 ohm)	Very high (200 to 750 K to 10 ohm)
Output Impedance	Very high (1 to 10 Mega Ohm)	Medium (50 to 500 kilo ohm)	Very low (up to 50 ohm)
Input Current	Emitter Current or I_E	Base Current or I_B	Base Current or I_B
Output Current	Collector Current or I_C	Collector Current or I_C	Emitter Current or I_E

10) Derive the relation between α and β

11) a)

$$I_E = I_B + I_C$$

Divide by I_C

$$\frac{I_E}{I_C} = \frac{I_B}{I_C} + 1$$

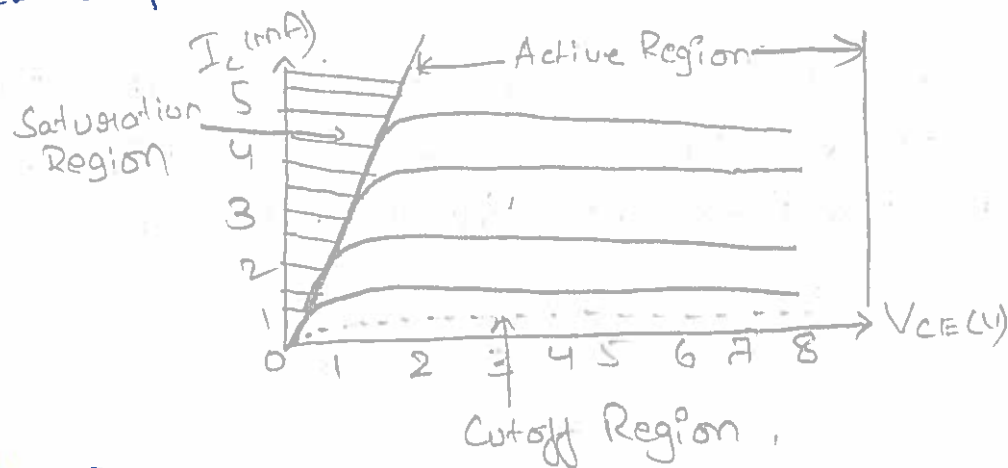
$$\frac{I_C}{I_E} = \alpha, \frac{I_C}{I_B} = \beta$$

$$\text{So, } \frac{1}{\alpha} = \frac{1}{\beta} + 1$$

$$\boxed{\beta = \frac{\alpha}{1-\alpha}}$$

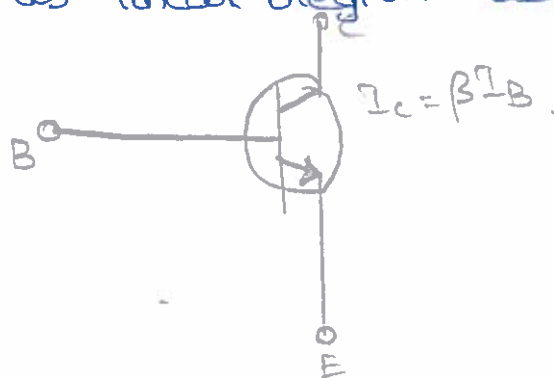
11) b) Sketch the family of CE output characteristics of a transistor. Explain cutoff, active and saturation region?

11) b) CE output characteristics of a transistor.



Active Region

The region in which transistors have many applications are called as linear region. acts as an Amplifier.

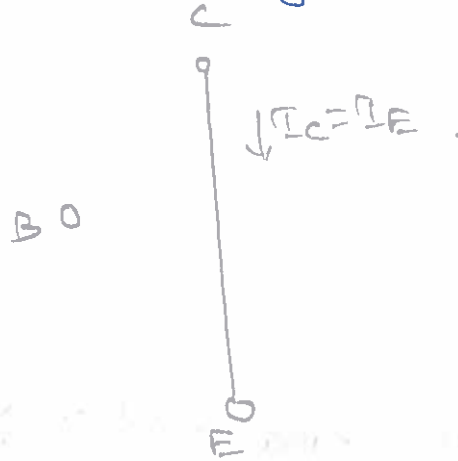


* The region lies b/w saturation & cutoff.

* The transistor operates in active region when the emitter junction is forward biased & collector junction is reverse biased.

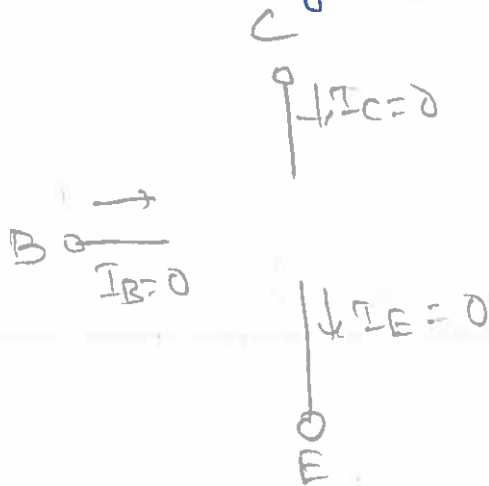
Saturation Region:

- * The region in which transistor tends to behave as a closed switch.
- * The transistor has effect of its collector & emitter being shorted.
- * The transistor operates in saturation region when both emitter & collector junctions are forward biased



Cutoff Region:

- * The region in which transistor tends to behave as a open switch
- * The transistor has effect of its collector & base being opened.
- * The transistor operates in cutoff region when emitter & collector junctions are reverse-biased,



12a) Obtain an expression of stability factor for fixed bias.

Fixed bias.

Stability factor.

$$S = \frac{\partial I_c}{\partial I_{C0}} \bigg|_{V_{BE} \text{ \& } \beta \text{ constant}}$$

We know that

$$I_B = \frac{V_{CC} - V_{BE}}{R_B}$$

No collector current present in this equation so $\frac{\partial I_B}{\partial I_c} = 0$. → (i)

We know I_c

$$I_c = \beta I_B + (1 + \beta) I_{C0}$$

differentiate I_c with respect to I_c we get

$$\frac{\partial I_c}{\partial I_c} = \beta \frac{\partial I_B}{\partial I_c} + (1 + \beta) \frac{\partial I_{C0}}{\partial I_c} \quad \therefore \frac{\partial I_c}{\partial I_{C0}} = S$$

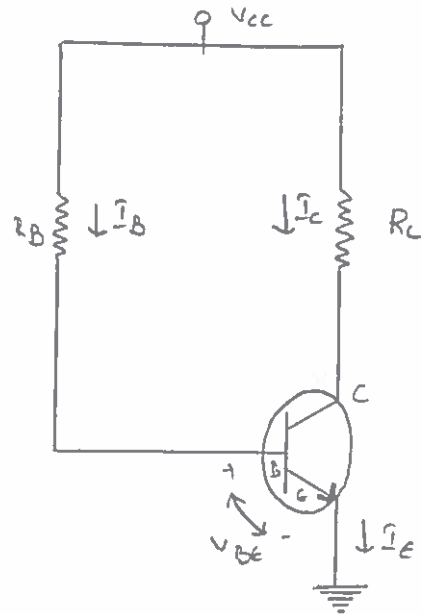
$$1 = \beta \frac{\partial I_B}{\partial I_c} + (1 + \beta) \frac{1}{S}$$

$$1 - \beta \frac{\partial I_c}{\partial I_c} = \frac{1 + \beta}{S}$$

$$S = \frac{1 + \beta}{1 - \beta \frac{\partial I_B}{\partial I_c}} \quad \rightarrow (2)$$

Substitute (i) in (2)

$$S = \frac{1 + \beta}{1 - \beta(0)} \quad [S = 1 + \beta]$$



1. The first part of the paper is devoted to the study of the properties of the function $f(x)$ defined by the equation

$$f(x) = \int_0^x f(t) dt$$

It is shown that the function $f(x)$ is continuous and differentiable on the interval $[0, 1]$.

$$f'(x) = f(x) \quad \text{for } x \in [0, 1]$$

It is also shown that the function $f(x)$ satisfies the initial condition $f(0) = 0$.

$$f(0) = 0$$

It is then shown that the function $f(x)$ is the unique solution of the initial value problem

$$f'(x) = f(x), \quad f(0) = 0$$

$$f(x) = 0 \quad \text{for } x \in [0, 1]$$

It is then shown that the function $f(x)$ is the unique solution of the initial value problem

$$f'(x) = f(x), \quad f(0) = 0$$

$$f(x) = 0 \quad \text{for } x \in [0, 1]$$

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It is then shown that the function $f(x)$ is the unique solution of the initial value problem

$$f'(x) = f(x), \quad f(0) = 0$$

12(5) Distinguish between thermal runaway & thermal stability?

Thermal runaway :

The Collector Current for CE circuit is given by $I_C = \beta I_B + (1 + \beta) I_{CO}$

→ The three Equations are, I_B, I_C, I_{CO}

→ In BJT, when temperature increases the I_{CO} current increases. This roughly doubles every 10 degree temperature raises. This in turn increases the I_C , the collector current.

→ The collector current increases the $\propto$ power dissipation,

Since power dissipation $P_{DC} = I_C \times V_{CE}$

→ Now, the increase in P_D again increases the I_{CO} . That is again come back to starting point and the cycle reaction starts.

→ At some point the transistor **damages**. This is known as thermal runaway.

Thermal stability

→ The thermal stability is defined as the ability of a fluid to resist breaking down under heat stress. The maximum temperature at suggested maximum temperature to which the fluid can be heated before the fluid begins to break down or degrade at an appreciable rate.

- When the Junction temperature of a transistor increases
 - the collector current increases as a result of increases
- In Saturation currents and dc conductances, thermal stability occurs.
- When Junction Temperature and collector current increase in regenerative and uncontrollable fashion.
- It is an important property of dielectric polymer materials for high density electric energy storage applications.

13) a) With suitable expressions explain self bias of BJT

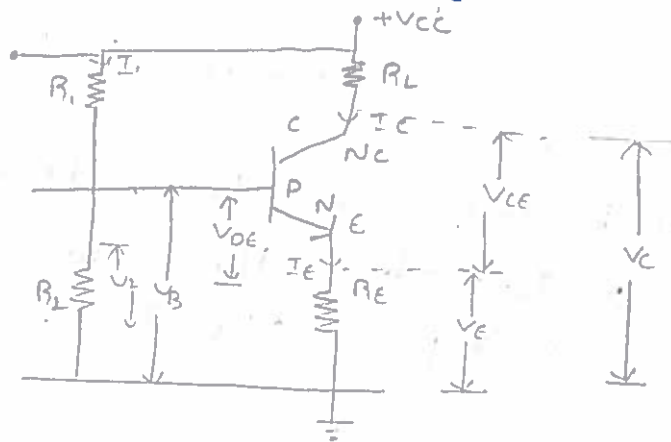
A) Self bias (or) emitter bias, is a very commonly used biasing arrangement.

It is also known as voltage divider bias.

* In this method, let us take a circuit in which two resistance R_1 & R_2 are connected across the supply voltage V_{CC} and provide biasing

* The emitter resistance R_E provides stabilization

* It causes a voltage drop in a direction so, as to reverse bias the emitter junction.



from the above circuit, the current I_1 flows through R_1 , as the base current I_B is very small, the current flowing through R_2 can also be taken as I_1

$$\therefore I_1 = \frac{V_{CC}}{(R_1 + R_2)}$$

The voltage at R_2 is given by

$$V_2 = \left(\frac{V_{CC}}{R_1 + R_2} \right) R_2$$

Apply KVL to base circuit

$$V_2 = V_{BE} + V_E = V_{BE} + I_E R_E$$

$$V_2 = V_{BE} + I_E R_E$$

$$\text{as } I_E \approx I_C$$

$$V_2 = V_{BE} + I_C R_E$$

$$I_C = \frac{V_2 - V_{BE}}{R_E}$$

$\therefore I_c$ is almost independent of transistor parameters

Now we calculate V_{ce}

Applying KVL to the collector side

$$V_{cc} = I_c R_L + V_{ce} + I_e R_e$$

$$= I_c R_L + V_{ce} + I_c R_e$$

$$= I_c (R_L + R_e) + V_{ce}$$

$$V_{ce} = V_{cc} - I_c (R_L + R_e)$$

- * If the temperature rises, there will be a rise in I_{co} and the I_c increases
- * If the current in R_e increases, the I_e decreases
- * Thus the presence of R_e reduces the increase in I_c and improves the operating point stability.

⑬ b) Explain about Thermistor and Sensistor bias compensation techniques

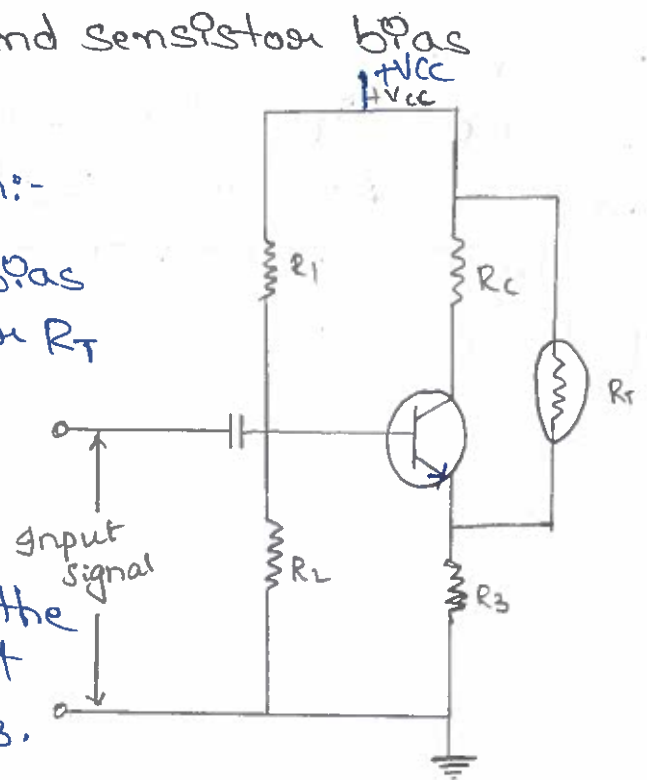
A) Thermistor Compensation:-

The figure shows that self bias CE Amplifier with thermistor R_T . The Thermistor has negative temperature coefficient of resistance.

The R_T is used to minimize the increase in collector current due to change in I_{CO} , V_{BE} or β .

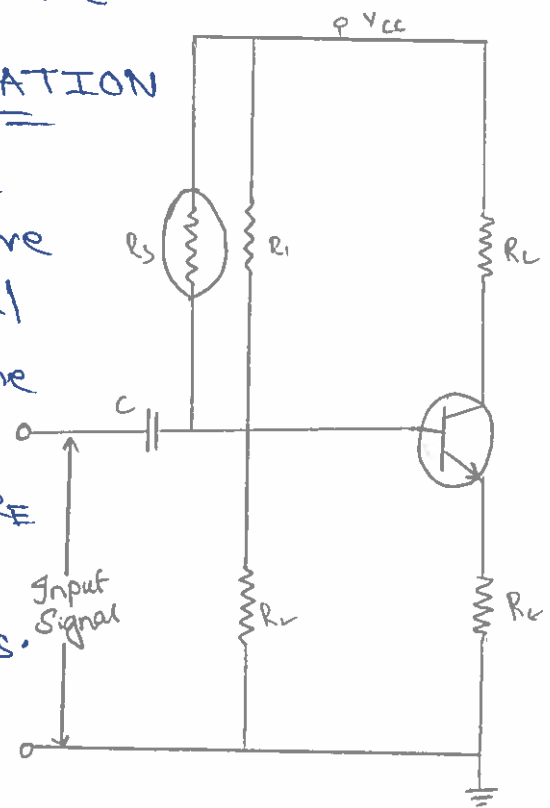
The Temperature increases, the resistance R_T of the thermistor decreases. The current fed to R_T into R_E increases so the voltage developed R_E increases in magnitude.

This voltage reverse biases the emitter junction the forward bias provided by resistors R_1 and R_2 . As a the collector current reduces. In this way the temperature sensitivity of R_T acts so as to compensate the increase in I_C due to temperature.



SENSISTOR COMPANSATION

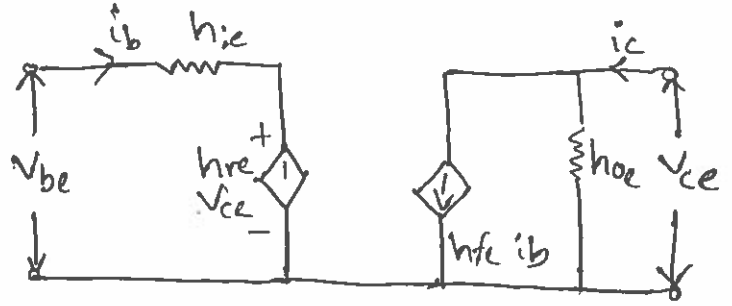
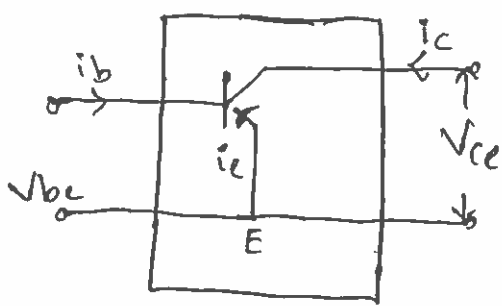
Sensistor is a temperature sensitive resistor having positive temperature coefficient of resistance like a metal. It is heavily doped semiconductor the sensistor may be used either in parallel with R_1 or in parallel with R_E . As the temperature increases, the resistance of the sensistor increases. Thus the resistance of the parallel combination ($R_1 \parallel R_2$) increases.



Now the voltage drop across R_2 decrease due to the decrease of the voltage, the net forward emitter bias decreases. As a result I_C decreases, this reduce compensates for the increased I_C caused by the increases I_{CO} , V_{BE} or β due to temperature.

14(a). Draw the hybrid parameters Equivalent circuit for an NPN Common Emitter transistor and explain.

Ans



$$V_{be} = h_{ie} i_b - h_{re} V_{ce}$$

$$i_c = h_{fe} i_b - V_{ce} \cdot h_{oe}$$

Case (i) :- when $V_{ce} = 0$

$$V_{be} = h_{ie} i_b - h_{re}(0)$$

$$V_{be} = h_{ie} i_b$$

$$h_{ie} = \frac{V_{be}}{i_b}$$

$$i_c = h_{fe} i_b - (0) h_{oe}$$

$$i_c = h_{fe} i_b$$

$$h_{fe} = \frac{i_c}{i_b}$$

Case (ii) :- when $i_b = 0$

$$V_{be} = h_{ie}(0) - h_{re} V_{ce}$$

$$V_{be} = -h_{re} V_{ce}$$

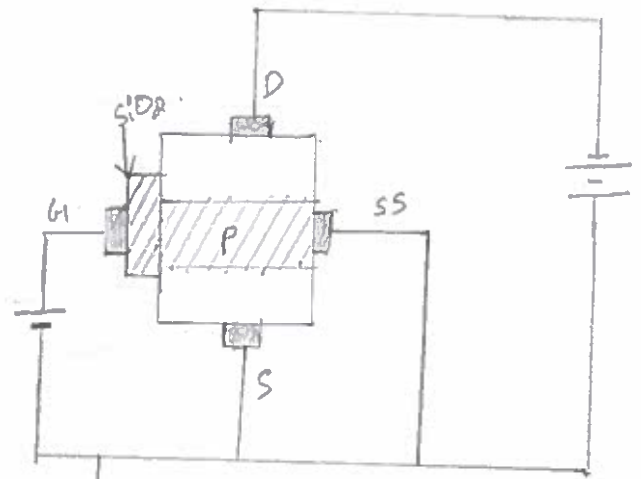
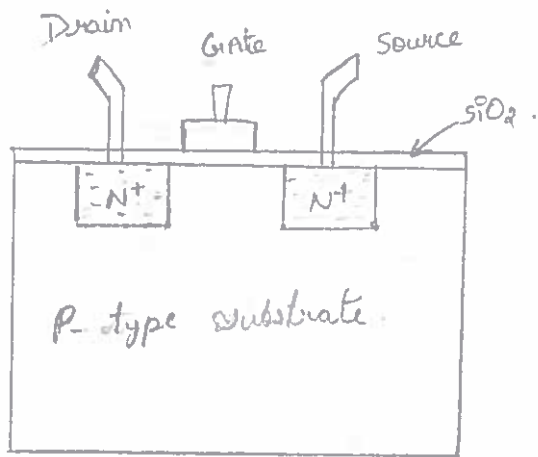
$$h_{re} = \frac{-V_{be}}{V_{ce}}$$

$$i_c = h_{fe}(0) - V_{ce} h_{oe}$$

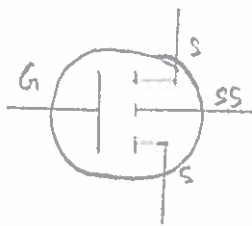
$$i_c = -V_{ce} h_{oe}$$

$$V_{ce} = \frac{-i_c}{h_{oe}}$$

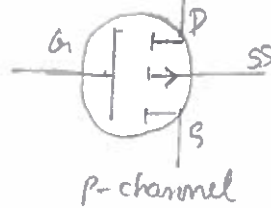
Enhancement type MOSFET:



Symbols:



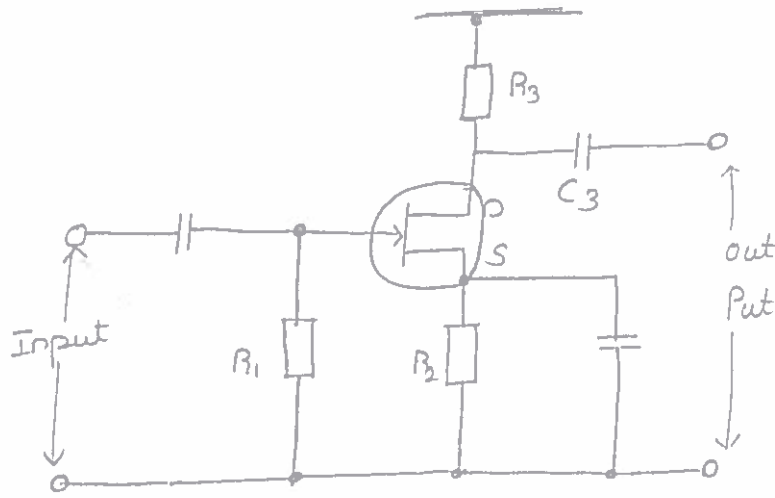
N-channel



- In enhancement mode of operation gate is +ve.
- when V_{GS} is $\uparrow$ no. of $\bar{e}$ is sent to P-type, then the layer becomes inversion layer.
- When inversion layer is formed; current passes.
- +ve charge to the capacitor another plate becomes induced.

15(a) with neat sketch, discuss about common source FET amplifiers.

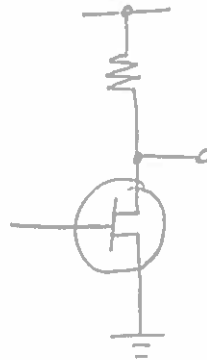
Ans:



In electronics, a Common Source amplifier is one of three basic single-stage Field-effect Transistors.

(FET) amplifiers

Typically used as a voltage or transconductance amplifiers. The easiest way to tell if a FET is Common Source, Common drain (or) Common gate.



Basic N-channel JFET
Common-source circuit

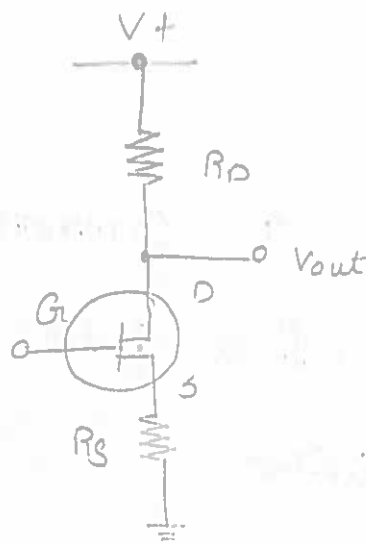
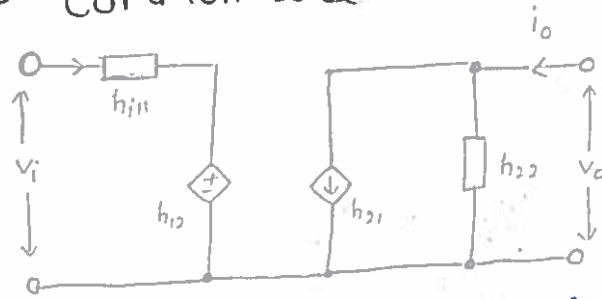


Fig-2 Basic N-channel JFET
Common-source circuit with
Source degeneration.

15(5)

Draw the hybrid Parameters equivalent circuit of PNP Common base Transistor and explain



The NPN-H parameters of Common base Transistor.

$$V_i = h_{ib} I_i + h_{rb} V_c \rightarrow (1)$$

$$I_c = h_{fb} I_e + h_{ob} V_c \rightarrow (2)$$

here $V_i = V_c =$ Input Voltage

$V_o = V_c =$ output Voltage

$I_e =$ input current

$I_c =$ output current.

$h_{ib} =$ Input impedance with output short circuited.

$h_{rb} =$ Reverse Voltage ratio with open circuited input

$h_{fb} =$ Current gain when output shorted.

$h_{ob} =$ output admittance with input open circuited.

$\therefore$ when input is open

$$I_e = 0 \quad \text{put } I_e = 0 \text{ in (1) \& (2)}$$

$$V_e = h_{rb} V_c$$

$$I_c = h_{ob} V_c$$

$$h_{rb} = \frac{V_e}{V_c} = \frac{\text{Input voltage}}{\text{output voltage}}$$

$$h_{ob} = \frac{I_c}{V_c} = \frac{\text{Output current}}{\text{output current}}$$

When Output is shorted $V_o = 0$

$$V_e = h_{ib} I_e$$

$$I_c = h_{fb} i_e$$

$$h_{ib} = \frac{V_e}{i_e} = \frac{\text{Input voltage}}{\text{Input current}}$$

$$h_{fb} = \frac{i_c}{i_e} = \frac{\text{Output current}}{\text{Input current}}$$

S. Jaya Ram
16/02/22

Shrey
16/2/22

Semester End Examination, February, 2022

Degree	B. Tech. (U. G.)	Program	ECE	Academic Year	2021 - 2022
Course Code	20EC303	Test Duration	3 Hrs. Max. Marks 70	Semester	III
Course	Signals and Systems				

Part A (Short Answer Questions 5 x 2 = 10 Marks)

No.	Questions (1 through 5)	Learning Outcome (s)	DoK
1	Give the Mathematical equation of impulse signal and plot its waveform	20EC303.1	L1
2	State Dirichlet's conditions	20EC303.2	L1
3	Give two properties of Convolution operation	20EC303.3	L1
4	List any two filter characteristics of linear systems	20EC303.4	L1
5	Define ROC of the laplace transform	20EC303.5	L3

Part B (Long Answer Questions 5 x 12 = 60 Marks)

No.	Questions (6 through 15)	Marks	Learning Outcome (s)	DoK
6 (a)	What are Continuous and Discrete Signals? Give examples	6M	20EC303.1	L1
6 (b)	What is System? Classify the System with suitable examples	6M	20EC303.1	L1
OR				
7	Derive the Response for Impulse and Step response of a System	12M	20EC303.1	L1
8(a)	Derive an expression for mean square error using the expression of a function using orthogonal signal space	6M	20EC303.2	L3
8 (b)	Find the Fourier transform of a Full wave rectified output whose fundamental period is π	6M	20EC303.2	L3
OR				
9 (a)	State and prove any three properties of Fourier Transform	6M	20EC303.2	L2
9 (b)	Find the Fourier Transform of the following signals. a) $\delta[t]$ b) $U(t)$ c) e^{-at}	6M	20EC303.2	L3
10 (a)	Perform the convolution of the two sequences $x[n]=\{1 \ 2 \ 3 \ 4\}$ and $h[n]=\{1,-1,1,-1\}$	4M	20EC303.3	L3
10 (b)	Explain and define Auto correlation function, write its properties and prove any two of them	8M	20EC303.3	L4
OR				
11 (a)	State and prove Parseval's theorem for energy / power signals	8M	20EC303.3	L4
11 (b)	Perform the convolution of $h(t)=e^{-2}u(t)$ and $x(t)=e^{-3}u(t)$	4M	20EC303.3	L3
12	Derive the Relationship between Bandwidth and Rise time	12M	20EC303.4	L2
OR				
13 (a)	State and Prove sampling Theorem	6M	20EC303.4	L2
13 (b)	Explain distortionless transmission through a system	6M	20EC303.4	L2
14 (a)	Obtain the Z-transform of $x(n) = -n^2u(-n-1)$	6M	20EC303.5	L4
14 (b)	State and prove Linearity and time Shifting properties of z-transform	6M	20EC303.5	L2
OR				
15 (a)	Obtain the Laplace transform of the following signals i) $\delta[t]$ ii) $e^{-2}u(t)$ iii) $A \sin \omega t u(t)$	6M	20EC303.5	L4
15 (b)	State and Prove the initial and final value theorem of Laplace transform	6M	20EC303.5	L2

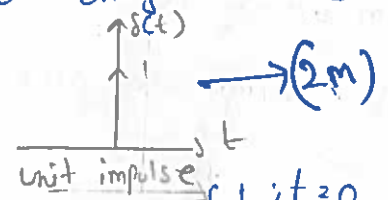
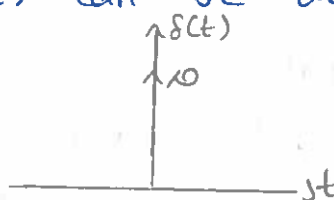
ANSWER KEY AND SCHEME OF EVALUATION

PART-A

① Give the Mathematical equation of impulse signal and plot its waveform.

A) Impulse function $\delta(t)$ can be defined only at $t=0$

(i.e) $\delta(t) = \begin{cases} \infty & ; t=0 \\ 0 & ; t \neq 0 \end{cases}$



for unit impulse function $\delta(t) = \begin{cases} 1 & ; t=0 \\ 0 & ; t \neq 0 \end{cases}$

② State Dirichlet's conditions.

A) (i) $x(t)$ is a absolute integral $\int_{-\infty}^{\infty} |x(t)| dt < \infty$

(ii) $x(t)$ having finite maxima & minimum values

(iii) $x(t)$ having finite discontinuity. $\rightarrow (2M)$

③ Give two properties of convolution operation.

A) (i) Commutative property

$$x_1(t) * x_2(t) = x_2(t) * x_1(t)$$

(ii) Distributive property:-

$$x_1(t) * [x_2(t) + x_3(t)] = x_1(t) * x_2(t) + x_1(t) * x_3(t).$$

(iii) Associative property:-

$$x_1(t) * [x_2(t) * x_3(t)] = [x_1(t) * x_2(t)] * x_3(t)$$

$\rightarrow (2M)$

④ List any two filter characteristics of linear systems.

A) Low Pass filter (LPF):- the filter transmits, without

any distortion, all of the signals of frequencies below a certain frequency ω_c radians per second.

High pass filter (HPF):- An ideal high pass filter with out any distortion, all of the signals of frequencies above certain frequency ω_c second. $\rightarrow (2m)$

ω_c is cutoff frequency.

5) Define ROC of the Laplace transform?

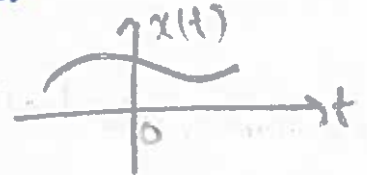
A) The range of ' σ ', for which the Laplace transform converges is termed the "Region of convergence (ROC)". $\rightarrow (2m)$

PART-B

6(a) What are continuous and discrete signals? Give examples.

A) Continuous-time signal:-

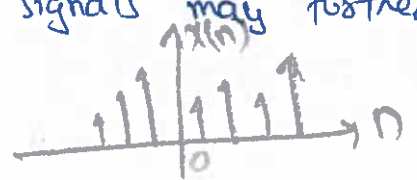
$\Rightarrow$ It is defined as, the independent variable is continuous and thus these signals are defined for a continuous of values of the independent variable.



Discrete-time signal:-

$\Rightarrow$ These are defined only at discrete times and consequently, for these signals the independent variable taken on only a discrete set of values.

Both continuous and discrete time signals may further be classified as



1) Deterministic and random signals:-

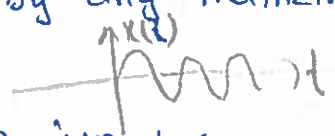
$\Rightarrow$ A signal exhibiting no uncertainty of its magnitude and phase at any given instant of time is called "deterministic signal".

The deterministic signal has a regular pattern and can be completely represented by mathematical equation at any time.

Ex:- sine wave, exponential signals.

⇒ A signal characterized by uncertainty about the occurrence is called random signal.

A random signal cannot be represented by any mathematical equation.



⇒ The pattern of such a signal is quite irregular.

⇒ Its amplitude and phase at any time instant cannot be predicted in advance.



Ex:- thermal noise generated in electric circuit.

2) Periodic and non-periodic signals:-

⇒ A signal which has a definite pattern and which repeats itself at regular intervals of time is called "periodic signal".

⇒ A signal which does not repeat at regular intervals of time is called a non-periodic signal.

Mathematically, a continuous time signal $x(t)$ is called periodic if and only if

$$x(t+T) = x(t) \text{ for all } t, \text{ (i.e.) for } -\infty < t < \infty$$

⇒ A signal is aperiodic if the above condition is not satisfied even for one value of t .

Accordingly the fundamental period ' T ' defines the duration of one complete cycle of $x(t)$.

The reciprocal of fundamental period (T) is called the fundamental frequency of the periodic signal $x(t)$.

It describes how frequently the periodic signal $x(t)$ repeats itself. $f = \frac{1}{T}$

$$\omega = 2\pi f = \frac{2\pi}{T}$$

Ex:- $\sin 12\pi t = x(t)$

$$\omega t = 12\pi t \Rightarrow \frac{2\pi}{T} = 12\pi$$

$$T = 1/6$$

Since T is a ratio of two integers

$\therefore x(t)$ is periodic with fundamental period of $T = \frac{1}{6}$ sec

Energy and power signals:-

Consider a voltage $v(t)$ developed across a resistor R , producing of current $i(t)$.

The instantaneous power dissipated in this resistor is defined by $p(t) = \frac{v^2(t)}{R}$

Accordingly in signal analysis, it is customary to define power in terms of a 1-ohm resistor, so that regardless of whether a given signal $x(t)$ represents a voltage or a current, we may express the instantaneous power of the signal as

$$p(t) = x^2(t)$$

on the basis of this convention, we define the total energy of the continuous time signal $x(t)$ as

$$E = \lim_{T \rightarrow \infty} \int_{-T/2}^{T/2} x^2(t) dt$$

$$E = \int_{-\infty}^{\infty} x^2(t) dt$$

and its time averaged (or) average power as

$$P = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t) dt$$

Ex:- $x(t) = 7 \cos(20t + \frac{\pi}{2})$

It is in the form $A \cos(\omega t + \phi)$

$$x(t) = \frac{A^2}{2}$$

$$A = 7$$

$$= \frac{49}{2}$$

Power signal

Even and odd signals:-

→ A continuous time signal $x(t)$ is said to be an even signal if it satisfies the condition

$$x(t) = x(-t)$$

Ex: cosine wave.

→ A continuous time signal $x(t)$ is said to be odd signal if it satisfies the condition

$$x(-t) = -x(t)$$

ex: sine wave.

Causal and non causal signals

→ A continuous time signal $x(t)$ is said to be causal if $x(t) = 0$ for $t < 0$, otherwise the signal is non-causal.

A continuous time signal $x(t)$ is said to be anti-causal if $x(t) = 0$ for $t > 0$.

Causal signal does not exist for negative time.

an anti-causal signal does not exist for positive time.

ex: $x(t) = e^{2t} u(t-1)$.

6(b) what is system? classify the system with suitable example.

A) system:- A system is formally defined as an entity that manipulates one or more signals to accomplish a function there by yielding new signals.

Static and Dynamic systems:-

A system is said to be memoryless (or) static if its output for each value of the independent variable at a given time is dependent only on the input at that time.

Ex:- $y(t) = x(t)$

2) A system is said to be memory or dynamic if its output signal depends on past or future values of the input signal.

Ex 1. $i(t) = \frac{1}{T} \int_{-\infty}^t v(\tau) d\tau$

Causal and non-causal systems:-

A system is said to be causal if the o/p of the system at any time 't' depends only on the present and past values of the i/p but not on future i/p.

A causal system are real time systems. They are physically realizable.

Ex:- $y(t) = x^2(t) + x(t-4)$

⇒ A system is said to be non-causal if the o/p of the system at any time 't' depends on future inputs.

Ex:- $y(t) = x(t+2) + x(t)$.

Time invariant and time variant:-

⇒ A system is said to be time invariant if its input/output characteristics do not change with time (i.e) if a time shift in the i/p results in a corresponding time shift in the output as shown in

$$\begin{aligned} x(t) &\longrightarrow y(t) \\ x(t-T) &\longrightarrow y(t-T) \end{aligned}$$

⇒ The time invariant property of the given continuous time system can be tested as follows.

$$y(t)/_{t \rightarrow t-t_0} = y(t)/_{x(t-t_0)}.$$

Ex:- $y(t) = x(t^2)$.

Linear and non linear system:-

⇒ A system which obeys the principle of superposition and principle of homogeneity is called a linear system.

and a system which does not obey the principle of superposition and homogeneity is called non linear system.

$$ax_1(t) + bx_2(t) \rightarrow ay_1(t) + by_2(t) \rightarrow \text{Continuous}$$

$$ax_1(n) + bx_2(n) \rightarrow ay_1(n) + by_2(n) \rightarrow \text{discrete}$$

Ex 1- $y(n) = n^2 x(n)$

Stable and unstable system:-

⇒ A system is said to be bounded input, bounded output. stable if and only if every bounded input results in the bounded output.

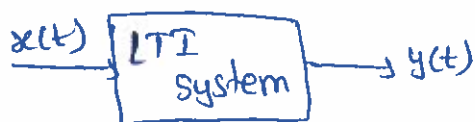
⇒ A bounded signal is a signal whose magnitude is always a finite value.

Ex 1- sine wave is a bounded signal. $\rightarrow (6M)$

7) Derive a Response for Impulse and step response of a system

A) Impulse response:-

Impulse response is the o/p of the system for a unit impulse input. for understanding the system behaviour, unit impulse response is very important.



$$y(t) = x(t) * h(t)$$

$$y(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau$$

$$y(t) = \int_{-\infty}^{\infty} x(t-\tau) h(\tau) d\tau$$

$$x(t) = \delta(t)$$

$$x(t-\tau) = s(t-\tau)$$

$$\therefore y(t) = \int_{-\infty}^{\infty} s(t-\tau) h(\tau) d\tau$$

$$= \int_{-\infty}^{\infty} s(-\tau-t) h(\tau) d\tau$$

$$= \int_{-\infty}^{\infty} h(\tau) s(\tau-t) d\tau$$

→ (6m)

$$\boxed{y(t) = h(t)}$$

Step response:-

The step response is the o/p of the system for a unit step input

→ The step response of an LTI continuous time system can be obtained by using convolution integral.



$$s(t) = x(t) * h(t)$$

$$s(t) = \int_{-\infty}^{\infty} x(t-\tau) h(\tau) d\tau$$

$$x(t) = u(t)$$

$$x(t-\tau) = u(t-\tau)$$

$$\therefore s(t) = \int_{-\infty}^{\infty} u(t-\tau) h(\tau) d\tau$$

$$\therefore s(t) = \int_{-\infty}^t h(\tau) d\tau$$

$$= \int_{-\infty}^t 1 \cdot h(\tau) d\tau$$

$$s(t) = \int_{-\infty}^t h(\tau) d\tau$$

$$\text{step response} = \int_{-\infty}^t \text{impulse response } d\tau \quad \rightarrow (6m)$$

8(a) Derive an expression for mean square error using the expression of a function using orthogonal signal space.

A) vector can be expressed as a sum of its components along 'n' mutually orthogonal vectors. provided these vectors formed a complete set of coordinate system.

let us consider a set of n functions $g_1(t), g_2(t), \dots, g_n(t)$ which are orthogonal to one another over an interval t_1 to t_2

$$\int_{t_1}^{t_2} g_j(t) g_k(t) dt = 0 ; j \neq k$$

$$\text{let } \int_{t_1}^{t_2} g_j^2(t) dt = K_j$$

let an arbitrary function $f(t)$ be over an interval (t_1, t_2)

$$f(t) = c_1 g_1(t) + c_2 g_2(t) + \dots + c_k g_k(t) + \dots + c_n g_n(t)$$

$$= \sum_{j=1}^n c_j g_j(t)$$

$$f_e(t) = f(t) - \sum_{j=1}^n c_j g_j(t)$$

$$E = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} \left(f(t) - \sum_{j=1}^n c_j g_j(t) \right)^2 dt$$

$$\text{let us consider } \frac{dE}{dc_j} = 0$$

$$\frac{d}{dc_j} \left(\frac{1}{t_2 - t_1} \int_{t_1}^{t_2} \left(f(t) - \sum_{j=1}^n c_j g_j(t) \right)^2 dt \right) = 0$$

$$\frac{d}{dc_j} \int_{t_1}^{t_2} f^2(t) dt \neq \frac{d}{dc_j} \int_{t_1}^{t_2} c_j^2 g_j^2(t) dt = \frac{d}{dc_j} \int_{t_1}^{t_2} c_j f(t) g_j(t) dt = 0$$

$$\int_{t_1}^{t_2} (-2f(t) g_j(t) + 2c_j g_j^2(t)) dt = 0$$

$$\int_{t_1}^{t_2} 2c_j g_j^2(t) dt = \int_{t_1}^{t_2} 2f(t) g_j(t) dt$$

$$c_g = \frac{\int_{t_1}^{t_2} f(t) g_g(t) dt}{\int_{t_1}^{t_2} g_g^2(t) dt}$$

$$c_g = \frac{1}{K_g} \int_{t_1}^{t_2} f(t) g_g(t) dt$$

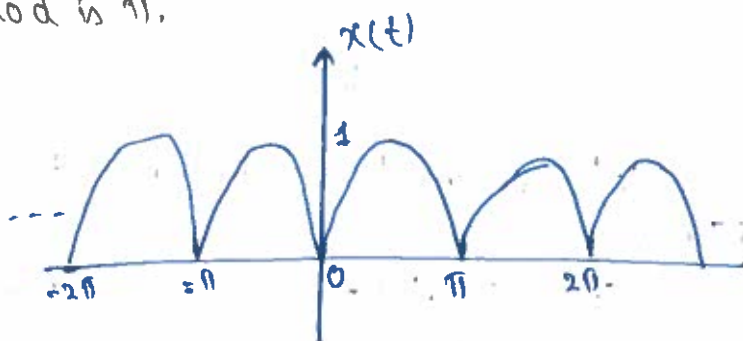
$$f(t) \approx c_1 g_1(t) + c_2 g_2(t) + \dots + c_n g_n(t)$$

$$f(t) \approx \sum_{g=1}^n c_g g_g(t)$$

→ (6M)

8(b) Find the Fourier transform of a full wave rectified o/p whose fundamental period is π .

(A)



$$\text{Given } x(t) = |\sin t| ; \text{ for } 0 < t < \pi$$

$$\omega = \frac{2\pi}{T}$$

where T = fundamental time period = π

$$\therefore \omega = \frac{2\pi}{\pi} = 2$$

Find Exponential Fourier series

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega t}$$

$$C_n = \frac{1}{T} \int_0^T x(t) e^{-jn\omega t} dt$$

$$= \frac{1}{\pi} \int_0^{\pi} \sin t \cdot e^{-jn\omega t} dt$$

$$= \frac{1}{\pi} \int_0^{\pi} \left[\frac{e^{jt} - e^{-jt}}{2j} \right] e^{-jn2t} dt$$

$$= \frac{1}{2\pi j} \int_0^{\pi} \left[e^{jt(1-2n)} - e^{-jt(1+2n)} \right] dt$$

$$= \frac{1}{2\pi j} \left(\frac{e^{\frac{j\omega(1-2n)}{1-4n^2}}}{j(1-2n)} \Big|_0 + \frac{e^{\frac{j\omega(1+2n)}{1-4n^2}}}{j(1+2n)} \Big|_0 \right)$$

$$= \frac{1}{2\pi j} \left(\frac{-2}{j(1-2n)} - \frac{2}{j(1+2n)} \right) = \frac{1}{2\pi} \left(\frac{4}{1-4n^2} \right) = \frac{2}{\pi(1-4n^2)}$$

$$\therefore x(t) = \sum_{n=-\infty}^{\infty} \frac{2}{\pi(1-4n^2)} e^{j2nt}$$

Apply Fourier transform on both side

$$F[x(t)] = F\left[\sum_{n=-\infty}^{\infty} \frac{2}{\pi(1-4n^2)} e^{j2nt}\right]$$

$$X(\omega) = \sum_{n=-\infty}^{\infty} \frac{2}{\pi(1-4n^2)} F[e^{j2nt}]$$

$$= \sum_{n=-\infty}^{\infty} \frac{2}{\pi(1-4n^2)} \cdot 2\pi \delta(\omega - 2n) = \sum_{n=-\infty}^{\infty} \frac{4\pi \delta(\omega - 2n)}{\pi(1-4n^2)} \quad \rightarrow (6M)$$

$$F[e^{j\omega_0 t}] = 2\pi \delta(\omega - \omega_0)$$

Q(a) state and prove any three properties of Fourier transform.

A) Linearity property:-

$$F[a_1 g_1(t) + a_2 g_2(t)] = a_1 F[g_1(t)] + a_2 F[g_2(t)]$$

Proof:- $F[a_1 g_1(t) + a_2 g_2(t)]$

$$= \int_{-\infty}^{\infty} (a_1 g_1(t) + a_2 g_2(t)) e^{-j\omega t} dt$$

$$= a_1 \int_{-\infty}^{\infty} g_1(t) e^{-j\omega t} dt + a_2 \int_{-\infty}^{\infty} g_2(t) e^{-j\omega t} dt$$

$$= a_1 G_1(\omega) + a_2 G_2(\omega) = \text{RHS.}$$

Time scaling :- If $g(t) \xrightarrow{FT} G(\omega)$ then

$$g(at) \longleftrightarrow \frac{1}{|a|} G\left(\frac{\omega}{a}\right).$$

Proof:- $F[g(t)] = \int_{-\infty}^{\infty} g(t) e^{-j\omega t} dt$

$$F[g(at)] = \int_{-\infty}^{\infty} g(at) e^{-j\omega t} dt \quad \left[\begin{array}{l} \text{put } at = \tau \\ a dt = d\tau \end{array} \right]$$

$$= g(\tau) e^{-j\frac{\omega}{a}\tau} d\tau$$

$$= \frac{1}{|a|} \int_{-\infty}^{\infty} g(\tau) e^{-j\frac{\omega}{a}\tau} d\tau$$

$$= \frac{1}{|a|} G\left(\frac{\omega}{a}\right).$$

$\rightarrow (2M)$

Time shifting:-

$$g(t) \xleftrightarrow{FT} G(\omega)$$

$$g(t-t_0) \xleftrightarrow{FT} e^{-j\omega t_0} G(\omega)$$

Proof:-

$$G(\omega) = F[g(t)] = \int_{-\infty}^{\infty} g(t) e^{-j\omega t} dt$$

$$F[g(t-t_0)] = \int_{-\infty}^{\infty} g(t-t_0) e^{-j\omega t} dt$$

$$k = t - t_0 \Rightarrow dt = dk$$

$$= \int_{-\infty}^{\infty} g(k) e^{-j\omega(k+t_0)} dk$$

$$= e^{-j\omega t_0} \int_{-\infty}^{\infty} g(k) e^{-j\omega k} dk$$

$$= e^{-j\omega t_0} \cdot G(\omega)$$

Frequency delay:-

$$g(t) \xleftrightarrow{FT} G(\omega)$$

$$e^{j\omega_c t} g(t) \xleftrightarrow{FT} G(\omega - \omega_c)$$

Proof:-

$$F[g(t)] = G(\omega) = \int_{-\infty}^{\infty} g(t) e^{-j\omega t} dt$$

$$F[g(t)e^{j\omega_c t}] = \int_{-\infty}^{\infty} g(t) e^{j\omega_c t} e^{-j\omega t} dt$$

$$= \int_{-\infty}^{\infty} g(t) e^{-j(\omega - \omega_c)t} dt$$

$$= G(\omega - \omega_c)$$

$$[2M \times 3 = 6M]$$

$$[2M \times 3M]$$

Q(b) Perform find the Fourier transform of the following signals

a) $\delta(t)$.

$$F[\delta(t)] = X(\omega) = \int_{-\infty}^{\infty} \delta(t) e^{-j\omega t} dt$$

$$x(t) = \delta(t)$$

$$F[\delta(t)] = \int_{-\infty}^{\infty} \delta(t) e^{-j\omega t} dt = \int_{-\infty}^{\infty} \delta(t-0) e^{-j\omega t} dt = e^{-j\omega(0)} = 1$$

c) $e^{-at} = x(t)$

$$FT(e^{-at} u(t)) = \int_{-\infty}^{\infty} e^{-at} u(t) e^{j\omega t} dt$$

$$= \int_{-\infty}^{\infty} e^{-at} e^{j\omega t} dt$$

$$= \int_0^{\infty} e^{-(a+j\omega)t} dt$$

$$= \left[\frac{e^{-(a+j\omega)t}}{-(a+j\omega)} \right]_0^{\infty}$$

$$= \frac{-1}{(a+j\omega)} [e^{-\infty} - e^0]$$

$$= \frac{1}{a+j\omega}$$

b) $x(t) = u(t)$

$$u(t) = \frac{1 + \text{sgn}(t)}{2}$$

$$FT(u(t)) = FT\left(\frac{1 + \text{sgn}(t)}{2}\right)$$

$$= \frac{1}{2} FT[1] + \frac{1}{2} FT[\text{sgn}(t)]$$

$$= \frac{1}{2} \left[2\pi\delta(\omega) + \frac{2}{j\omega} \right]$$

$$FT(u(t)) = \pi\delta(\omega) + \frac{1}{j\omega}$$

$$\rightarrow [2m \times 3 = 6m]$$

10a) Perform the convolution of two sequence $x(n) = [1 \ 2 \ 3 \ 4]$

and $h(n) = [1, -1, 1, -1]$

Ans

$$x(n) * h(n) = \sum_{m=-\infty}^{\infty} x(m) h(n-m)$$

The input sequence $x(n)$ is arranged as a system and the impulse response is arranged as a row as shown below the element of the two dimensional arrays we obtained by multiplying the corresponding

row element with the column element. of the two dimensional array we obtained by multiplying the corresponding row element using with column element.

$$y(0)=1; y(1)=2-1=1, y(2)=2$$

$$y(2)=2, y(4)=-3, y(3)=1, y(6)=-4$$

$$x(n) \begin{array}{c} \begin{array}{cccc} 1 & -1 & 1 & -1 \\ 1 & -1 & 1 & -1 \\ 2 & -2 & 2 & -2 \\ 3 & -3 & 3 & -3 \\ 4 & -4 & 4 & -4 \end{array} \end{array}$$

(c)

arrange two given sequence in a row one after another then multiply $h(n)$ with $x(n)$ with each element then continue upto given element.

then add all this elements in -

a column that is gives the samples of $y(n)$

$$x(n) = 1 \ 2 \ 3 \ 4$$

$$h(m) = 1 \ -1 \ 1 \ -1$$

$$\begin{array}{cccc} 1 & 2 & 3 & 4 \\ -1 & -2 & -3 & -4 \end{array}$$

$$\begin{array}{cccc} & & 1 & 2 & 3 & 4 \end{array}$$

$$\begin{array}{cccc} & & & & -1 & -2 & -3 & -4 \\ 1 & 1 & 2 & 2 & -3 & -3 & 1 & -4 \end{array}$$

$$\therefore y(n) = \{1, 1, 2, 2, -3, 1, -4\}$$

10(b) Explain and define Auto correlation function, write its properties and prove any two of them.

A) Auto correlation:-

→ The Auto correlation function gives the measure of match of similarity or relatedness or coherence b/w a signal and time delayed version.

→ This means that the auto correlation function is a special form of cross correlation function

$$R(\tau) = \int_{-\infty}^{\infty} x(t)x(t-\tau)dt$$

→ It is defined as the correlation of a signal with itself

Auto correlation for energy signals:-

The auto correlation of an energy signal $x(t)$ is

$$\text{given by } R(\tau) = \int_{-\infty}^{\infty} x(t)x^*(t-\tau)dt.$$

properties of ACF of energy signal:-

① The ACF exhibits conjugate symmetry

$$R(\tau) = R^*(-\tau)$$

Proof:- from definition of autocorrelation of an energy signal $x(t)$ is given by

$$R(\tau) = \int_{-\infty}^{\infty} x(t) x^*(t-\tau) dt$$

$$R^*(\tau) = \int_{-\infty}^{\infty} x^*(t) x(t-\tau) dt$$

$$\tau \rightarrow -\tau$$

$$R^*(\tau) = \int_{-\infty}^{\infty} x^*(t) x(t+\tau) dt$$

$$R^*(-\tau) = R(\tau)$$

② The value of ACF of an energy signal at origin is equal to the total energy of that signal $R(0) = E = \int_{-\infty}^{\infty} |x(t)|^2 dt$

Proof:- from definition

$$R(\tau) = \int_{-\infty}^{\infty} x(t) x^*(t-\tau) dt$$

$$\text{Let } \tau = 0$$

$$R(0) = \int_{-\infty}^{\infty} x(t) x^*(t-\tau) dt$$

$$= \int_{-\infty}^{\infty} |x(t)|^2 dt$$

$$R(0) = E = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

Autocorrelation for power signals:-

The ACF of a periodic signal with any period T is given by

$$R(\tau) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t) x^*(t-\tau) dt$$

properties:-

1) The ACF exhibits conjugate symmetry $R(\tau) = R^*(-\tau)$

Proof:- $R(\tau) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t) x^*(t-\tau) dt$

$$R^*(\tau) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} x^*(t) x(t-\tau) dt$$

$$R^*(-\tau) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} x^*(t) x(t+\tau) dt$$

$$\boxed{R^*(-\tau) = R(\tau)}$$

2) The ACF at origin is equal to the average power of that signal $R(0) = P$

$$\text{Proof!- } R(\tau) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t) x^*(t-\tau) dt$$

$$\tau = 0$$

$$R(0) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t) x^*(t) dt$$

$$R(0) = P$$

→ 8m

11(a) state and prove Parseval's theorem for energy/power signals.

A) Parseval's theorem for energy signals!- It defines the energy of a signal in terms of its Fourier transform. using Parseval's theorem the energy of a signal $x(t)$ can be evaluated directly from the frequency spectrum $x(\omega)$ without the knowledge of its time domain version

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |x(\omega)|^2 d\omega = \int_{-\infty}^{\infty} |x(f)|^2 df$$

Proof!- consider a function $x(t)$ such that

$$x(t) \xleftrightarrow{F.T} X(\omega)$$

$$x^*(t) \xleftrightarrow{F.T} X^*(-\omega)$$

The energy of a signal $x(t)$ is given by

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

$$= \int_{-\infty}^{\infty} x(t) x^*(t) dt.$$

$$\text{IFT} [X(\omega)] = x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega$$

$$E = x^*(t) \int_{-\infty}^{\infty} \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega dt$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) d\omega \left[\int_{-\infty}^{\infty} x^*(t) e^{-j(\omega^*)t} dt \right]^*$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) d\omega X^*(-\omega) d\omega$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) X^*(-\omega) d\omega$$

$$E = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(\omega)|^2 d\omega$$

→ (4m)

Power spectral density:-

Parseval's power theorem:- It defines the power of a signal in terms of its fourier series coefficients (i.e) in terms of the harmonic components present in the signal.

$$P = \sum_{n=-\infty}^{\infty} |c_n|^2$$

proof:- consider a function $x(t)$

$$|x(t)|^2 = x(t)x^*(t)$$

The average power for one cycle is

$$P = \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt$$

$$= \frac{1}{T} \int_{-T/2}^{T/2} x(t)x^*(t) dt$$

$$x(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega t}$$

$$P = \frac{1}{T} \sum_{n=-\infty}^{\infty} c_n \int_{-T/2}^{T/2} x^*(t) e^{jn\omega t} dt$$

$$= \sum_{n=-\infty}^{\infty} c_n c_n^*$$

$$P = \sum_{n=-\infty}^{\infty} |c_n|^2$$

→ (um)

11(b) perform the convolution of $h(t) = e^{-2t} u(t)$ and $x(t) = e^{-3t} u(t)$

A) Given $x(t) = e^{-3t} u(t)$, $h(t) = e^{-2t} u(t)$

w.k.T

$$x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau$$

$$\stackrel{(or)}{=} \int_{-\infty}^{\infty} x(t-\tau) h(\tau) d\tau$$

$$= \int_{-\infty}^{\infty} e^{-3(t-\tau)} u(t-\tau) e^{-2\tau} u(\tau) d\tau$$

$$u(\tau) = 1 \text{ for } \tau > 0$$

$$u(t-\tau) = 1 \text{ for } (t-\tau) \geq 0$$

$$\therefore x(t) * h(t) = \int_0^t e^{-3t} e^{-2\tau} e^{3\tau} d\tau$$

$$= e^{-3t} \int_0^t e^{\tau} d\tau$$

$$= e^{-3t} \left[\frac{e^{\tau}}{1} \right]_0^t$$

$$= e^{-3t} [e^t - e^0]$$

$$= e^{-3t} e^t - e^{-3t}$$

$$x(t) * h(t) = [e^{-2t} - e^{-3t}] u(t)$$

→ (um)

12) Derive the Relationship b/w Bandwidth and Rise time.

Ans) we know that the transfer function of an ideal LPF is given $H(\omega) = |H(\omega)| e^{-j\omega t_d}$.

$$\text{where } H(\omega) = \begin{cases} 1 & |\omega| < \omega_c \\ 0 & |\omega| > \omega_c \end{cases}$$

$$H(\omega) = 1 e^{-j\omega t_d}, |\omega| < \omega_c$$

$$= 0, |\omega| > \omega_c$$

$\therefore$ the impulse response $h(t)$ of the LPF is obtained by taking the inverse fourier transform of the transfer function $H(\omega)$

$$\therefore h(t) = F^{-1}\{H(\omega)\}$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} H(\omega) e^{j\omega t} d\omega$$

$$= \frac{1}{2\pi} \int_{-\omega_c}^{\omega_c} e^{-j\omega t_d} e^{j\omega t} d\omega$$

$$= \frac{1}{2\pi} \left[\int_{-\omega_c}^{\omega_c} e^{j\omega(t-t_d)} d\omega \right]$$

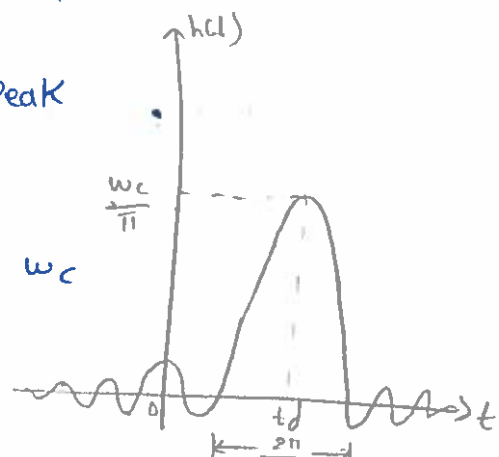
$$= \frac{1}{2\pi} \left[\frac{e^{j\omega(t-t_d)}}{j(t-t_d)} \right]_{-\omega_c}^{\omega_c}$$

$$= \frac{1}{\pi(t-t_d)} \left[\sin(\omega_c(t-t_d)) \right]$$

$$h(t) = \frac{\omega_c}{\pi} \frac{\sin \omega_c(t-t_d)}{\omega_c(t-t_d)}$$

The impulse response of ideal LPF is shown below.

$\Rightarrow$ The impulse response has a peak value at $t = t_d$. This value $\frac{\omega_c}{\pi}$ is proportional to cutoff frequency ω_c



if the ^{impulse} step response is known the step response can be obtained by convolution.

$$\therefore \text{the step response } y(t) = h(t) * u(t) = \int_{-\infty}^t h(\tau) d\tau$$

$$y(t) = \int_{-\infty}^t \frac{\omega_c}{\pi} \frac{\sin \omega_c(\tau - t_d)}{\omega_c(\tau - t_d)} d\tau$$

$$\text{let } x = \omega_c(\tau - t_d)$$

$$dx = \omega_c d\tau$$

$$\text{let } \tau \rightarrow -\infty \Rightarrow x = -\infty$$

$$\tau \rightarrow t \Rightarrow x = \omega_c(t - t_d)$$

$$\therefore y(t) = \int_{-\infty}^{\omega_c(t-t_d)} \frac{\omega_c}{\pi} \frac{\sin x}{x} \frac{dx}{\omega_c}$$

$$y(t) = \frac{1}{\pi} \int_{-\infty}^{\omega_c(t-t_d)} \frac{\sin x}{x} dx$$

$$y(t) = \frac{1}{\pi} \left[\text{Si}(x) \right]_{-\infty}^{\omega_c(t-t_d)}$$

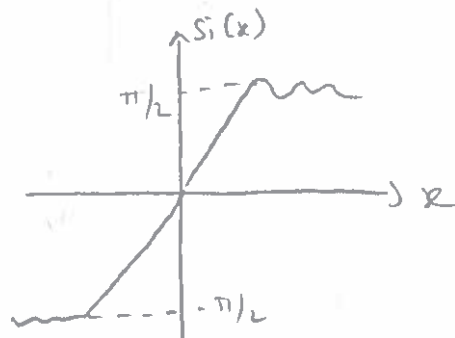
(i) $\text{Si}(x)$ is an odd function, that is

$$\text{Si}(-x) = -\text{Si}(x)$$

$$\text{Si}(0) = 0$$

$$\text{Si}(\infty) = \pi/2$$

$$\text{Si}(-\infty) = -\pi/2$$



$$y(t) = \frac{1}{\pi} \left[\text{Si}(x) \right]_{-\infty}^{\omega_c(t-t_d)}$$

$$= \frac{1}{\pi} \left[\text{Si}(\omega_c(t-t_d)) - \text{Si}(-\infty) \right]$$

$$= \frac{1}{\pi} \left[\text{Si}(\omega_c(t-t_d)) + \frac{\pi}{2} \right]$$

$$y(t) = \frac{1}{2} + \frac{1}{\pi} \text{Si}(\omega_c(t-t_d))$$

if $\omega_c \rightarrow \infty$ then the response

$$y(t) = \frac{1}{2} + \frac{1}{\pi} \sin(\infty)$$

$$= \frac{1}{2} + \frac{1}{\pi} \left(\frac{\pi}{2} \right)$$

$$y(t) = 1$$

if $\omega_c \rightarrow -\infty$ then the response

$$y(t) = \frac{1}{2} + \frac{1}{\pi} \sin(-\infty)$$

$$= 0$$

$\therefore$ The step response of LPF is shown in below.

from figure we have

$$\left. \frac{dy(t)}{dt} \right|_{t=t_d} = \frac{1-0}{t_r} = \frac{1}{t_r}$$

$$y(t) = \int_{-\infty}^t \frac{\omega_c}{\pi} \frac{\sin(\omega_c(t-t_d))}{\omega_c(t-t_d)} dt$$

$$\left. \frac{dy(t)}{dt} \right|_{t=t_d} = \frac{\omega_c}{\pi} \frac{\sin \omega_c(t-t_d)}{\omega_c(t-t_d)} \Big|_{t=t_d}$$

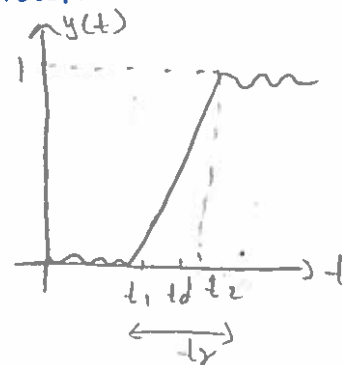
$$\left. \frac{dy(t)}{dt} \right|_{t=t_d} = \frac{\omega_c}{\pi}$$

$$\frac{1}{t_r} = \frac{\omega_c}{\pi}$$

$$t_r = \frac{\pi}{\omega_c}$$

$$t_r = \frac{\pi}{B.W}$$

$$\text{rise time} = \frac{\pi}{\text{Bandwidth.}}$$



→ (12 M)

13(a) state and prove sampling theorem.

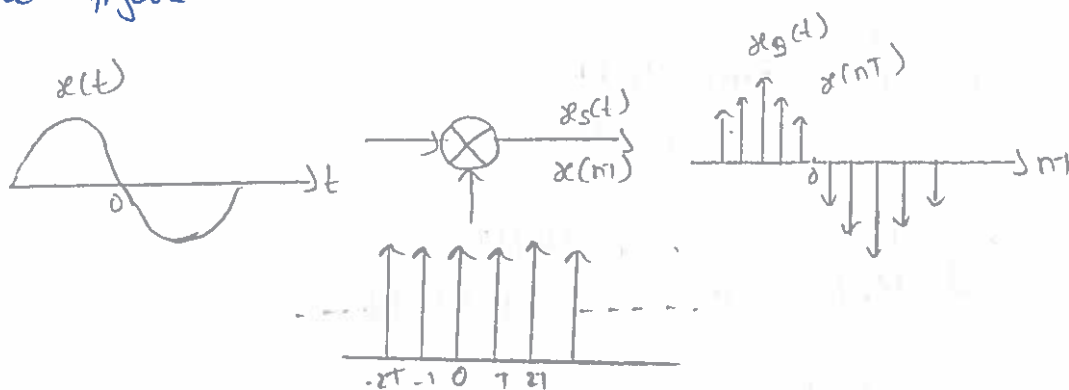
Ans: Sampling theorem:-

⇒ The sampling theorem is one of the most useful theorem since it applies to digital communications system.

⇒ The sampling theorem states that A band limited signal $x(t)$ with $x(\omega) = 0$ for $|\omega| \geq \omega_m$. The sampling frequency $f_s \geq 2f_m$ where f_m is the highest frequency component present in it.

This theorem is known as uniform sampling theorem since it pertains to the specification of a given signal by its sample at uniform intervals of $\frac{1}{f_s}$ sec.

Proof:- The sampling operation can be represented as shown below figure.



$x(t)$ is a continuous time band limited signal to be sampled which has no spectral component above f_m cycle per sec.

$$f_s = \frac{1}{T}$$

$$x_s(t) = x(t) \cdot \delta_T(t) \quad \text{where} \quad \delta_T(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$$

$$\delta_T(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_s t}$$

$$c_n = \frac{1}{T} \int_{-T/2}^{T/2} \delta(t) e^{-jn\omega_s t} dt = \frac{1}{T}$$

$$\therefore \delta(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$$

$$x_s(t) = x(t) \delta_T(t) = \frac{1}{T} \sum_{n=-\infty}^{\infty} x(t) e^{jn\omega_s t}$$

$$F.T[x_s(t)] = FT\left[\frac{1}{T} \sum_{n=-\infty}^{\infty} x(t) e^{jn\omega_s t}\right]$$

$$= \frac{1}{T} \sum_{n=-\infty}^{\infty} x(t) e^{jn\omega_s t}$$

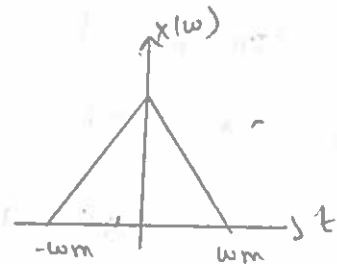
$$= \frac{1}{T} \sum_{n=-\infty}^{\infty} x(\omega - n\omega_s)$$

$$= \frac{1}{T} \sum_{n=-\infty}^{\infty} x\left(\omega - \frac{2\pi}{T}n\right)$$

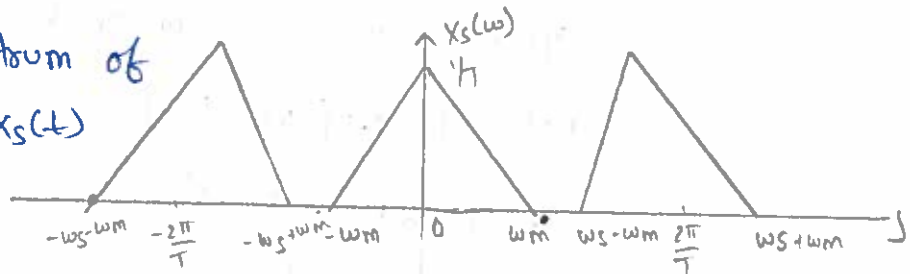
$$X_s(f) = f_s \sum_{n=-\infty}^{\infty} x(f - nf_s)$$

→ The signal $x(t)$ is band limited to f_m . The term $x(\omega - (\frac{2\pi}{T})n)$ is the shifting of $x(\omega)$ from $\omega=0$ to $\omega - (\frac{2\pi}{T})n$.

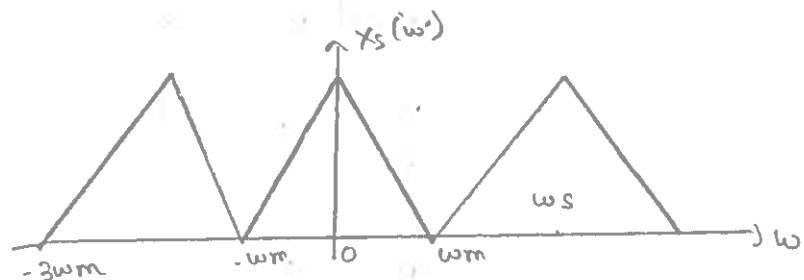
→ frequency spectrum of continuous time signal $x(t)$.



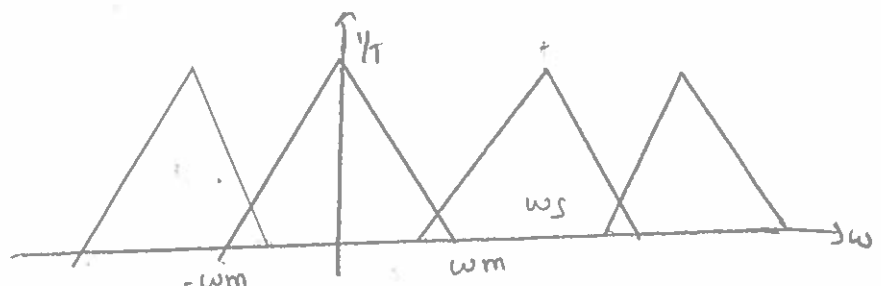
→ frequency spectrum of sampled signal $x_s(t)$ when $\frac{\pi}{T} > \omega_m$



→ when $\frac{\pi}{T} = \omega_m$



→ when $\frac{\pi}{T} < \omega_m$



$$f_s - f_m \geq f_m$$

$$f_s \geq 2f_m$$

$$\frac{\pi}{T} \geq \omega_m$$

$$\pi f_s \geq 2\pi f_m$$

$$\boxed{f_s \geq 2f_m}$$

→ BP(6M)

13(b) explain distortionless transmission through a system.

A) ⇒ The change of shape of the signal when it is transmitted through a system is called distortion.

⇒ Transmission of a signal through is said to be 'distortionless' if the o/p is an replica of i/p signal.

⇒ A constant change in magnitude at a constant time delay are not considered as distortion.

$$y(t) = kx(t - t_d)$$

apply fourier transform on b.s

$$FT[y(t)] = FT[kx(t - t_d)]$$

$$Y(\omega) = K FT[x(t - t_d)]$$

$$Y(\omega) = K e^{-j\omega t_d} X(\omega)$$

$$\frac{Y(\omega)}{X(\omega)} = K e^{-j\omega t_d}$$

$$H(\omega) = K e^{-j\omega t_d}$$

$$|H(\omega)| = |K e^{-j\omega t_d}| = K |e^{-j\omega t_d}|$$

$$= K(1) = K$$

$$|H(\omega)| = K$$

$$\theta(\omega) = \angle H(\omega) = -\omega t_d$$

the

linear phase system

for distortionless transmission then should not be any phase distortion. $\rightarrow$ (GM)

No phase distortion means the phase should be linear

for linear phase system

$$H(\omega) = |H(\omega)| e^{-j\omega t_d}$$

$\downarrow$ apply I.F.T

$$h(t) = \text{IFT}(|H(\omega)| e^{j\omega t_d})$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} |H(\omega)| e^{j\omega t_d} e^{j\omega t} d\omega$$

$$= \frac{1}{2\pi} \left\{ \int_0^{\infty} |H(\omega)| e^{j\omega(t-t_d)} d\omega + \int_0^{\infty} |H(\omega)| e^{j\omega(t-t_d)} d\omega \right\}$$

$$= \frac{1}{2\pi} \left[\int_0^{\infty} |H(\omega)| \left(e^{j\omega(t-t_d)} + e^{j\omega(t-t_d)} \right) d\omega \right]$$

$$h(t) = \frac{1}{\pi} \int_0^{\infty} |H(\omega)| \cos \omega(t-t_d) d\omega$$

$$t \rightarrow t_d + t$$

$$h(t_d + t) = \frac{1}{\pi} \int_0^{\infty} |H(\omega)| \cos(\omega(t_d + t - t_d)) d\omega$$

$$h(t_d + t) = \frac{1}{\pi} \int_0^{\infty} |H(\omega)| \cos \omega t d\omega \quad - (1)$$

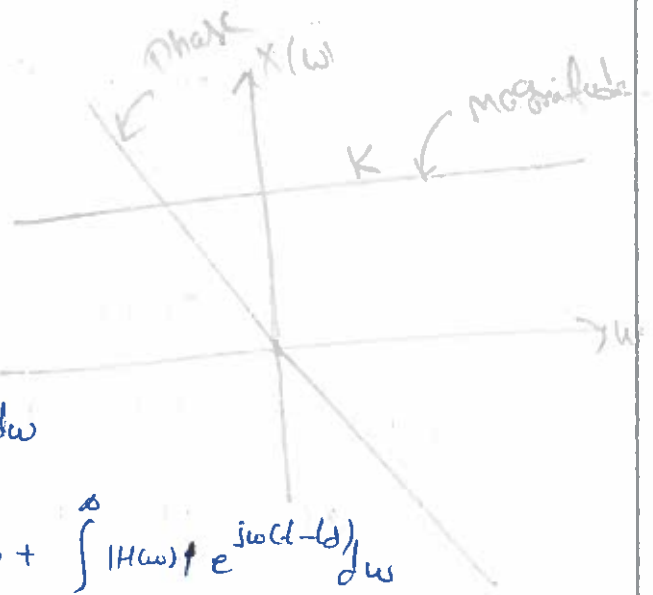
$$t \rightarrow t_d - t$$

$$h(t_d - t) = \frac{1}{\pi} \int_0^{\infty} |H(\omega)| \cos \omega t d\omega \quad - (2)$$

from (1) & (2)

we can say

$$h(t_d + t) = h(t_d - t)$$



14(a) obtain the Z-transform $x(n) = -n^2 u(n-1)$

Ans) Given $x(n) = -n^2 u(n-1)$

$$Z[u(n-1)] = \frac{z}{z-1}$$

using multiplication by n property we have

$$Z[(-n)u(n-1)] = -z \frac{d}{dz} \left(z[u(n-1)] \right)$$

$$= -z \frac{d}{dz} \left(\frac{z}{z-1} \right)$$

$$Z[-nu(n-1)] = \frac{z}{(z-1)^2}$$

again using multiplication by n property we have

$$Z[-n^2 u(n-1)] = -z \frac{d}{dz} \left[Z[-nu(n-1)] \right]$$

$$= -z \frac{d}{dz} \left[\frac{z}{(z-1)^2} \right]$$

$$= -z \left(\frac{(z-1)(z-1-2z)}{(z-1)^4} \right)$$

$$= -z \frac{(-z-1)}{(z-1)^3} = \frac{z(z+1)}{(z-1)^3} \rightarrow (6M)$$

14(b) state and prove linearity and time shifting properties of Z-transform.

Ans) Linearity Property:-

The linearity property of Z-transform states that the Z-transform of a weighted sum of two signals is equal to the weighted sum of individual Z-transform. That is the linearity property.

$$x_1(n) \xrightarrow{ZT} X_1(z) \quad \text{with } \text{ROC} = R_1$$

$$x_2(n) \xrightarrow{ZT} X_2(z) \quad \text{with } \text{ROC} = R_2$$

$$\text{then } ax_1(n) + bx_2(n) \xrightarrow{ZT} aX_1(z) + bX_2(z) \quad \text{with } \text{ROC} = R_1 \cap R_2$$

proof:-

$$\text{w.k.t } Z[x(n)] = X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$\begin{aligned}
 Z[ax_1(n) + bx_2(n)] &= \sum_{n=-\infty}^{\infty} (ax_1(n) + bx_2(n)) z^{-n} \\
 &= \sum_{n=-\infty}^{\infty} ax_1(n) z^{-n} + \sum_{n=-\infty}^{\infty} bx_2(n) z^{-n} \\
 &= a \sum_{n=-\infty}^{\infty} x_1(n) z^{-n} + b \sum_{n=-\infty}^{\infty} x_2(n) z^{-n} \\
 &= ax_1(z) + bx_2(z); \text{ROC}, R_1 \cap R_2
 \end{aligned}$$

$$ax_1(n) + bx_2(n) \xrightarrow{ZT} ax_1(z) + bx_2(z)$$

Time shifting property:-

The time shifting property of z transform states that $x(n) \xrightarrow{ZT} X(z)$ with zero initial condition with ROC = R

$$x(n-m) \xrightarrow{ZT} z^{-m} X(z)$$

$$Z[x(n)] = X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$Z[x(n-m)] = \sum_{n=-\infty}^{\infty} x(n-m) z^{-n}$$

$$n-m = p \Rightarrow n = m+p$$

$$Z[x(n-m)] = \sum_{p=-\infty}^{\infty} x(p) z^{-(m+p)}$$

$$= z^{-m} \sum_{p=-\infty}^{\infty} x(p) z^{-p}$$

$$= z^{-m} X(z)$$

$$x(n-m) \xrightarrow{ZT} z^{-m} X(z)$$

$$x(n+m) \xrightarrow{ZT} z^m X(z) \longrightarrow (2 \times 3m = 6m)$$

15(a) obtain the Laplace transform of the following signals.

(i) $\delta(t)$.

$$x(t) = \delta(t)$$

$$\begin{aligned} \text{LT}[x(t)] &= \int_{-\infty}^{\infty} x(t) e^{-st} dt = x(s) \\ &= \int_{-\infty}^{\infty} \delta(t) e^{-st} dt \\ &= e^{-s(0)} = 1 \text{ dc value} \end{aligned}$$

(ii) $e^{-2t} u(t)$

$$x(t) = e^{-2t} u(t)$$

$$\begin{aligned} \text{LT}[x(t)] &= \int_{-\infty}^{\infty} e^{-2t} u(t) e^{-st} dt \\ &= \int_0^{\infty} e^{-(2+s)t} dt \\ &= \left[\frac{e^{-(2+s)t}}{-(2+s)} \right]_0^{\infty} \\ &= \frac{1}{s+2} \end{aligned}$$

$$s+2 > 0, \quad s > -2 \quad \text{Re}\{s\} > -2$$

(iii) $A \sin \omega t u(t)$.

$$x(t) = A \sin \omega t u(t)$$

$$\begin{aligned} \text{LT}[A \sin \omega t u(t)] &= \text{LT} \left[A \cdot \frac{e^{j\omega t} - e^{-j\omega t}}{2j} \cdot u(t) \right] \\ &= \frac{A}{2j} \text{LT}[e^{j\omega t} u(t)] - \frac{A}{2j} \text{LT}[e^{-j\omega t} u(t)] \end{aligned}$$

$$\text{LT}[A \sin \omega t u(t)] = \frac{A}{2j} \left[\frac{1}{s-j\omega} \right] - \frac{A}{2j} \left[\frac{1}{s+j\omega} \right]$$

$$= \frac{A}{2j} \left[\frac{2j\omega}{s^2 + \omega^2} \right]$$

$$\text{LT}[A \sin \omega t u(t)] = \frac{A\omega}{s^2 + \omega^2} \quad (3 \times 2M = 6M)$$

15(b) state and prove the initial and final value theorem of Laplace transform.

A) initial value theorem :-

It enables to calculate the initial value of a function $x(t)$

$$\lim_{t \rightarrow 0} x(t) = x(0) = \lim_{s \rightarrow \infty} sX(s)$$

Proof :- given $\mathcal{L}\{x(t)\} = X(s)$

$$\mathcal{L}\left\{\frac{dx(t)}{dt}\right\} = \int_0^{\infty} \frac{dx(t)}{dt} e^{-st} dt = sX(s) - x(0)$$

apply $\lim_{s \rightarrow \infty}$ on both sides

$$\lim_{s \rightarrow \infty} \int_0^{\infty} \frac{dx(t)}{dt} e^{-st} dt = \lim_{s \rightarrow \infty} (sX(s) - x(0))$$

$$x(0) = \lim_{s \rightarrow \infty} sX(s)$$

$$\therefore \lim_{t \rightarrow 0} x(t) = x(0) = \lim_{s \rightarrow \infty} sX(s)$$

final value theorem :-

The final value theorem enables to determine the final value of a function $x(t)$

$$\lim_{t \rightarrow \infty} x(t) = x(\infty) = \lim_{s \rightarrow 0} sX(s)$$

Proof :- given $\mathcal{L}\{x(t)\} = X(s)$

$$\mathcal{L}\left\{\frac{dx(t)}{dt}\right\} = \int_0^{\infty} \frac{dx(t)}{dt} e^{-st} dt = sX(s) - x(0)$$

apply $\lim_{s \rightarrow 0}$ on both sides

$$\lim_{s \rightarrow 0} \int_0^{\infty} \frac{dx(t)}{dt} e^{-st} dt = \lim_{s \rightarrow 0} [sx(s) - x(0)]$$

$$[x(t)]_0^{\infty} = \lim_{s \rightarrow 0} sx(s) - x(0)$$

$$\therefore \boxed{x(\infty) = \lim_{t \rightarrow \infty} x(t) = \lim_{s \rightarrow 0} sx(s)} \rightarrow [2 \times 3\text{m} = 6\text{m}]$$

Scheme of Evaluation

Part-A

1. for equation $\rightarrow 1m$
wave form $\rightarrow 1m$
 2. Dirichlet's condition $\rightarrow 2m$
 3. two properties of convolution $\rightarrow 1m+1m$
 4. two filter characteristics $\rightarrow 1m+1m$
 5. Definition of ROC $\rightarrow 2m$
- } 10m

Part-B

- 6(a) for each type of signals $\rightarrow 1.5m \Rightarrow$ total 6m
- 6(b) for each type of system $\rightarrow 5m$
definition to system $\rightarrow 1m$ } 6m
7. Derivation of Response for impulse $\rightarrow 6m$
Derivation of Response for step $\rightarrow 6m$ } 12m
- 8(a) expression for mean square error $\rightarrow 6m$
for initial formula $\rightarrow 1m$
procedure $\rightarrow 4m$
find expression $\rightarrow 1m$ }
- 8(b) for full wave rectified wave form
& define $x(t) \rightarrow 1m$
getting $G_n \rightarrow 3m$
solving $X(\omega) \rightarrow 2m$ } 6m
- 9(a) for any three properties of FS
each property 2m $\Rightarrow 3 \times 2m$ } 6m

- 9(b) (a) $\delta(t) \rightarrow 2m$
 (b) $u(t) \rightarrow 2m$
 (c) $e^{at} \rightarrow 2m$ } $6m$

- 10(a) Convolution formula } $1m$
 given data } $4m$
 procedure/solution } $2m$
 final value $\rightarrow 1m$

- 10(b) definition of ACF $\rightarrow 1m$
 ACF Properties for Energy signal $\rightarrow 3.5m$
 ACF Properties for Power signal $\rightarrow 3.5m$ } $8m$

- 11(a) Parseval's theorem for energy signal $\rightarrow 4m$
 Parseval's theorem for power signal $\rightarrow 4m$ } $8m$

- 11(b) Convolution formula
 given data $\rightarrow 1m$
 Solution $\rightarrow 2m$
 final answer $\rightarrow 1m$ } $4m$

- 12 Derivation for six time & Bandwidth $\rightarrow 12m$

- 13(a) Statement for sampling theorem $\rightarrow 1m$
 proof $\rightarrow 5m$ } $6m$

- (b) Distribution for signal for magnitude $3m$
 phase $3m$ } $6m$

- 14(a) given data & Z-Transform formula $\rightarrow 2m$
 linearity $\rightarrow 3m$; time shift $\rightarrow 3m$ } $6m$

- 15(a) (i) $\delta(t) \rightarrow 2m$
 (ii) $e^{at}u(t) \rightarrow 2m$ } $6m$

- (b) (i) $\rightarrow$ initial theorem $\rightarrow 3m$
 final theorem $\rightarrow 3m$ } $6m$

Semester End Examination, January/February, 2022

Degree	B. Tech. (U. G.)	Program	ECE	Academic Year	2021 - 2022
Course Code	20EC304	Test Duration	3 Hrs.	Max. Marks	70
Course	Random Variables and Stochastic Process			Semester	III

Part A (Short Answer Questions 5 x 2 = 10 Marks)

No.	Questions (1 through 5)	Learning Outcome (s)	DoK
1	State Baye's theorem of probability	20EC304.1	L1
2	List any two properties of moment generating function	20EC304.2	L1
3	State Central Limit Theorem	20EC304.3	L1
4	List any four properties of Power Spectral Density	20EC304.4	L1
5	Define effective noise temperature	20EC304.5	L1

Part B (Long Answer Questions 5 x 12 = 60 Marks)

No.	Questions (6 through 15)	Marks	Learning Outcome (s)	DoK
6 (a)	Define Joint Probability, Marginal Probability and Conditional Probability and also derive the relation among them.	6M	20EC304.1	L2
6 (b)	In a family consisting of two children, probability for a child to be a boy is 0.5. Find the probability that at least one of the two children is a boy	6M	20EC304.1	L3

OR

7	A random variable X has pdf shown below. i) Find the value of k. ii) Find $P(1/4 < X < 1/2)$. $f(x) = \begin{cases} kx^2(1-x^3), & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$	12M	20EC304.1	L2

8	Find the mean and variance of uniformly distributed random variable	12M	20EC304.2	L2

OR

9 (a)	Let $Y=2X+3$, If the random variable 'X' is uniformly distributed over $[-1, 2]$, determine pdf of random variable 'Y', $f_Y(y)$	6M	20EC304.2	L2
9 (b)	Explain about the characteristic function and state its properties	6M	20EC304.2	L2

10 (a)	List any four properties of joint density function Two random variables X and Y have a joint probability density function	2M	20EC304.3	L1

10 (b)	$f_{xy}(x,y) = \begin{cases} \frac{5}{16} x^2 y, & 0 < x < 2 \\ & 0 < y < x \\ 0, & \text{else} \end{cases}$ a) Find the marginal density functions of X and Y b) Are X and Y statistically independent?	10M	20EC304.3	L3

OR

11 (a)	Find the density function of $W=X+Y$, where the densities of X and Y are assumed to be: $f_x(x)=4u(x)e^{-4x}$ and $f_y(y)=5u(y)e^{-5y}$	6M	20EC304.3	L2
11(b)	The joint characteristic function of two random variables X and Y is given by $\phi_{xy}(\omega_1, \omega_2) = \exp(-\omega_1^2 - 4\omega_2^2)$ Check whether X and Y are uncorrelated or not	6M	20EC304.3	L3
12 (a)	State and explain four properties of auto-correlation of a random process ' $X(t)$ '	6M	20EC304.4	L2
12 (b)	Find the mean and variance of a stationary random process whose auto-correlation function is given by $R_{xx}(\tau) = 16+3/(8+\tau^2)$	6M	20EC304.4	L4
OR				
13 (a)	Prove that the random process $X(t)=A \cos(\omega t+\Theta)$ is wide sense stationary if it is assumed that A and ω are constants and Θ is uniformly distributed random variable in the interval $[0,2\pi]$	8M	20EC304.4	L4
13 (b)	State and explain the Wiener-Khintchine relation	4M	20EC304.4	L2
14 (a)	Derive the relation between PSDs of input and output random process of an LTI system	6M	20EC305.5	L2
14 (b)	If $Y(t) = X(t+a) - X(t-a)$ then show that $S_{YY}(\omega) = 4 S_{XX}(\omega) \sin^2(a\omega)$ where ' $X(t)$ ' is the input random process and ' $Y(t)$ ' is the output random process of an LTI system.	6M	20EC304.5	L3
OR				
15 (a)	Define Band limited and Narrow band Processes	2M	20EC304.5	L1
15 (b)	Derive the expression for average noise figure of cascaded two port networks	10M	20EC304.5	L3

ANSWER KEY AND SCHEME OF EVALUATION

PART-A

1. Baye's theorem statement - 2m
2. Two Properties of mgf - 2m
3. Central limit theorem statement - 2m
4. Properties of PSD - each carry $\frac{1}{2}m$ - $4 \times \frac{1}{2} = 2m$
5. Definition of noise temperature - 2m

PART-B

- 6) a) Definitions each carry - 2m
 $3 \times 2m \rightarrow 6m$
- b) Problem - 6m
- 7) ~~def~~ Problem
Find value of k - 6m
find interval b/w $P(\frac{1}{4} < x < \frac{1}{2})$ - 6m.
- 8) Uniform distribution derivation
mean - 6m
Variance $\rightarrow 6m$.
- 9) a) Problem - 6m
b) Properties of characteristic function - 6m

10) a) Properties of joint density function $\rightarrow 2m$

b) a) problem, marginal density function of $x - 4m$
marginal density function of $y - 4m$

b) relation of x and y are statistically independent $- 2m$.

11) a) Problem - formula $- 1m$
Calculation $- 5m$ } $6m$

b) Problem on joint characteristic function
formulas $- 2m$
Calculation $- 4m$ } $6m$.

12) a) Properties of auto correlation
each carry $- 1\frac{1}{2} m$.
four properties $- 4 \times 1.5 = 6m$.

b) Problem on auto correlation function
find mean $- 3m$
variance $- 3m$

13) (a) Problem $- 8m$
formulas $- 2m$
Calculation $- 6m$ } $14m$

(b) Wiener - Khintchine relation $- 4m$



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ANSWER KEY AND SCHEME OF EVALUATION

- 14) a) Derivation of PSD Input and output
random process - 6m
- b) Problem - formula - 1m }
derivation - 5m } 6m
- 15) a) Definitions - each carry - 1m
Band limited - 1m }
Narrow band - 1m } 2m
- b) Derivation of average noise figure
formula - 1m
diagram - 1m
expression calculation - 8m.

ANSWER KEY AND SCHEME OF EVALUATION

PART- A

1. State Baye's theorem of probability

Ans: Baye's theorem:

If $B_1, B_2, B_3, \dots, B_n$ are n mutually exclusive and exhaustive events such that $P(B_i) > 0$ ($i=1, 2, 3, \dots, n$) in a sample space S and A is any other event in S intersecting with every B_i (i.e. A can only occur in combination with every any one of the events $B_1, B_2, B_3, \dots, B_n$) such that $P(A) > 0$ then the conditional probability of B_i given A is

$$P\left(\frac{B_i}{A}\right) = \frac{P(B_i) P(A/B_i)}{\sum_{i=1}^n P(B_i) P(A/B_i)} \quad i=1, 2, 3, \dots, n$$

2. List any two properties of moment generating function

Ans: Properties of moment generating function:

1. The moment generating function at $v=0$ is unity. It is given as $M_x(v)/_{v=0} = M_x(0) = 1$.
2. Let x be a random variable with moment generating function $M_x(v)$. Then the moment generating function

for $y = ax + b$ is given by

$$M_y(v) = e^{bv} M_x(av)$$

3. If $M_x(v)$ is a moment generating function of a random variable x , then $M_x(cv) = M_{cx}(v)$, where c is a real constant.

4. If x_1 and x_2 are two independent random variables with moment generating function $M_{x_1}(v)$ and $M_{x_2}(v)$:

$$M_{x_1+x_2}(v) = M_{x_1}(v) M_{x_2}(v)$$

3. State Central limit theorem

Ans: Statement: According to the Central limit theorem (CLT) the distribution of random process which is cumulative effect of a large number of independent noise sources can be assumed to be gaussian.

(or)
It states that the probability density function of a sum of N independent random variables approaches the gaussian density function as N tends to infinity. Although the theorem can be applied to some statistically dependent random variables, in most applications, we use it only for statistically independent random variables.

ANSWER KEY AND SCHEME OF EVALUATION

4. List any four properties of power spectral density?

Ans: Properties of power spectral density:

1. The power density spectrum $S_{xx}(\omega)$ is a real value and non-negative function. i.e. $S_{xx}(\omega) \geq 0$

2. The power density spectrum is an even function in ω .
i.e. $S_{xx}(\omega) = S_{xx}(-\omega)$

3. The power spectral density at zero frequency is equal to the area under the curve of the auto correlation $R_{xx}(\tau)$.

$$\text{i.e. } S_{xx}(0) = \int_{-\infty}^{\infty} R_{xx}(\tau) d\tau$$

4. $S_{xx}(\omega)$ is always a real function.

5. If $S_{xx}(\omega)$ is a psd of the WSS random process $x(t)$,

$$\text{then } \frac{1}{2\pi} \int_{-\infty}^{\infty} S_{xx}(\omega) d\omega = A \{E[x^2(t)]\} = R_{xx}(0)$$

(or) the time average of the mean square value of a WSS random process equals the area under the curve of the power spectral density.

6. If $x(t)$ is a WSS random process with psd $S_{xx}(\omega)$, then the psd of the derivative of $x(t)$ is equal to ω^2 times the psd $S_{xx}(\omega)$.

5. Define effective noise temperature

Ans: Definition of Effective noise temperature:

It is defined as the noise temperature with respect to the available noise power or power spectral density. For a passive RLC network, then noise temperature is given by

$$T = \frac{P_a}{k_B} \text{ kelvin.}$$

This is the same as physical temperature ^{or}. For active devices, where the power or power spectral density is amplified, the temperature increases as the density increases. This temperature is called equivalent (or) effective noise temperature.

$$T_n = \frac{P_a}{k_B}$$

ANSWER KEY AND SCHEME OF EVALUATION

PART - B

- 6 a) Define Joint probability, Marginal probability and Conditional probability and also derive the relation among them.

Ans: Joint probability:

if a sample space consists of two events A and B which are not mutually exclusive, then the probability of these events occurring jointly or simultaneously is called joint probability

$$P(A \cap B) = \lim_{n \rightarrow \infty} \frac{n(A \cap B)}{n}$$

Marginal Probability: Marginal probability is the probability of an event irrespective of the outcome of another variable

Conditional probability:-

Let A and B are any two events of a sample spaces then the conditional probability is defined as the probability of the event A occurring on the

condition that the probability event B is already known.

It is denoted by $P(A/B)$.

$$\text{i.e. } P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A \cap B)}{P(B)}, \quad P(B) \neq 0$$

Similarly, the conditional probability of occurrence of B when the event A is given can be expressed as

$$P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{P(A \cap B)}{P(A)}, \quad P(A) \neq 0$$

6) b) In a family consisting of two children, probability for a child to be a boy is 0.5. Find the probability that at least one of the two children is a boy.

Ans:-

There are two children then

Sample Space

$$S = \{GG, BB, GB, BG\}$$

where G - girl, B - boy $n(S) = 4$

Now we have to find the probability, the favourable cases is at least one boy

$$A = \{GB, BG, BB\} \text{ i.e. } n(A) = 3$$

$\therefore$ The probability of at least one boy in family

$$P = \frac{n(A)}{n(S)} = \frac{3}{4}$$

$$P = \frac{3}{4}$$

ANSWER KEY AND SCHEME OF EVALUATION

7. A random variable x has pdf shown below : i) Find the value of k (ii) Find $P(\frac{1}{4} < x < \frac{1}{2})$

$$f_x(x) = \begin{cases} k x^2(1-x^3), & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

Ans:

$$\text{Given } f_x(x) = \begin{cases} k x^2(1-x^3), & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

Now we have to find the value of 'k'.

We know that $\int_{-\infty}^{\infty} f_x(x) dx = 1$



$$\int_{-\infty}^{\infty} f_x(x) dx + \int_0^1 f_x(x) dx + \int_1^{\infty} f_x(x) dx = 1$$

$$0 + \int_0^1 k(x^2(1-x^3)) dx + 0 = 1$$

$$k \int_0^1 x^2(1-x^3) dx = 1$$

$$k \int_0^1 (x^2 - x^5) dx = 1$$

$$K \left[\frac{x^3}{3} - \frac{x^6}{6} \right]_0^1 = 1$$

$$K \left[\left(\frac{1}{3} - \frac{1}{6} \right) - (0 - 0) \right] = 1$$

$$K \left[\frac{1}{3} - \frac{1}{6} \right] = 1$$

$$K \left(\frac{2-1}{6} \right) = 1$$

$$K \left(\frac{1}{6} \right) = 1$$

$$\boxed{K = 6}$$

(ii) we know that $P(x_1 < x < x_2) = \int_{x_1}^{x_2} f_x(x) dx$.

$$P\left(\frac{1}{4} < x < \frac{1}{2}\right) = \int_{1/4}^{1/2} f_x(x) dx$$

$$= \int_{1/4}^{1/2} Kx^2(1-x^3) dx$$

$$= K \int_{1/4}^{1/2} (x^2 - x^5) dx$$

$$= K \left[\frac{x^3}{3} - \frac{x^6}{6} \right]_{1/4}^{1/2}$$

$$= 6 \left[\left(\frac{(1/2)^3}{3} - \frac{(1/2)^6}{6} \right) - \left(\frac{(1/4)^3}{3} - \frac{(1/4)^6}{6} \right) \right]$$

$$= 6 \left[\left(\frac{1/8}{3} - \frac{1/64}{6} \right) - \left(\frac{1/64}{3} - \frac{1/256}{6} \right) \right]$$

ANSWER KEY AND SCHEME OF EVALUATION

$$= 6 \left[\frac{0.125}{3} - \frac{0.15625}{6} - \frac{0.015625}{3} + \frac{0.00390625}{6} \right]$$

$$= 6 \left[0.0416 - 0.0260 - 0.00520 + 0.00065 \right]$$

$$= 6 \left[-0.03575 \right]$$

$$= -0.2145.$$

$$\therefore \boxed{P\left(\frac{1}{4} < x < \frac{1}{2}\right) = -0.2145}$$

8. Find the mean and variance of uniformly distributed random variable

Ans:

mean of uniformly distribution function:-

For a uniform distribution, we know that

$$f_x(x) = \begin{cases} \frac{1}{b-a} & a < x < b \\ 0 & \text{otherwise} \end{cases}$$

$$\text{mean } \mu(x) \quad E[x] = \int_{-\infty}^{\infty} x f_x(x) dx \quad \rightarrow \text{---} \frac{+}{a} \frac{+}{b} \text{---} \infty$$

$$= \int_{-\infty}^a x f_x(x) dx + \int_a^b x f_x(x) dx + \int_b^{\infty} x f_x(x) dx$$

$$= 0 + \int_a^b x \left(\frac{1}{b-a}\right) dx + 0$$

$$= \frac{1}{b-a} \int_a^b x \, dx$$

$$= \frac{1}{b-a} \left[\frac{x^2}{2} \right]_a^b$$

$$= \frac{1}{2} \frac{1}{b-a} [b^2 - a^2]$$

$$= \frac{1}{2} \frac{1}{b-a} ((b+a)(b-a))$$

$$= \frac{1}{2} (b+a)$$

$$\therefore E[x] = \frac{b+a}{2} \text{ (or) } \frac{a+b}{2}$$

Variance of the uniform distribution function!

$$\text{Variance } \sigma_x^2 = E[x^2] - [E[x]]^2 \rightarrow \textcircled{1}$$

Now find

$$E[x^2] = \int_{-\infty}^{\infty} x^2 f_x(x) \, dx$$



$$= \int_{-\infty}^a x^2 f_x(x) \, dx + \int_a^b x^2 f_x(x) \, dx + \int_b^{\infty} x^2 f_x(x) \, dx$$

$$= 0 + \int_a^b x^2 \left(\frac{1}{b-a}\right) \, dx + 0$$

$$= \frac{1}{b-a} \int_a^b x^2 \, dx$$

$$= \frac{1}{b-a} \left[\frac{x^3}{3} \right]_a^b$$

$$= \frac{1}{3} \cdot \frac{1}{b-a} [b^3 - a^3]$$

ANSWER KEY AND SCHEME OF EVALUATION

$$= \frac{1}{3} \left[\frac{1}{b-a} \right] \left[(b-a)(b^r + a^r + ab) \right] \quad [\because a^3 - b^3 = (a-b)(a^2 + b^2 + ab)]$$

$$E[x^r] = \frac{1}{3} [a^r + b^r + ab]$$

from ①

$$\sigma_x^2 = E[x^2] - [E[x]]^2$$

$$= \frac{a^r + b^r + ab}{3} - \left(\frac{a+b}{2} \right)^2$$

$$= \frac{a^r + b^r + ab}{3} - \frac{(a+b)^2}{4}$$

$$= \frac{a^r + b^r + ab}{3} - \frac{a^2 + b^2 + 2ab}{4}$$

$$= \frac{4(a^r + b^r + ab) - 3(a^2 + b^2 + 2ab)}{12}$$

$$= \frac{4a^r + 4b^r + 4ab - 3a^2 - 3b^2 - 6ab}{12}$$

$$= \frac{a^r + b^r - 2ab}{12}$$

$$\left[\sigma_x^2 = \frac{(a-b)^2}{12} \text{ or } \frac{(b-a)^2}{12} \right]$$

9(a) Let $Y = 2X + 3$, If the random variable X is uniformly distributed over $[-1, 2]$ determine pdf of random variable ' Y ', $f_Y(Y)$.

Ans!

Given $Y = 2X + 3$.

If X is a random variable uniformly distributed over $[-1, 2]$

We know that $f_X(x) = \begin{cases} \frac{1}{b-a}, & a < x < b \\ 0, & \text{otherwise} \end{cases}$

$$f_X(x) = \begin{cases} \frac{1}{2-(-1)}, & -1 < x < 2 \\ 0, & \text{otherwise} \end{cases}$$

$$f_X(x) = \begin{cases} \frac{1}{3}, & -1 < x < 2 \\ 0, & \text{otherwise} \end{cases}$$

Now $Y = 2X + 3$

$$\Rightarrow 2X = Y - 3$$

$$X = \frac{1}{2}(Y - 3)$$

Now it can be written as

$$X = \frac{1}{2}(Y - 3)$$

$$\begin{aligned} \text{So } \frac{dx}{dy} &= \frac{d}{dy} \left[\frac{1}{2}(Y - 3) \right] \\ &= \frac{1}{2} \left[\frac{d}{dy}(Y) - \frac{d}{dy}(3) \right] \end{aligned}$$

$$\begin{aligned} \frac{dx}{dy} &= \frac{1}{2}(1 - 0) = \frac{1}{2} \\ \text{i.e. } \boxed{\frac{dx}{dy} = \frac{1}{2}} \end{aligned}$$

ANSWER KEY AND SCHEME OF EVALUATION

Now the pdf of Y is

$$f_Y(y) = f_X(x) \left| \frac{dx}{dy} \right|$$

$$= \frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$$

$$\boxed{f_Y(y) = \frac{1}{6}}$$

$$\therefore f_Y(y) = \begin{cases} \frac{1}{6}, & -1 < x < 2 \\ 0, & \text{otherwise} \end{cases}$$

(b) Explain about characteristic function and state its properties.

Ans! Characteristic Function:-

Consider a random variable x with a p.d.f $f_X(x)$, then the expected value of the function $e^{j\omega x}$ is called characteristic function. It is expressed as

$$\phi_X(\omega) = E[e^{j\omega x}]$$

It is a function of a real variable, $-\infty < \omega < \infty$ where j is an imaginary operator

$$\text{i.e. } \phi_X(\omega) = E[e^{j\omega x}] = \sum_i e^{j\omega x_i} P(x_i) \quad (\text{for discrete})$$

$$\phi_X(\omega) = \int_{-\infty}^{\infty} e^{j\omega x} f_X(x) dx \quad (\text{for continuous})$$

Properties of Characteristic Function:

1. The characteristic function is unity at $\omega = 0$ and given by $\phi_x(\omega)/_{\omega=0} = \phi_x(0) = 1$
2. The maximum amplitude of the characteristic function is unity at $\omega = 0$,
i.e. $|\phi_x(\omega)| \leq \phi_x(0)$ (or) $|\phi_x(\omega)| \leq 1$
3. $\phi_x(\omega)$ is a continuous function of ω in the range $-\infty < \omega < \infty$
4. $\phi_x(-\omega)$ and $\phi_x(\omega)$ are conjugate functions. i.e.
$$\phi_x(-\omega) = \phi_x^*(\omega) \text{ and } \phi_x^*(-\omega) = \phi_x(\omega)$$
5. If $\phi_x(\omega)$ is a characteristic function of a random variable x , then the characteristic function of $y = ax + b$ is given by
$$\phi_y(\omega) = e^{j\omega b} \phi_x(a\omega), \text{ where } a \text{ and } b \text{ are real constants.}$$
6. If $\phi_x(\omega)$ is a characteristic function of a random variable x , then $\phi_x(c\omega) = \phi_x(\omega)$, where c is a real constant.
7. If x_1 and x_2 are two independent random variables, then
$$\phi_{x_1+x_2}(\omega) = \phi_{x_1}(\omega) \phi_{x_2}(\omega).$$

ANSWER KEY AND SCHEME OF EVALUATION

10) a) List any four properties of joint density function?

Ans:

Properties of joint density function:

1. $f_{xy}(x, y) \geq 0$ i.e. a joint probability density function is always non-negative

2.
$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_{xy}(x, y) dx dy = 1$$

3. The joint distribution function is equal to

$$F_{xy}(x, y) = \int_{-\infty}^x \int_{-\infty}^y f_{xy}(x, y) dx dy$$

4. The probability of the joint event $\{x_1 < x \leq x_2, y_1 < y \leq y_2\}$ is given as $P\{x_1 < x \leq x_2, y_1 < y \leq y_2\} = \int_{x_1}^{x_2} \int_{y_1}^{y_2} f_{xy}(x, y) dx dy$

(b) Two random variables x and y have a joint probability density function $f_{xy}(x, y) = \frac{5}{16} xy, 0 < x < 2, 0 < y < 2$
 $= 0, \text{ else.}$

(a) Find the marginal density functions of x and y

(b) are x and y statistically independent?

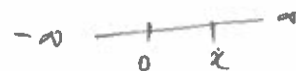
Ans:

Given
$$f_{xy}(x, y) = \frac{5}{16} xy, 0 < x < 2, 0 < y < 2$$

 $= 0, \text{ else.}$

Now find marginal density functions

$$f_x(x) = \int_{-\infty}^{\infty} f_{xy}(x, y) dy$$



$$= \int_{-\infty}^{\infty} \frac{5}{16} x^2 y dy$$

$$= \int_{-\infty}^0 f_{xy}(x, y) dy + \int_0^2 f_{xy}(x, y) dy + \int_2^{\infty} f_{xy}(x, y) dy$$

$$= 0 + \int_0^2 \frac{5}{16} x^2 y dy + 0$$

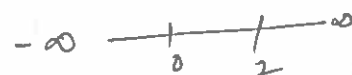
$$= \frac{5}{16} x^2 \int_0^2 y dy$$

$$= \frac{5}{16} x^2 \left[\frac{y^2}{2} \right]_0^2$$

$$= \frac{5}{32} x^2 [x^2 - 0]$$

$$\boxed{f_x(x) = \frac{5}{32} x^2}$$

$$f_y(y) = \int_{-\infty}^{\infty} f_{xy}(x, y) dx$$



$$= \int_{-\infty}^0 f_{xy}(x, y) dx + \int_0^2 f_{xy}(x, y) dx + \int_2^{\infty} f_{xy}(x, y) dx$$

$$= 0 + \int_0^2 \frac{5}{16} x^2 y dx + 0$$

$$= \frac{5}{16} y \int_0^2 x^2 dx$$

$$= \frac{5}{16} y \left[\frac{x^3}{3} \right]_0^2$$

$$= \frac{5}{48} y \left[\frac{8}{3} - 0 \right]$$

$$= \frac{5}{48} y \times \frac{8}{3}$$

$$\boxed{f_y(y) = \frac{5}{18} y}$$

ANSWER KEY AND SCHEME OF EVALUATION

(b) $\therefore$ Now $f_x(x) = \frac{5}{32} x^4$

$$f_y(y) = \frac{5}{18} y.$$

If x and y are Statistically independent

We know that $f_{xy}(x, y) = f_x(x) f_y(y)$

Now find $f_x(x) f_y(y) = \frac{5}{32} x^4 \times \frac{5}{18} y$

$$= \frac{25}{576} x^4 y$$

$$f_x(x) f_y(y) \neq f_{xy}(x, y)$$

$\therefore x$ and y are not independent.

11. a) Find the density function of $w = x + y$, Where the densities of x and y are assumed to be:

$$f_x(x) = 4u(x) e^{-4x}, \text{ and } f_y(y) = 5u(y) e^{-5y}.$$

Ans :

Given $f_x(x) = 4u(x) e^{-4x}$

$$f_y(y) = 5u(y) e^{-5y}$$

If x and y are independent the density function of $w = x + y$ is the convolution of $f_x(x)$ and $f_y(y)$

$$f_w(w) = f_x(x) * f_y(y)$$

$$= \int_0^{\infty} f_y(y) f_x(w-y) dy \rightarrow (1)$$

$$\text{Here } f_y(y) = 5u(y) e^{-5y}$$

$$f_x(w-y) = 4e^{-4(w-y)} u(w-y)$$

from (1)

$$f_w(w) = \int_0^{\infty} 5u(y) e^{-5y} \cdot 4e^{-4(w-y)} u(w-y) dy$$

$$= 20 \int_0^{\infty} e^{-5y} e^{-4(w-y)} u(y) u(w-y) dy$$

$$= 20 \int_0^w e^{-5y} e^{-4w+4y} u(y) u(w-y) dy$$

$$= 20 \int_0^w e^{-y} \cdot e^{-4w} dy, \quad w > 0$$

$$= 20 e^{-4w} \int_0^w e^{-y} dy, \quad w > 0$$

$$= 20 e^{-4w} \left[\frac{e^{-y}}{-1} \right]_0^w$$

$$= \frac{20}{-1} e^{-4w} [e^{-w} - e^0] u(w)$$

$$= -20 e^{-4w} [e^{-w} - 1] u(w)$$

$$\boxed{f_w(w) = 20 e^{-4w} [1 - e^{-w}] u(w)}$$

ANSWER KEY AND SCHEME OF EVALUATION

- (b) The joint characteristic function of two random variables x and y is given by $\phi_{xy}(\omega_1, \omega_2) = \exp(-\omega_1^2 - 4\omega_2^2)$ check whether x and y are uncorrelated or not

Ans:

Given characteristic function

$$\begin{aligned}\phi_{xy}(\omega_1, \omega_2) &= \exp(-\omega_1^2 - 4\omega_2^2) \\ &= e^{-\omega_1^2 - 4\omega_2^2}\end{aligned}$$

the joint moments of order $n+k$ are

$$m_{nk} = (-j)^{n+k} \frac{\partial^{n+k} \phi_{xy}(\omega_1, \omega_2)}{\partial \omega_1^n \partial \omega_2^k} \bigg|_{\omega_1 = \omega_2 = 0}$$

- (i) The first order moments are

$$\begin{aligned}m_{10} = E[x] &= (-j) \frac{\partial}{\partial \omega_1} [\phi_{xy}(\omega_1, \omega_2)] \bigg|_{\omega_1 = \omega_2 = 0} \\ &= -j \frac{\partial}{\partial \omega_1} [e^{-\omega_1^2 - 4\omega_2^2}] \bigg|_{\omega_1 = \omega_2 = 0} \\ &= -j e^{-\omega_1^2 - 4\omega_2^2} (-2\omega_1) \bigg|_{\omega_1 = \omega_2 = 0}\end{aligned}$$

$$\therefore m_{10} = -j e^0 [-2(0)]$$

$$\boxed{m_{10} = 0}$$

The mean value of x is zero.

$$m_{01} = E[Y] = (-j) \frac{\partial}{\partial \omega_2} [\phi_{xy}(\omega_1, \omega_2)] \bigg|_{\omega_1=\omega_2=0}$$

$$= -j \frac{\partial}{\partial \omega_2} [e^{-2\omega_1^2 - 4\omega_2^2}] \bigg|_{\omega_1=\omega_2=0}$$

$$= -j e^{-\omega_1^2 - 4\omega_2^2} (-8\omega_2) \bigg|_{\omega_1=\omega_2=0}$$

$$E[Y] = m_{01} = -j e^0 [-8(0)]$$

$$\boxed{m_{01} = 0}$$

$\therefore$ mean value of Y is zero.

Now

Correlation of x and y

$$C_{xy} = R_{xy} = E[XY] = m_{11}$$

$$m_{11} = (-j)^2 \frac{\partial^2 \phi_{xy}(\omega_1, \omega_2)}{\partial \omega_1 \partial \omega_2} \bigg|_{\omega_1=\omega_2=0}$$

$$= -j^2 \frac{\partial^2}{\partial \omega_1 \partial \omega_2} [e^{-\omega_1^2 - 4\omega_2^2}] \bigg|_{\omega_1=\omega_2=0}$$

$$= - \frac{\partial}{\partial \omega_1} \left[\frac{\partial}{\partial \omega_2} (e^{-\omega_1^2 - 4\omega_2^2}) \right] \bigg|_{\omega_1=\omega_2=0}$$

$$= - \frac{\partial}{\partial \omega_1} [(e^{-\omega_1^2 - 4\omega_2^2}) (-8\omega_2)] \bigg|_{\omega_1=\omega_2=0}$$

$$= +8\omega_2 \frac{\partial}{\partial \omega_1} [e^{-\omega_1^2 - 4\omega_2^2}] \bigg|_{\omega_1=\omega_2=0}$$

$$= 8\omega_2 e^{-\omega_1^2 - 4\omega_2^2} (-2\omega_1) \bigg|_{\omega_1=\omega_2=0}$$

$$= -16\omega_1\omega_2 e^{-\omega_1^2 - 4\omega_2^2} \bigg|_{\omega_1=\omega_2=0}$$

$$C_{xy} = R_{xy} = 0$$

$$\text{i.e. } \boxed{m_{11} = 0}$$

$\therefore$ x and y are uncorrelated.

ANSWER KEY AND SCHEME OF EVALUATION

12. (a) State and explain four properties of auto correlation of a random process $x(t)$.

Ans:

Properties of Auto correlation:

Auto correlation function:

The auto correlation function between two random processes is given by $R_{xx}(t, t+\tau) = E[x(t) x(t+\tau)]$.

Properties of Auto correlation function:

1. The mean square value $x(t)$ is $E[x^2(t)] = R_{xx}(0)$

Proof:

$$R_{xx}(\tau) = E[x(t_1) x(t_2)]$$

$$= E[x(t) x(t+\tau)]$$

$$\text{if } \tau=0, \quad = E[x(t) x(t)]$$

$$R_{xx}(\tau) = E[x^2(t)].$$

2. Auto correlation function is maximum at the origin.

$$|R_{xx}(\tau)| \leq R_{xx}(0) \quad \text{or} \quad R_{xx}(0) \geq |R_{xx}(\tau)|.$$

Proof! Consider $[x(t_1) \pm x(t_2)]^2 \geq 0$.

$$[x^2(t_1) + x^2(t_2) \pm 2x(t_1)x(t_2)] \geq 0$$

$$E[x^2(t_1) + x^2(t_2) \pm 2x(t_1)x(t_2)] \geq 0$$

from property ①

$$R_{xx}(0) + R_{xx}(0) \pm 2R_{xx}(\tau) \geq 0$$

$$2R_{xx}(0) \pm 2R_{xx}(\tau) \geq 0$$

$$2[R_{xx}(0) \pm R_{xx}(\tau)] \geq 0$$

$$R_{xx}(0) \pm R_{xx}(\tau) \geq 0$$

$$R_{xx}(0) \geq \pm R_{xx}(\tau)$$

$$R_{xx}(0) \geq |R_{xx}(\tau)|$$

$$\therefore \boxed{R_{xx}(\tau) \leq R_{xx}(0)}$$

3. Auto correlation is an even function of τ .

$$R_{xx}(\tau) = R_{xx}(-\tau)$$

$$R_{xx}(-\tau) = E[x(t)x(t-\tau)] \quad \begin{array}{l} t-\tau = u \\ t = u+\tau \end{array}$$

$$R_{xx}(-\tau) = E[x(u+\tau)x(u)]$$

$$\boxed{R_{xx}(-\tau) = R_{xx}(\tau)}$$

ANSWER KEY AND SCHEME OF EVALUATION

4. If a Random process $x(t)$ has non-zero mean value and ergodic with no periodic component - then.

$$\lim_{|T| \rightarrow \infty} R_{xx}(T) = (\bar{x})^2$$

Proof:- Consider a random process $x(t)$ with s.v's $x(t_1)$ & $x(t_2)$.

Given the mean value is $E[x(t)] = \bar{x} \neq 0$.

The auto correlation function is

$$R_{xx}(T) = E[x(t)x(t+T)] = E[x(t_1)x(t_2)]$$

Since the process has no periodic components as $|T| \rightarrow \infty$ the random variables become independent.

$$\text{i.e. } \lim_{|T| \rightarrow \infty} R_{xx}(T) = E[x(t_1)] E[x(t_2)]$$

Since $x(t)$ is ergodic

$$E[x(t_1)] = E[x(t_2)] = \bar{x}$$

$$\lim_{|T| \rightarrow \infty} R_{xx}(T) = \bar{x} \cdot \bar{x} = (\bar{x})^2$$

12) b) Find the mean and variance of a stationary random process whose autocorrelation function is given by

$$R_{xx}(\tau) = 16 + \frac{3}{8 + \tau^2}$$

Ans:

Given $R_{xx}(\tau) = 16 + \frac{3}{8 + \tau^2}$

(i) Mean value $\overline{x^2} = \lim_{T \rightarrow \infty} R_{xx}(\tau)$

We know that

$$\overline{x^2} = \lim_{T \rightarrow \infty} |R_{xx}(\tau)|$$

$$= \lim_{\tau \rightarrow \infty} \left(16 + \frac{3}{8 + \tau^2} \right)$$

$$= 16 + \frac{3}{\infty}$$

$$= 16 + 0$$

$$\overline{x^2} = 16$$

mean value $\boxed{\overline{x} = 4}$

(ii) Variance $\sigma_x^2 = E[x^2] - [E[x]]^2$ or $E[x^2] - \overline{x^2} \rightarrow (1)$

Now find $E[x^2] = ?$

We know that $E[x^2] = R_{xx}(0)$ (from property)

$$= \left(16 + \frac{3}{8 + \tau^2} \right)_{\tau=0}$$

$$= 16 + \frac{3}{8 + 0}$$

$$= 16 + 0.375$$

$$E[x^2] = 16.375$$

from (1)

$$\sigma_x^2 = 16.375 - 16$$

$$\boxed{\sigma_x^2 = 0.375}$$

ANSWER KEY AND SCHEME OF EVALUATION

13 (a) Prove that the random process $x(t) = A \cos(\omega t + \theta)$ is wide sense stationary if it is assumed that A and ω are constants and θ is uniformly distributed random variable in the interval $[0, 2\pi]$

Ans:-

Given $x(t) = A \cos(\omega t + \theta)$

We know that $P_{xx}(\tau) = A^2 \{ E[x^2(t)] \}$

and $f_{\theta}(\theta) = \begin{cases} \frac{1}{2\pi}, & 0 < \theta \leq 2\pi \\ 0, & \text{otherwise} \end{cases}$

$$E[x^2(t)] = \int_{-\infty}^{\infty} x^2(t) f_{\theta}(\theta) d\theta$$

$$= \int_{-\infty}^{\infty} x^2(t) f_{\theta}(\theta) d\theta + \int_0^{2\pi} x^2(t) f_{\theta}(\theta) d\theta + \int_{2\pi}^{\infty} x^2(t) f_{\theta}(\theta) d\theta$$

$$= 0 + \int_0^{2\pi} (A^2 \cos^2(\omega t + \theta)) \left(\frac{1}{2\pi}\right) d\theta$$

$$= \frac{1}{2\pi} \int_0^{2\pi} A^2 \cos^2(\omega t + \theta) d\theta$$

$$= \frac{A^2}{2\pi} \int_0^{2\pi} \frac{1 + \cos(2(\omega t + \theta))}{2} d\theta$$

$$= \frac{A^2}{2\pi} \int_0^{2\pi} \left[\frac{1 + \cos 2(\omega t + \theta)}{2} \right] d\theta \quad \left[\because \cos \theta = \frac{1 + \cos 2\theta}{2} \right]$$

$$= \frac{A^2}{4\pi} \int_0^{2\pi} [1 + \cos(2\omega t + 2\theta)] d\theta$$

$$= \frac{A^2}{4\pi} \left[\int_0^{2\pi} 1 d\theta + \int_0^{2\pi} \cos(2\omega t + 2\theta) d\theta \right]$$

$$= \frac{A^2}{4\pi} \left[\left[\theta \right]_0^{2\pi} + \left[\frac{\sin(2\omega t + 2\theta)}{2} \right]_0^{2\pi} \right]$$

$$= \frac{A^2}{4\pi} \left[(2\pi - 0) + \frac{1}{2} [\sin(2\omega t + 4\pi) - \sin 2\omega t] \right]$$

$$= \frac{A^2}{4\pi} \left[2\pi - \frac{1}{2} \sin(2\omega t) \right]$$

$$E[x^2(t)] = \frac{A^2}{2} - \frac{A^2}{8\pi} \sin 2\omega t$$

$\therefore$ The time average power is

$$P_{xx} = A[E[x^2(t)]] = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T \left[\frac{A^2}{2} - \frac{A^2}{8\pi} \sin 2\omega t \right] dt$$

$$= \frac{1}{2T} \frac{A^2}{2} \int_{-T}^T 1 dt$$

$$= \frac{A^2}{4T} [t]_{-T}^T$$

$$= \frac{A^2}{4T} [T + T]$$

$$= \frac{A^2}{4T} (2T)$$

$$\boxed{P_{xx} = \frac{A^2}{2}}$$

ANSWER KEY AND SCHEME OF EVALUATION

13) b) State and explain Wiener khintchine relation.

Ans! Wiener khintchine relation!

The power density spectrum and the time average of the auto correlation function form a fourier transform pair.

$$\text{i.e. } S_{xx}(\omega) = \int_{-\infty}^{\infty} R_{xx}(\tau) e^{-j\omega\tau} d\tau \text{ and } R_{xx}(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S_{xx}(\omega) e^{j\omega\tau} d\omega$$

Proof! we know that

$$A[R_{xx}(t, t+\tau)] = A\{E[x(t)x(t+\tau)]\}.$$

for WSS random process

$$A[R_{xx}(t, t+\tau)] = R_{xx}(\tau)$$

$$\text{Power Spectral density is } S_{xx}(\omega) = \lim_{T \rightarrow \infty} E\left[\frac{R_T(\omega)}{2T}\right]$$

$$\text{W.K.T } |R_T(\omega)|^2 = X_T(\omega) X_T^*(\omega)$$

$$X_T(\omega) = \int_{-T}^T x(t) e^{-j\omega t} dt, \quad X_T^*(\omega) = \int_{-T}^T x(t) e^{j\omega t} dt$$

$$S_{xx}(\omega) = \lim_{T \rightarrow \infty} E\left[\frac{1}{2T} \int_{-T}^T x(t) e^{-j\omega t} dt \int_{-T}^T x(t) e^{j\omega t} dt\right]$$

$$S_{xx}(\omega) = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T \int_{-T}^T E[x(t)x(t)] e^{-j\omega(t_1-t)} dt dt,$$

taking inverse fourier transform

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} S_{xx}(\omega) e^{j\omega\tau} d\omega = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T \int_{-T}^T R_{xx}(t_1-t_2) \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{j\omega\tau} e^{-j\omega(t_1-t_2)} d\omega dt_1 dt_2$$

$$= \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T \int_{-T}^T R_{xx}(t, \tau) \delta(t_1 - t - T) dt d\tau,$$

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} S_{xx}(\omega) e^{j\omega T} d\omega = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T R_{xx}(t, \tau) \int_{-T}^T \delta(t_1 - t - T) dt d\tau$$

[$\because \int_{-T}^T \delta(t_1 - t - T) dt = 1$]

$$\therefore \frac{1}{2\pi} \int_{-\infty}^{\infty} S_{xx}(\omega) e^{j\omega T} d\omega = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T R_{xx}(t, t+T) dt$$

$$= A [R_{xx}(t, t+T)]$$

$$\therefore A [R_{xx}(t, t+T)] = \frac{1}{2\pi} \int_{-\infty}^{\infty} S_{xx}(\omega) e^{j\omega T} d\omega$$

Taking Fourier transform on b.s

$$S_{xx}(\omega) = \int_{-\infty}^{\infty} A [R_{xx}(t, t+T)] e^{-j\omega T} dT$$

$$\text{For a w.s.s, } R_{xx}(t, t+T) = R_{xx}(T)$$

$R_{xx}(T)$ and $S_{xx}(\omega)$ are a Fourier transform pair.

ANSWER KEY AND SCHEME OF EVALUATION

14) a) Derive the relation between PSDs of input and output random process of an LTI system.

Ans: Power Density Spectrum of Response:

Consider that a random process $X(t)$ is applied on an LTI system having a transfer function $H(\omega)$. The output response is $Y(t)$.

If the power spectrum of the input is $S_{XX}(\omega)$, then the power spectrum of the output response is given by

$$S_{YY}(\omega) = |H(\omega)|^2 S_{XX}(\omega)$$

Proof: Let $R_{YY}(\tau)$ be the autocorrelation of the output response $Y(t)$. Then the power spectrum of the response is the Fourier transform of $R_{YY}(\tau)$.

$$\begin{aligned}\therefore S_{YY}(\omega) &= F[R_{YY}(\tau)] \\ &= \int_{-\infty}^{\infty} R_{YY}(\tau) e^{-j\omega\tau} d\tau\end{aligned}$$

we know that $R_{yy}(T) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} R_{xx}(T+T_1-T_2) h(T_1) h(T_2) dT_1 dT_2$

Then
$$S_{yy}(\omega) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} R_{xx}(T+T_1-T_2) h(T_1) h(T_2) dT_1 dT_2 e^{j\omega T} dT$$

$$= \int_{-\infty}^{\infty} h(T_1) \int_{-\infty}^{\infty} h(T_2) \int_{-\infty}^{\infty} R_{xx}(T+T_1-T_2) e^{j\omega T} dT dT_2 dT_1$$

$$= \int_{-\infty}^{\infty} h(T_1) e^{j\omega T_1} \int_{-\infty}^{\infty} h(T_2) e^{j\omega T_2} \int_{-\infty}^{\infty} R_{xx}(T+T_1-T_2) e^{j\omega T} dT dT_2 dT_1$$

Let $T+T_1-T_2 = t$, $dT = dt$

$$S_{yy}(\omega) = \int_{-\infty}^{\infty} h(T_1) e^{j\omega T_1} dT_1 \cdot \int_{-\infty}^{\infty} h(T_2) e^{j\omega T_2} dT_2 \int_{-\infty}^{\infty} R_{xx}(t) e^{j\omega t} dt$$

we know that $S_{yy}(\omega) = H^*(\omega) H(\omega) S_{xx}(\omega) = |H(\omega)|^2 S_{xx}(\omega)$

$$S_{yy}(\omega) = |H(\omega)|^2 S_{xx}(\omega)$$

14) b) If $y(t) = x(t+a) - x(t-a)$ then show that $S_{yy}(\omega) = 4 S_{xx}(\omega) \sin^2(a\omega)$ where $x(t)$ is the input random process and $y(t)$ is the output random process of an LTI system.

Ans! Given $y(t) = x(t+a) - x(t-a)$

$$R_{yy}(T) = E[y(t) y(t+T)]$$

ANSWER KEY AND SCHEME OF EVALUATION

$$\begin{aligned}
 &= E\{[x(t+a) - x(t-a)][x(t+a+\tau) - x(t-a+\tau)]\} \\
 &= E[x(t+a)x(t+a+\tau)] - E[x(t-a)x(t+a+\tau)] \\
 &\quad - E[x(t+a)x(t-a+\tau)] + E[x(t-a)x(t-a+\tau)] \\
 &= R_{xx}(\tau) - R_{xx}(\tau+2a) - R_{xx}(\tau-2a) + R_{xx}(\tau) \\
 &= 2R_{xx}(\tau) - R_{xx}(\tau+2a) - R_{xx}(\tau-2a)
 \end{aligned}$$

Taking Fourier transform on both sides, we get

$$S_{yy}(\omega) = 2S_{xx}(\omega) - F[R_{xx}(\tau+2a)] - F[R_{xx}(\tau-2a)]$$

Using time shifting property of Fourier transforms, we can write

$$F[R_{xx}(\tau+2a)] = e^{j\omega 2a} S_{xx}(\omega)$$

$$F[R_{xx}(\tau-2a)] = e^{-j\omega 2a} S_{xx}(\omega)$$

$$S_{yy}(\omega) = 2S_{xx}(\omega) - S_{xx}(\omega)[e^{j2a\omega} + e^{-j2a\omega}]$$

$$= 2S_{xx}(\omega) - 2S_{xx}(\omega) \cos 2\omega a$$

$$= 2S_{xx}(\omega) [1 - \cos 2\omega a]$$

$$= 2S_{xx}(\omega) \times 2\sin^2(a\omega)$$

$$S_{yy}(\omega) = 4S_{xx}(\omega) \sin^2(a\omega)$$

15) a) Define Band limited and Narrowband processes

Ans: Band limited Process:-

If the bandpass PSD of $N(t)$ is limited to a frequency bandwidth of ω (rad/sec) i.e. $S_{NN}(\omega)$ is zero outside frequency bandwidth of ω and it does not include $\omega=0$, the process is called band limited process.

Narrow Band Process:

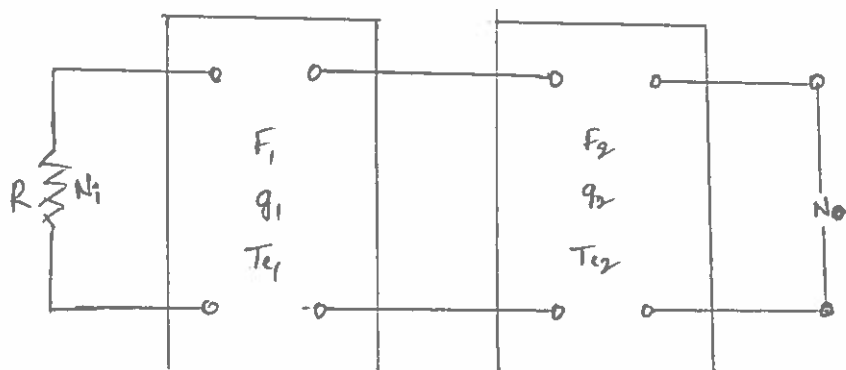
When the bandwidth ' ω ' of band limited process is very less than ω_0 i.e. if $\omega \ll \omega_0$. where ω_0 is the frequency at which power spectrum is maximum, then it is said to be narrow band process.

(b) Derive the expression for average noise figure of cascaded two port networks

Ans: Noise figure in terms of equivalent input noise temperature is

$$F = 1 + \frac{T_e}{T}$$

Consider Two Networks in Cascade.



ANSWER KEY AND SCHEME OF EVALUATION

Where F_1 and F_2 are noise figures of first and second stage respectively. g_1 and g_2 are the available gains of first and second stages respectively. T_{e1} and T_{e2} are the effective noise temperature of first and second stages respectively.

Let N_i = Input noise power generated by R.

Case-I : If Noise contributed by the stages is zero.

Then $N_{o1} = g_1 g_2 N_i$

where N_{o1} = Output noise power due to N_i .

Case II : Consider the noise contributed by the first stage. The contribution of the noise at the output of first stage is

$$= g_1 (F_1 - 1) N_i$$

$$= g_1 \left(\frac{T_{e1}}{T} \right) N_i$$

Then it is amplified by second stage

$$N_{o2} = g_1 (F_2 - 1) N_i = g_2 \frac{T_{e2}}{T} N_i$$

Total output noise power

$$N_o = N_{o1} + N_{o2} + N_{o3}$$

$$N_o = g_1 g_2 N_i + g_1 g_2 \left(\frac{T_{e1}}{T} \right) N_i + g_2 \left(\frac{T_{e2}}{T} \right) N_i$$

$$\frac{1}{g_1 g_2} \frac{N_o}{N_i} = 1 + \frac{T_{e1}}{T} + \frac{T_{e2}}{T} \times \frac{1}{g_1}$$

The overall noise figure (for cascade of two networks)

$$F = \frac{1}{g_1 g_2} \times \frac{N_o}{N_i}$$

$$= 1 + \frac{T_{e1}}{T} + \frac{T_{e2}}{T} \times \frac{1}{g_1}$$

$$1 + \frac{T_e}{T} = 1 + \frac{T_{e1}}{T} + \frac{T_{e2}}{T} \times \frac{1}{g_1}$$

T_e = overall effective noise temperature of two networks in cascade

$$T_e = T_{e1} + \frac{T_{e2}}{g_1}$$

From the relation b/w noise figure and effective noise temperature we have, $T_e = T(F-1)$, $T_{e1} = T(F_1-1)$, $T_{e2} = T(F_2-1)$

By substituting all the corresponding values in the expression of effective noise temperature of cascading system of '2' stages,

we get

$$T_e = T_{e1} + \frac{T_{e2}}{g_1}$$

$$T(F-1) = T(F_1-1) + \frac{T(F_2-1)}{g_1}$$

$$(F-1) = (F_1-1) + \frac{F_2-1}{g_1}$$

$$\boxed{\therefore F = F_1 + \frac{F_2-1}{g_1}}$$

This is the required expression of the noise figure of cascade system having 2 stages.

Semester End Examination, February, 2022

Degree	B. Tech. (U. G.)	Program	ECE	Academic Year	2021 - 2022
Course Code	20EC305	Test Duration	3 Hrs.	Max. Marks	70
Course	Digital System Design	Semester	III		

Part A (Short Answer Questions 5 x 2 = 10 Marks)

No.	Questions (1 through 5)	Learning Outcome (s)	DoK
1	Convert: $(FABC)_{16} = ()_8$	20EC305.1	L1
2	Define pair quad and octet in K-Maps and give examples	20EC305.2	L1
3	What are PLAs and PALs?	20EC305.3	L1
4	State the function table of T-Flip-Flop	20EC305.4	L1
5	Draw VHDL design flow	20EC305.5	L2

Part B (Long Answer Questions 5 x 12 = 60 Marks)

No.	Questions (6 through 15)	Marks	Learning Outcome (s)	DoK
6 (a)	Convert the following i) $(53.625)_{10}$ to $(?)_2$ ii) $(3FD)_{16}$ to $(?)_2$ iii) $(A69.8)_{16}$ to $(?)_{10}$	6M	20EC305.1	L2
6 (b)	Show that the dual of the exclusive OR is also its complement	6M	20EC305.1	L2
OR				
7 (a)	(i) Convert the following binary 1010011 into gray code (ii) Convert the following gray code 101011 into its equivalent binary	6M	20EC305.1	L2
7 (b)	(i) What are 2's complement and 9's complement of a numbers? Give examples (ii) Subtract the following using 1's and 2's complement $(101)_2 - (10110)_2$	6M	20EC305.1	L2
8 (a)	(i) Simplify the following Boolean function, using a four variable Karnaugh map method and implement the simplified function using NAND gates $F(A,B,C,D) = \sum m(0,2,4,5,6,7,8,10,13,15)$	6M	20EC305.2	L2
8 (b)	Solve the given expression using consensus theorem $A'B' + AC + BC + B'C + AB$	6M	20EC305.2	L2
OR				
9 (a)	Obtain the simplified expression in POS (product of sums) of $F(W,X,Y,Z) = M(1,2,4,7,12,14,15)$ using K-maps	6M	20EC305.2	L2
9 (b)	Obtain the simplified expression in product of sums. (i) $F(A,B,C,D) = \pi(0,1,2,3,4,10,11)$ (ii) $F(A,B,C,D) = \pi(1,3,5,7,13,15)$	6M	20EC305.2	L2
10 (a)	Organize the procedure for multiplexers and de-multiplexers and draw the logic diagram of a 4-to-1 line multiplexer with logic gates	6M	20EC305.3	L3
10 (b)	Model a circuit for 2-bit magnitude comparator using gates	6M	20EC305.3	L3
OR				
11 (a)	Design odd parity generator for 3-bit using a decoder	6M	20EC305.3	L3
11 (b)	Construct the functions and applications of PLAs in memory addressing and implement the following two Boolean functions with a PLA: $F1(A,B,C) = \sum (0,1,3,5)$ and $F2(A,B,C) = \sum (1,2,4,7)$	6M	20EC305.3	L3

12 (a)	Explain the working of JK Flip Flop	6M	20EC305.4	L2
12 (b)	Demonstrate the conversion of an SR Flip-Flop into D Flip-Flop	6M	20EC305.4	L2
OR				
13 (a)	Relate and explain Johnson counter	6M	20EC305.4	L2
13 (b)	Illustrate the shift Right and Left with examples	6M	20EC305.4	L2
Inference the VHDL code for the following				
14 (a)	(i) 3 bit up Counter (ii) Contrast the architecture body? Write the syntax for architecture body	6M	20EC305.5	L4
14 (b)	Analyze overloading? Explain overloading in VHDL with suitable example	6M	20EC305.5	L4
OR				
15 (a)	Discover the dataflow design style of VHDL programming with suitable example	6M	20EC305.5	L4
15 (b)	Contrast in detail about various data types and operators used in VHDL	6M	20EC305.5	L4



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SONTYAM, ANANDAPURAM, VISAKHAPATNAM - 531 173

ANSWER KEY AND SCHEME OF EVALUATION

PART- A :-

1. Conversion $(FABC)_{16}$ to $(5274)_8$ - 2m
2. K-map pair and Octedt - 1m
Example - 1m
3. PLA - 1m
PAL - 1m
4. T-Flip Flop Function table - 2m
5. VHDL Design flow diagram - 2m

PART- B :-

- 6.(a) (i) Conversion $(53.625)_{10}$ to $(?)_2$ → 2m
(ii) Conversion $(3FD)_{16}$ to $(?)_2$ → 2m
(iii) Conversion $(A69.8)_{16}$ to $(?)_{10}$ → 2m
- (b) Ex-OR gate → 3m
Ex NOR gate → 3m
- 7.(a) (i) Conversion binary 1010011 into Gray code → 3m
(ii) Conversion Gray code 101011 into binary → 3m
- (b) (i) 2's complement numbers → 2m
1's complement numbers → 2m
Example → 2m
(ii) 1's complement → 3m

8. (a) K-map based simplification $\rightarrow 2m$

NAND Gate based simplification $\rightarrow 4m$

(b)

Consensus theorem statement $\rightarrow 2m$

theorem based $A'B' + AC + B'C + B'C + AB$ expression $\rightarrow 4m$

9. (a) Simplification K-map based $\rightarrow 4m$

POS form Design $\rightarrow 2m$

(b)

(i) POS form for $F(A, B, C, D) = \Pi(0, 1, 2, 3, 4, 10, 11) \rightarrow 3m$

(ii) POS form for $F(A, B, C, D) = \Pi(1, 3, 5, 7, 13, 15) \rightarrow 3m$

10. (a).

Procedure for mux and De-mux $\rightarrow 2m$

Logic Diagram of 4×1 line mux with logic gates $\rightarrow 4m$

(b).

2-Bit magnitude Comparator using Gates $\rightarrow 6m$

11. (a)

Odd parity generator $\rightarrow 2m$

3-Bit odd parity generator based Decoder $\rightarrow 4m$

(b)

PLA Construction for $F_1(A, B, C) = \Sigma(0, 1, 3, 5) \rightarrow 3m$

PLA Construction for $F_2(A, B, C) = \Sigma(1, 2, 4, 7) \rightarrow 3m$

12.

(a) JK-Flipflop Diagram $\rightarrow 2m$

Function table $\rightarrow 2m$

Function table based working $\rightarrow 2m$

(b)

SR-Flip-flop Excitation table $\rightarrow 1m$

D-Flipflop Excitation table $\rightarrow 1m$

Conversion table from SR-to-D $\rightarrow 4m$.

13. (a) Johnson Counter diagram $\rightarrow 2m$

Johnson Counter explanation $\rightarrow 4m$

(b) Shift Right explanation with example $\rightarrow 3m$
Left Shift explanation with example $\rightarrow 3m$

14. (a) (i) VHDL code for 3-bit up counter $\rightarrow 3m$

(ii) Architecture body diagram $\rightarrow 1m$

Architecture body syntax $\rightarrow 2m$

(b) Overloading Analysis $\rightarrow 2m$
VHDL code for overloading $\rightarrow 4m$

15. (a) Dataflow design style diagram $\rightarrow 2m$

VHDL program for combinational ckt (b) Sequential ckt $\rightarrow 4m$

(b) Data types in VHDL $\rightarrow 3m$

Operators in VHDL $\rightarrow 3m$.



N S RAJU INSTITUTE OF TECHNOLOGY

(AUTONOMOUS)

SONTYAM, ANANDAPURAM, VISAKHAPATNAM - 531 173

ANSWER KEY AND SCHEME OF EVALUATION

1. $(FABC)_{16} = (5274)_8$

$\Rightarrow$

$(FABC)_{16} =$

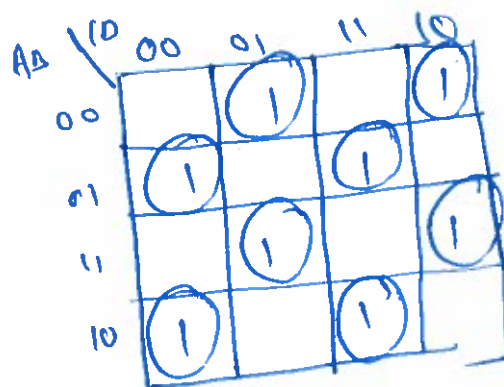
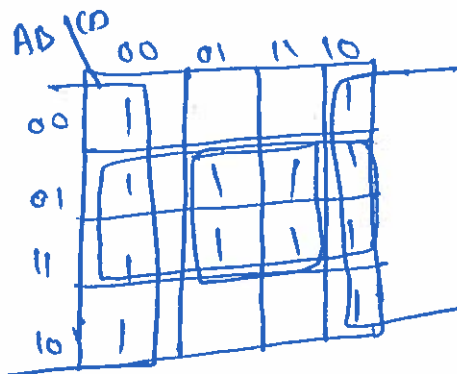
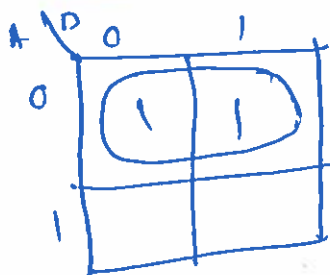
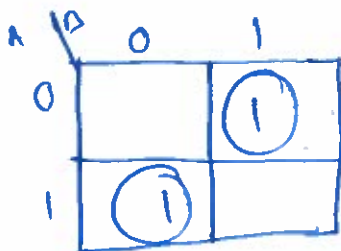
$$\begin{array}{cccc} 1111 & 1010 & 1011 & 1100 \\ \hline & 5 & 2 & 7 & 4 \end{array}$$

$(FABC)_{16} = (5274)_8$

$$\begin{aligned} A &= 1010 \\ B &= 1011 \\ C &= 1100 \\ D &= 1101 \\ E &= 1110 \\ F &= 1111 \end{aligned}$$

2. k-map

k-map is a method of simplifying Boolean algebra Equations. Maurice Karnaugh introduced it in 1953 as a refinement.



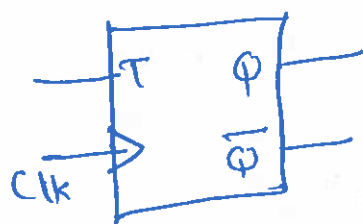
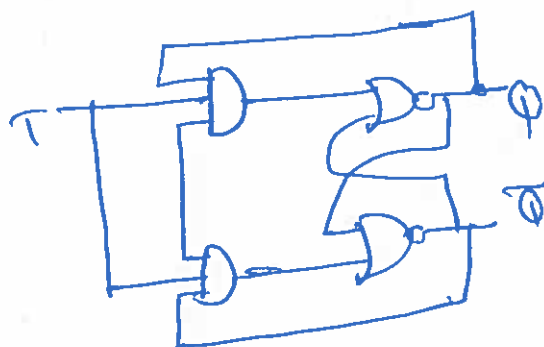
3. PLA:- [Programmable Logic Array].

The number of don't care conditions is excessive, it is more economical to use a second type of LSI Component called a PLA: It is similar to PROM. In PLA both AND and OR gates have fuses. at $\mu 81P$.

PAL:- (Programmable Array Logic).

It is a Programmable logic device with a fixed OR gate array and programmable AND array.

4. T-Flip-Flop:-



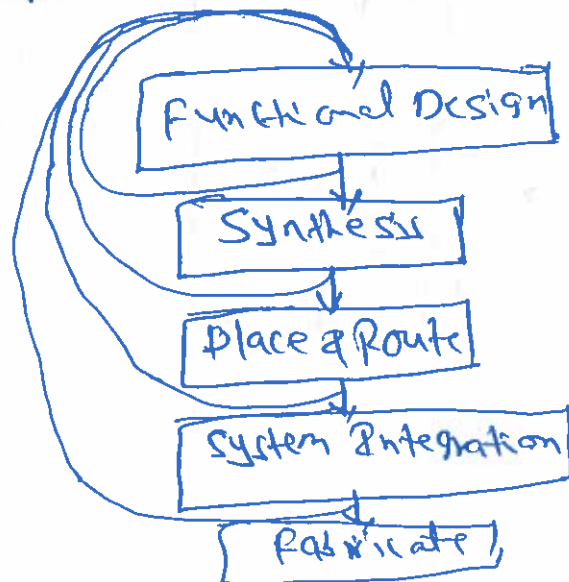
Q_n	T	Q_{n+1}
0	0	0
0	1	1
1	0	0
1	1	1

T	Q_{n+1}
0	Q_n
1	$\overline{Q_n}$

5. VHDL Design Flow:-

The first step in the design flow is writing the Synthesis register Transfer Level (RTL) VHDL circuit model.

VHDL model



⑥ a) Convert the following

(i) $53.625_{(10)}$

$$\begin{array}{r} 2 \overline{) 53} \\ 2 \overline{) 26} - 1 \\ 2 \overline{) 13} - 0 \\ 2 \overline{) 6} - 1 \\ 2 \overline{) 3} - 0 \\ 1 - 1 \end{array}$$

$110101_{(2)}$

$110101.10100_{(2)}$

Converting decimal point

$0.625 \times 2 = 1.30$

$0.30 \times 2 = 0.60$

$0.60 \times 2 = 1.20$

$0.20 \times 2 = 0.40$

$0.40 \times 2 = 0.80$

10100

(ii) $(3FD)_{16} = (?)_{10}$

$00111111101_{(2)}$

(iii) $(A69.8)_{16} = (?)_{10}$

$16^2 \ 16^1 \ 16^0$

$A \ 6 \ 9 \ . \ 8$

$(256 \times 10) + (16 \times 6) + (1 \times 9) + 8 \times 16^{-1}$

$2560 + 96 + 9 + 0.5$

$2665.5_{(10)}$

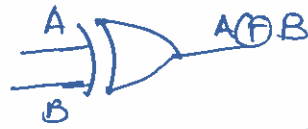
⑦ Show that the dual of Exclusive OR is also its complement.

Dual :- Rules :- ① Exchange '1's & '0's
② Exchange '.' & '+' operators

complement :- Rules :- In addition to the above two literals also changed

(Ex:- $A \rightarrow \bar{A}$, $B \rightarrow \bar{B}$)

Ex-OR :-



$$A \oplus B = \bar{A}B + A\bar{B}$$

Dual

$$\begin{aligned} (i) & (\bar{A} + B) \cdot (A + \bar{B}) \\ &= A\bar{A} + \bar{A}\bar{B} + AB + B\bar{B} \\ &= \bar{A}\bar{B} + AB \end{aligned}$$

Complement

$$\begin{aligned} (i) & \bar{A}B + A\bar{B} \\ & (A + \bar{B}) \cdot (\bar{A} + B) \\ &= A\bar{A} + \bar{A}\bar{B} + AB + B\bar{B} \\ &= \bar{A}\bar{B} + AB \end{aligned}$$

Both complement and dual of Exclusive OR gate are same.

⑦

Q. Convert the following binary 1010011 into gray code.

MSB 1010011 LSB

Rules:-

- (i) Keep MSB as it is
- (ii) Perform Exclusive OR operation with adjacent bits

$$\begin{array}{ccccccc} \oplus & \oplus & \oplus & \oplus & \oplus & \oplus & \\ 1 & \rightarrow 0 & \rightarrow 1 & \rightarrow 0 & \rightarrow 0 & \rightarrow 1 & \rightarrow 1 \end{array} \quad (B)$$



$$\begin{array}{ccccccc} & 1 & 1 & 1 & 1 & 0 & 1 & 0 & (G) \\ \hline 1 & 0 & 1 & 0 & 0 & 1 & 1 & (B) & \Rightarrow & 1 & 1 & 1 & 1 & 0 & 1 & 0 & (G) \end{array}$$

(ii) Convert the following Gray code into its binary

$$101011(G) = ()_2$$

C

$$\begin{array}{cccccc} 1 & 0 & 1 & 0 & 1 & 1 \\ \downarrow & \oplus & \oplus & \oplus & \oplus & \oplus \\ 1 & 1 & 0 & 0 & 1 & 0 \end{array}$$

$$101011(G) = 110010(B)$$

7. (i) what are 2's complement & 9's complement of a given numbers give examples

x = radix

N = Number

n = no. of digit in the number.

2's complement

$$(2^n - N)$$

2's complement of 1001

$$\begin{array}{r} 0110 \\ + 1 \\ \hline 0111 \end{array}$$

(2-1)'s complement

$$(2^n - 1) - N$$

Find 9's complement of 43
~~10~~ $2=10$ $N=43$
 $n=2$

$$\begin{array}{r} (10^n - 1) - 43 \\ (100 - 1) - 43 \\ 99 - 43 \\ = 56 \end{array}$$

(ii) Subtract the following using 1's & 2's complement

Method.

Subtraction $(101)_2 - (10110)_2$
 using 1's complement

00101 - Minuend
 10110 - Subtrahend

Perform 1's complement on subtrahend

Add it to Minuend

carry not generated

so consider result is -ve

$$01001$$

$$\begin{array}{r} 00101 \\ + 01001 \\ \hline 01110 \end{array}$$

perform 1's comp
 10001(2)

Using 2's comp

Minuend - 00101

Subtrahend - 10110

perform 2's comp on subtrahend $\rightarrow$

Add it to Minuend

Carry not generated
So perform 2's comp on
Result & treat the
result as -ve

$$\begin{array}{r} 01001 \\ + 1 \\ \hline 01010 \\ + 00101 \\ + 01010 \\ \hline 01111 \end{array}$$

$$\begin{array}{r} 10000 \\ + 1 \\ \hline 10001 \end{array} \text{ (Ans)}$$

Q2 (i) Simplify Boolean functions using a four variable
k-map method & implement the simplified
function using NAND gates

$F(A, B, C, D) = \sum m(0, 2, 4, 5, 6, 7, 8, 10, 13, 15)$

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1			1
$\bar{A}B$	1	1	1	1
AB		1	1	
$A\bar{B}$	1			1

$$= BD + \bar{A}B + \bar{B}\bar{D}$$

9

(a) Obtain the simplified expression in pos of $F(A, B, C, D) = \sum (1, 2, 4, 7, 12, 14, 15)$

	$(C+D)$	$(C+\bar{D})$	$(C\bar{C}+D)$	$(\bar{C}+D)$
$(A+B)$	0	1	3	2
$(A+\bar{B})$	4	5	6	7
$(\bar{A}+B)$	12	13	14	15
$(\bar{A}+\bar{B})$	8	9	11	10

$$F = [B+C+D][B+\bar{C}+\bar{D}][A+B+C][A+B+C+D][A+B+C+D]$$

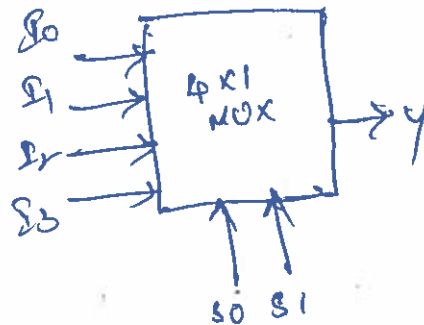
(b) Obtain the simplified expressions in product of sum $F(A, B, C, D) = \prod (0, 1, 2, 3, 4, 10, 11)$

	$(C+D)$	$(C+\bar{D})$	$(C\bar{C}+D)$	$(\bar{C}+D)$
$(A+B)$	0	1	3	2
$(A+\bar{B})$	4	5	6	7
$(\bar{A}+B)$	12	13	14	15
$(\bar{A}+\bar{B})$	8	9	11	10

$$(A+B)(A+C+D)(B+\bar{C})$$

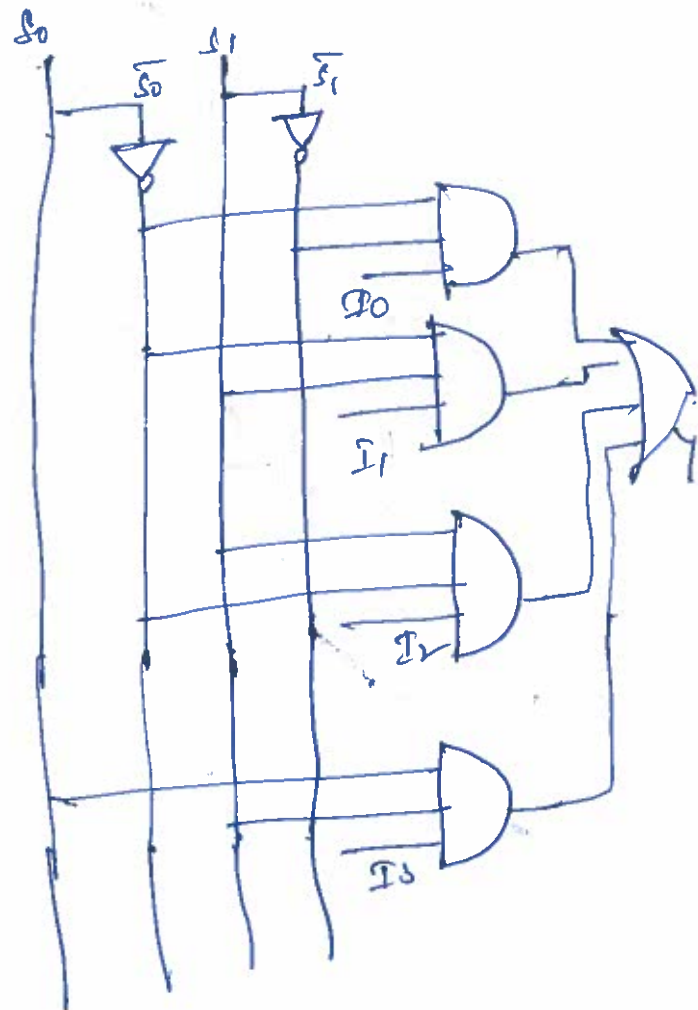
10 (a) organize the procedures for Multiplexers and de Multiplexers and draw the logic diagram of a 4 to 1 line Multiplexer with logic gates.

Multiplexer:- Multiplexer is a combinational logic circuit which send to multiplex the data of 2^n i/p lines on to a single o/p line depends on the 'n' selection lines.



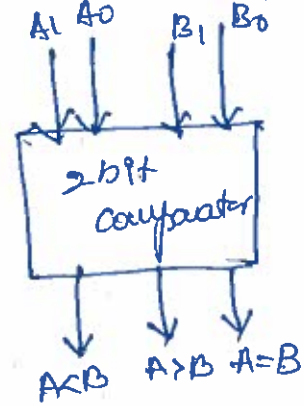
$$Y = \bar{S}_0 \bar{S}_1 I_0 + \bar{S}_0 S_1 I_1 + S_0 \bar{S}_1 I_2 + S_0 S_1 I_3$$

S_1	S_0	Y
0	0	I_0
0	1	I_1
1	0	I_2
1	1	I_3



⑤ Model a circuit for 2 bit - Magnitude Comparator using gates.

Comparator is a combinational logic ckt which is used to compare the magnitude of 2 no's & produces 3 o/p's $A > B$, $A < B$, $A = B$



A ₁	A ₀	B ₁	B ₀	A < B	A = B	A > B
0	0	0	0	0	1	0
0	0	0	1	1	0	0
0	0	1	0	1	0	0
0	0	1	1	1	0	0
0	1	0	0	0	0	1
0	1	0	1	0	1	0
0	1	1	0	1	0	0
0	1	1	1	1	0	0
1	0	0	0	0	0	1
1	0	0	1	0	0	1
1	0	1	0	0	1	0
1	0	1	1	1	0	0
1	1	0	0	0	0	1
1	1	0	1	0	0	1
1	1	1	0	0	0	1
1	1	1	1	0	1	0

	$\bar{B}_1 \bar{B}_0$	$\bar{B}_1 B_0$	$B_1 \bar{B}_0$	$B_1 B_0$
$\bar{A}_1 \bar{A}_0$	0	1	1	1
$\bar{A}_1 A_0$	0	0	1	1
$A_1 \bar{A}_0$	0	0	0	0
$A_1 A_0$	0	0	1	0

$$\bar{A}_1 B_1 + \bar{A}_1 \bar{A}_0 B_0 + \bar{A}_0 B_0 B_1$$

1			
	1		
		1	
			1

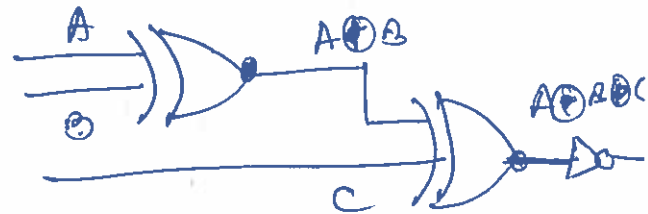
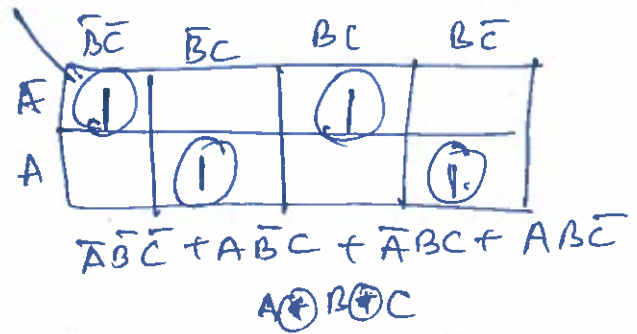
$$\bar{A}_1 \bar{B}_0 \bar{B}_1 \bar{B}_0 + \bar{A}_1 \bar{A}_0 \bar{B}_1 \bar{B}_0 + A_1 \bar{A}_0 B_1 \bar{B}_1 + A_1 \bar{A}_0 B_1 B_0$$

1			
1	1		1
1	1		

$$A_1 \bar{B}_1 + A_0 \bar{B}_1 \bar{B}_0 + A_1 \bar{A}_0 \bar{B}_0$$

① a) Design odd parity generator for 3 bit using decoder.

A	B	C	
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0



~~$\overline{C}(\overline{A}\overline{B})$~~

$$\overline{C}(\overline{A}\overline{B} + AB) + C(\overline{A}B + A\overline{B})$$

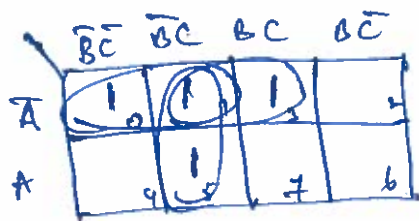
$$\overline{C}\overline{x} + Cx$$

$$C \oplus \overline{x}$$

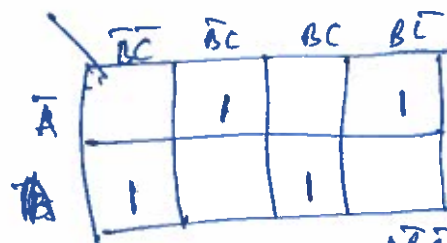
② Construct the functions and Applications of PLA's in memory addressing & implement the following two Boolean functions with PLA

$F_1(A, B, C) = \sum(0, 1, 3, 5)$ and $F_2(A, B, C) = \sum(1, 2, 4, 7)$

Programmable logical Array in which both AND gates & OR gates are programmable.
 PLA's are used to construct PLD's.
 In PLA both OR gates & AND gates have fuses & at the i/p



$$F_1 = \bar{A}\bar{B} + \bar{A}\bar{C} + \bar{B}C$$



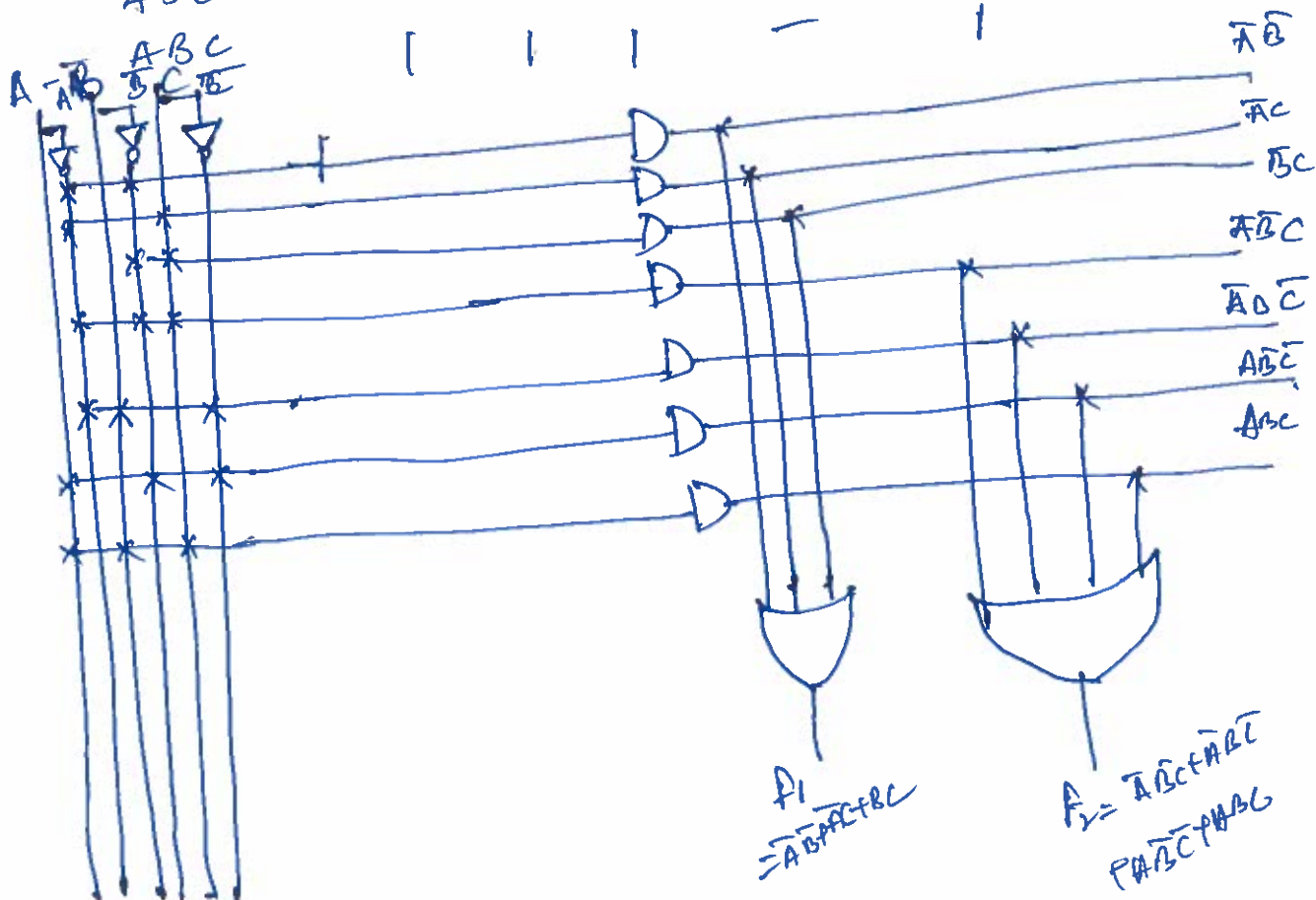
$$\bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C} + ABC$$

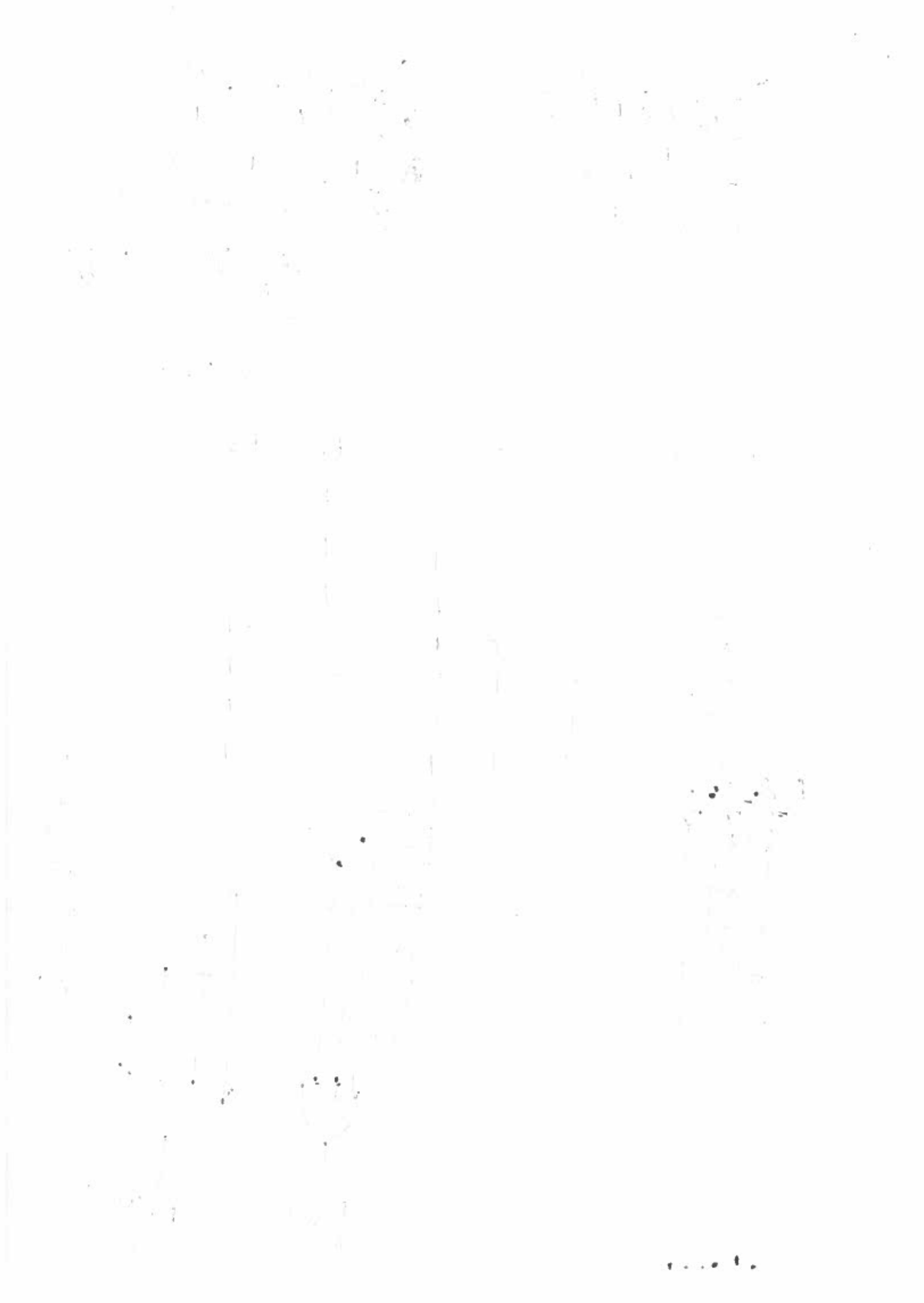
$$C(\bar{A}\bar{B} + A\bar{B}) + \bar{C}(\bar{A}B + AB)$$

$$C\bar{B} + \bar{C}B$$

$$C \oplus B \oplus A$$

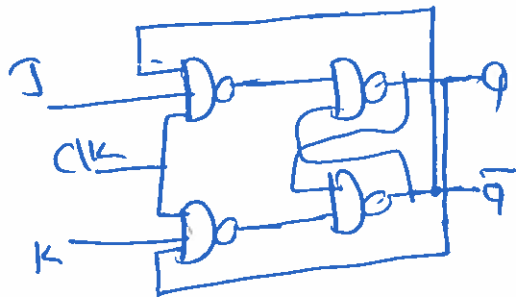
minterms	A	B	C	F_1	F_2
$\bar{A}\bar{B}$	0	0	-	1	-
$\bar{A}C$	0	-	1	1	-
$\bar{B}C$	-	0	1	1	-
$\bar{A}\bar{B}C$	0	0	1	-	1
$\bar{A}B\bar{C}$	0	1	0	-	1
$A\bar{B}\bar{C}$	1	0	0	-	1
ABC	1	1	1	-	1



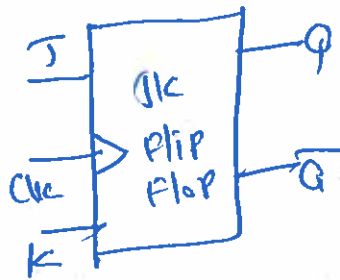


12. (a)

JK-Flip-flop



=



CLK	J	K	Q _n	Q _{n+1}
↑	0	0	0	0
↑	0	0	1	1
↑	0	1	0	0
↑	0	1	1	0
↑	1	0	0	1
↑	1	0	1	1
↑	1	1	0	1
↑	1	1	1	0

CLK	J	K	Q _{n+1}
↑	0	0	Q _n
↑	0	1	0
↑	1	0	1
↑	1	1	$\overline{Q_n}$

(b)

SR Flip-flop

CLK	S	R	Q _{n+1}
↑	0	0	Q _n
↑	0	1	0
↑	1	0	1
↑	1	1	X

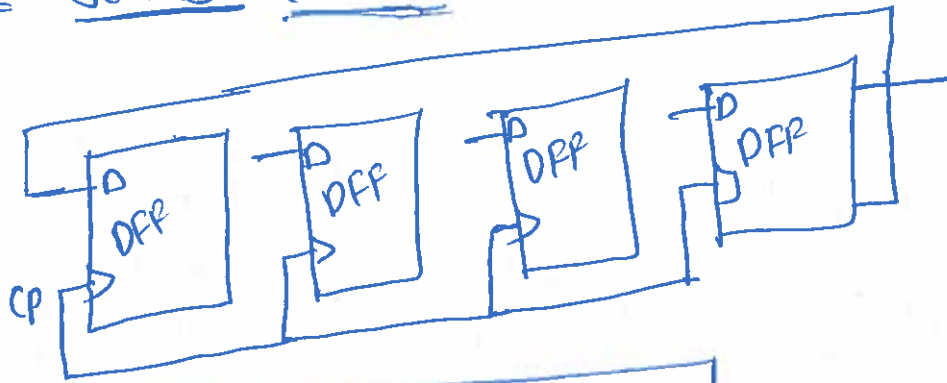
D Flip-flop

CLK	D	Q _{n+1}
⊗ ↑	0	0
↑	1	1

Conversion table

D	Q _n	Q _{n+1}	S	R
0	0	0	0	X
0	1	0	X	1
1	0	1	1	X
1	1	1	X	0

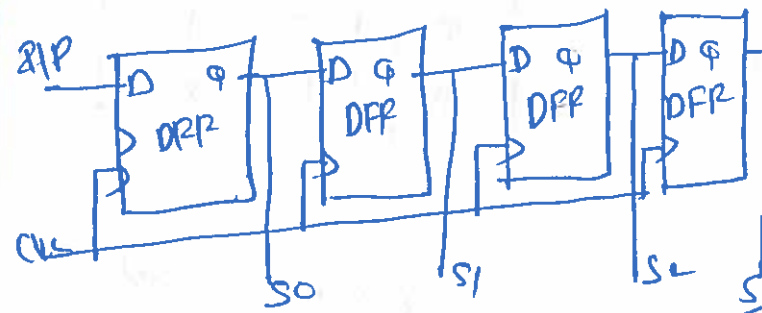
(13) (a) Johnson Counter :-



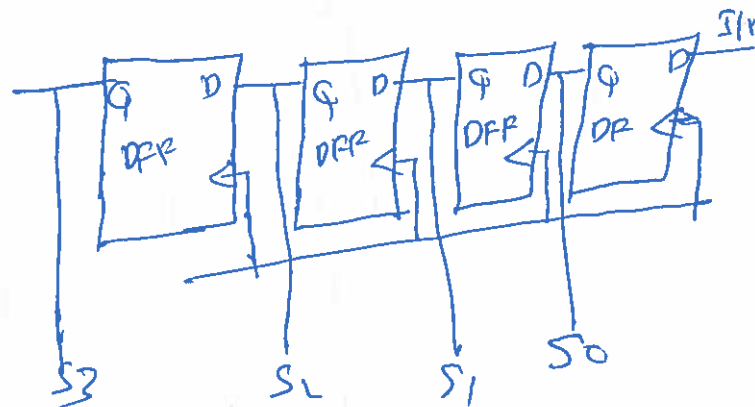
CP	Q1	Q2	Q3	Q4
0	0	0	0	0
1	-	0	0	0
2	-	-	0	0
3	-	-	-	0
4	0	-	-	-
5	0	0	-	-
6	0	0	0	-
7	0	0	0	0
8	0	0	0	0

(13).b

CP	D	S0	S1	S2	S3
↑	0	0	0	0	0
↑	-	-	0	0	0
↑	0	0	-	0	0
↑	-	0	0	-	0



CP	D	S3	S2	S1	S0
↑	0	0	0	0	0
↑	-	-	0	0	0
↑	0	0	-	0	0
↑	-	0	0	-	0



14. (a)

(i) Library IEEE;

USE IEEE.Std_logic_1164.all;

Entity UP_Counter IS

Generic (bits : 3);

Port (a2 IN BIT_VECTOR (bits downto 0));

CLK : IN BIT;

Count : OUT BIT_VECTOR (bits downto 0);

End UP_Counter

Architecture behaviour of UP_Counter IS

begin

Count <= 3'b000;

a <= 3'b001;

Process (CLK)

begin

Count <= Count + a;

End Process;

End Behaviour

(ii)

Architecture Architecture name of Entity - NAME IS

Constant sizeWidth : Positive := 16;

Signals S1, S2, S3 : bit_vector (0 to N-1)

Signal sel : Bit;

begin

Combinational Statement;

Sequential Statement

End Architecture - NAME;

14.(b) Overloading:

The operator is called overloaded if there is more than one function. If there is more than one function specifying it for different data and result types. VHDL allows defining operators of the same names as predefined operators.

Ex:-

type newtype is ('0', '1', 'z', 'x');

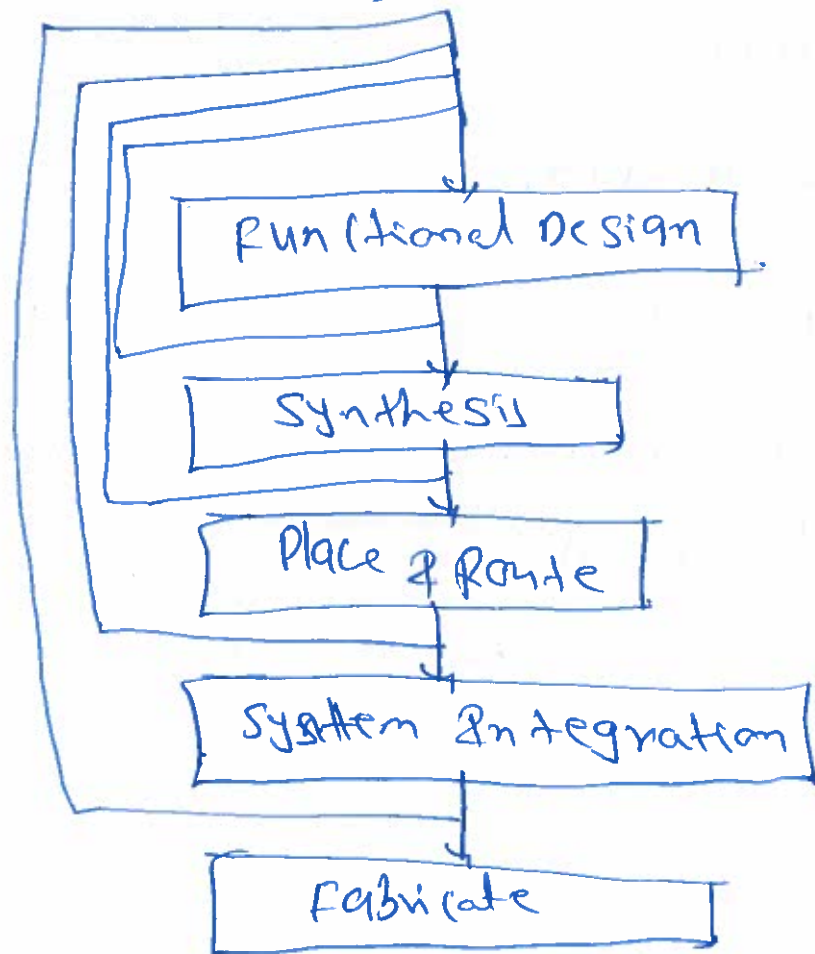
function "and" (left, right: newtype) return newtype;

function "or" (left, right: newtype) return newtype;

signal A, B, C: newtype;

C <= (A and B) or X;

15.(a)



* Example

```
Library IEEE;  
use IEEE - Std logic - 1164.all;  
Entity full_Adder is  
    Port (A, B, Cin: In std logic;  
          Carry, Sum: out std logic);  
End full_Adder
```

Architecture ~~dataflow~~ dataflow of full_Adder is

```
begin  
    Sum <= A xor B xor Cin  
    Carry <= (A AND B) OR (B AND Cin) OR (Cin AND A);  
End dataflow;
```

15. (b)

VHDL Data types

VHDL has a set of standard data types.
It is also possible to have user defined data types and subtypes.

Some of predefined data types in VHDL are

BIT
BOOLEAN
INTEGER

The STD-LOGIC and STD-LOGIC-VECTOR datatypes are not built-in VHDL datatypes, but are defined in the standard logic-1164 package.

BIT

BIT-VECTOR

STD-LOGIC

The Data values are

X - unknown

0 - logic-0

1 - logic-1

Z - high impedance / tri-state.

Semester End Examination, January/February, 2022

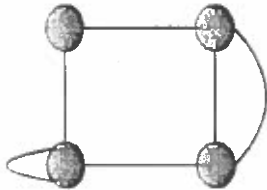
Degree	B. Tech. (U. G.)	Program	CSE, CSE (AI & ML) & CSE (DS)	Academic Year	2021 - 2022
Course Code	20BSX16	Test Duration	3 Hrs.	Max. Marks	70
Course	Mathematical Foundations for Computer Science	Semester	III		

Part A (Short Answer Questions 5 x 2 = 10 Marks)

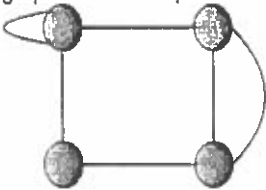
No.	Questions (1 through 5)	Learning Outcome (s)	DoK
1	Define Tautology and contradiction?	20BSX16.1	L1
2	Define subgroup	20BSX16.2	L2
3	Find the gcd of 615 and 1080, and find the integers x and y such that $\gcd(615, 1080) = 615x + 1080y$.	20BSX16.3	L1
4	Find the first 5 terms of the recurrence relation $a_n = a_{n-1} + a_{n-3}$, $a_0 = 1, a_1 = 2, a_2 = 0$	20BSX16.4	L2
5	Define complete Graph (Kn) and how many edges are in K6.	20BSX16.5	L1

Part B (Long Answer Questions 5 x 12 = 60 Marks)

No.	Questions (6 through 15)	Marks	Learning Outcome (s)	DoK
6 (a)	Define contingency. Prove that $(\neg p \rightarrow \neg q) \Leftrightarrow (q \Leftrightarrow r)$ is contingency.	6M	20BSX16.1	L2
6 (b)	Show that $(p \rightarrow q) \rightarrow q \Leftrightarrow p \vee q$	6M	20BSX16.1	L3
OR				
7 (a)	Without constructing the truth table (i) Obtain the principal conjunctive normal form of $(\neg p \rightarrow r) \wedge (q \leftrightarrow p)$. (ii) Obtain the PDNF for $(P \wedge Q) \vee (\neg P \wedge R) \vee (Q \wedge R)$	6M	20BSX16.1	L2
7 (b)	For each of the sets of the premises, what relevant conclusion or conclusions can be drawn? Explain the rules of Inference used to obtain each conclusion from the premises. "Every student has an Internet account". "Homer does not have an Internet account". "Maggie has an Internet account."	6M	20BSX16.1	L3
8 (a)	Define Equivalence relation. Let $X = \{1, 2, 3, \dots, 7\}$ and $R = \{ \langle x, y \rangle / x - y \text{ is divisible by } 3 \}$. Show that R is an equivalence relation.	6M	20BSX16.2	L3
8 (b)	Explain Hasse Diagram. Draw the Hasse Diagram of D_{24}	6M	20BSX16.2	L2
OR				
9 (a)	Define inverse of a function and find the inverse of $f(x) = \frac{2x+3}{5}$, $f(x) = \frac{3x+4}{5}$ for all x in R	6M	20BSX16.2	L3
9 (b)	Show that the set $\{1, 2, 3, 4, 5\}$ is not a group under addition and multiplication modulo 6.	6M	20BSX16.2	L2
10 (a)	State and prove Euler's theorem. Give an example	6M	20BSX16.3	L2
10 (b)	Prove that for all integers a, b, c, (i) if $a b$, then $a bc$ (ii) if $a b$ and $b c$ then $a c$ for all a, b, c integers	6M	20BSX16.3	L3
OR				
11 (a)	State and prove Fermat's theorem	6M	20BSX16.3	L2
11 (b)	Define G CD, LCM and co primes. Find the prime factorization of 243, 125.	6M	20BSX16.3	L3
12 (a)	Find the explicit solution of the Fibonacci recurrence relation with	6M	20BSX16.4	L3

- $F_0 = 0, F_1 = 1$.
- 12 (b) Solve the recurrence relation $a_n = 4a_{n-1} - 4a_{n-2}$, for $n \geq 2$ with initial conditions $a_0 = 6$, and $a_1 = 8$. 6M 20BSX16.4 L2
- OR
- 13 (a) Find the solution to the recurrence relation $a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$, with the initial conditions $a_0 = 2, a_1 = 5, a_2 = 15$ 6M 20BSX16.4 L2
- 13 (b) Find all the solutions of the recurrence relation $a_n = 2a_{n-1} + 2n^2$. 6M 20BSX16.4 L3
- 14 (a) Explain about the bipartite and complete bipartite Graphs with diagrams 6M 20BSX16.5 L3
- 14 (b) Determine whether the following graphs are isomorphic or not.
- 

G1



G2
- OR
- 15 (a) Define Hamiltonian graph and draw a graph with six vertices which is Hamiltonian but not Eulerian? 6M 20BSX16.5 L2
- 15 (b) Find minimal spanning tree using Kruskal's algorithm. Use a graph of your choice. 6M 20BSX16.5 L3

Scheme of Evaluation

NSRIT

Semester End Examination, Feb. 2022

Page no: 1

B.Tech, III Semester

Date of Exam: 14.02.2022

Course Code: 20BSX16,
Course: Mathematical Foundations for Computer Science

PART: A

1. Tautology: A compound proposition that is always true for all possible truth values of its variables or it contains only T in the last column of its truth table is called a Tautology.

$$p \vee \neg p \equiv T \quad \begin{array}{c|c} p & \neg p \\ \hline T & F \\ F & T \end{array} \quad \text{--- 1M}$$

Contradiction: A compound proposition that is always false for all possible truth values of its variables or it contains only F in the last column of its truth table is called a Contradiction.

$$p \wedge \neg p \equiv F \quad \begin{array}{c|c} p & \neg p \\ \hline T & F \\ F & T \end{array} \quad \text{--- 1M}$$

2. Let $(G, *)$ be a group and let H is a subset of G .
If $(H, *)$ is a group then then $(H, *)$ is said to be a subgroup — 2M

3. GCD of 615, 1080

$$\begin{aligned} 1080 &= 1 \cdot 615 + 465 \\ 615 &= 1 \cdot 465 + 150 \\ 465 &= 3 \cdot 150 + 15 \\ 150 &= 10 \cdot 15 + 0 \end{aligned}$$

$\therefore$ GCD of 1080 and 615 is 15.

Now write $15 = 465 - 3 \cdot 150$

$$= 465 - 3[615 - 1 \cdot 465]$$

$$= -3 \cdot 615 + 4 \cdot 465$$

$$= -3 \cdot 615 + 4[1080 - 615]$$

$$= 4 \cdot 1080 - 7 \cdot 615$$

$\therefore$ GCD of 1080, 615 is $15 = 615(-7) + 4(1080)$

$$= 615x + 1080y$$

$$\therefore x = -7, y = 4$$

$$\begin{array}{r} 615 \overline{) 1080} 1 \\ \underline{615} \\ 465 \overline{) 615} 1 \\ \underline{465} \\ 150 \overline{) 465} 3 \\ \underline{450} \\ 15 \overline{) 150} 10 \\ \underline{150} \\ 0 \end{array}$$

— 1M

$$\begin{array}{r} 1M \\ \hline 2M \end{array}$$

4. $a_n = a_{n-1} + a_{n-3}$, $a_0 = 1, a_1 = 2, a_2 = 0$ — (1M)

$$a_3 = a_2 + a_0 = 0 + 1 = 1$$

$$a_4 = a_3 + a_1 = 1 + 2 = 3$$

$$a_5 = a_4 + a_2 = 3 + 0 = 3$$

First five terms are

$$1, 2, 0, 1, 3 \quad \text{--- (2M)}$$

5. A Simple graph G is said to be Complete if every vertex in G is ^{Page No. 41} Connected with every other vertex and it is denoted by K_n . — 1M.

The number of edges in Complete graph K_n is $\frac{n(n-1)}{2}$

$\therefore$ The number of edges in K_6 , $\frac{6(6-1)}{2} = 15$. — $\frac{1M}{2M}$

PART: B

6. a. Contingency: A Compound proposition that is neither a tautology nor a Contradiction is called a Contingency. — 1M.

P	Q	R	$\sim P$	$\sim Q$	$\sim P \rightarrow \sim Q$	$Q \Rightarrow R$	$(\sim P \rightarrow \sim Q) \Leftrightarrow (Q \Rightarrow R)$
T	T	T	F	F	T	T	T
T	T	F	F	F	T	F	F
T	F	T	F	T	T	F	F
T	F	F	F	T	T	T	T
F	T	T	T	F	F	T	F
F	T	F	T	F	F	F	T
F	F	T	T	T	T	F	F
F	F	F	T	T	T	T	T

— 4M

The last column is neither Tautology nor Contradiction and hence

$(\sim P \rightarrow \sim Q) \Leftrightarrow (Q \Rightarrow R)$ is Contingency. — 1M
6M

6(b). Show that $(P \rightarrow Q) \rightarrow Q \Leftrightarrow P \vee Q$

P	Q	$(P \rightarrow Q)$	$(P \rightarrow Q) \rightarrow Q$	$P \vee Q$	$(P \rightarrow Q) \rightarrow Q \Leftrightarrow P \vee Q$
T	T	T	T	T	T
T	F	F	T	T	T
F	T	T	T	T	T
F	F	T	F	F	T

$\therefore (P \rightarrow Q) \rightarrow Q \Leftrightarrow (P \vee Q)$

(or)

Page no: 3.

$$\begin{aligned}
 (p \rightarrow q) \rightarrow q &\equiv (\neg p \vee q) \rightarrow q \\
 &\equiv \neg(\neg p \vee q) \vee q \\
 &\equiv \neg(\neg p) \wedge \neg q \vee q \\
 &\equiv p \wedge (\neg q \vee q)
 \end{aligned}$$

$$A \rightarrow B \equiv \neg A \vee B$$

$$q \vee \neg q$$

q	$\neg q$	$q \vee \neg q$
T	F	T
F	T	T

7 (a). PCNF of $(\neg p \rightarrow r) \wedge (q \leftrightarrow p)$.

(i)

$$(\neg p \rightarrow r) \wedge (q \leftrightarrow p) \equiv (\neg(\neg p) \vee r) \wedge ((q \rightarrow p) \wedge (p \rightarrow q))$$

$$(\because p \rightarrow q \equiv \neg p \vee q)$$

$$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$$

$$\equiv (p \vee r) \wedge [(q \rightarrow p) \wedge (p \rightarrow q)]$$

$$\equiv (p \vee r) \wedge [(\neg q \vee p) \wedge (\neg p \vee q)]$$

— 1M

$$\equiv (p \vee r) \vee (\neg q \wedge q) \wedge (\neg q \vee p) \wedge (\neg p \vee q) \vee (r \wedge \neg r) \wedge (\neg p \vee q) \vee (r \wedge \neg r)$$

using missing terms

$$\equiv (p \vee r \vee \neg q) \wedge (p \vee r \vee q) \wedge (\neg q \vee p \vee r) \wedge (\neg q \vee p \vee \neg p)$$

$$\wedge (\neg p \vee q \vee r) \wedge (\neg p \vee q \vee \neg r)$$

$$\equiv (p \vee r \vee q) \wedge (p \vee r \vee \neg q) \wedge (\neg q \vee p \vee r) \wedge (\neg p \vee q \vee r)$$

$$\wedge (\neg p \vee q \vee \neg r)$$

is the Conjunctive normal terms. which is the required PCNF — 3M

$$(ii) (p \wedge q) \vee (\neg p \wedge r) \vee (q \wedge r)$$

$$\equiv (p \wedge q) \wedge (r \vee \neg r) \vee (\neg p \wedge r) \wedge (q \vee \neg q) \vee (q \wedge r) \wedge (p \vee \neg p)$$

$$p \vee \neg p \equiv T$$

$$p \wedge T \equiv p$$

$$\equiv (p \wedge q \wedge r) \vee (p \wedge q \wedge \neg r) \vee (\neg p \wedge r \wedge q) \vee (\neg p \wedge r \wedge \neg q)$$

$$\vee (q \wedge r \wedge p) \vee (q \wedge r \wedge \neg p)$$

(By distributive law)

$$\equiv (p \wedge q \wedge r) \vee (p \wedge q \wedge \neg r) \vee (\neg p \wedge r \wedge q) \vee (\neg p \wedge r \wedge \neg q)$$

is the disjunction of minterms. which is the required PDNF — 1M

3M

7(b).

The given premises are "Every student has an internet account"

"Homer does not have an internet account"

"Maggie has an internet account"

Let x be student. $S(x)$: x is a student. $I(x)$: x has an internet account. $H(x)$: Homer is a student. $M(x)$: Maggie is a student.

Every student has an internet account.

 $\forall x, S(x) \rightarrow I(x)$

Homer does not have an internet account.

 $H(x) \wedge \neg I(x)$

Maggie has an internet account

 $M(x) \wedge I(x)$

Premise	Step	Statement formula	Rule	Justification
[1]	(1)	$\forall x, S(x) \rightarrow I(x)$	Rule P	Hypothesis
[1]	(2)	$S(a) \rightarrow I(a)$	Rule $\rightarrow$	Universal specification from step (1)
[1]	(3)	$S(a)$	Rule $\rightarrow$	from step 2
[1]	(4)	$I(a)$	Rule $\rightarrow$	step 1
[2]	(5)	$H(x) \wedge \neg I(x)$	Rule P	Hypothesis
[2]	(6)	$H(a) \wedge \neg I(a)$	Rule $\rightarrow$	G.E.S, Step 5)
[3]	(7)	$M(x) \wedge I(x)$	Rule P	Hyp.
[3]	(8)	$M(a)$	Rule $\rightarrow$	E-S (step 7)

8(a). A relation on a set A is called an equivalence relation if it is reflexive, symmetric and transitive.

i.e., $(a, a) \in R \forall a \in R$

$(a, b) \in R \Rightarrow (b, a) \in R$

If $(a, b) \in R$ and $(b, c) \in R$ then $(a, c) \in R$. — 2M.

Given $X = \{1, 2, 3, 4, 5, 6, 7\}$

Relation $R = \{(x, y) / x - y \text{ is divisible by } 3\}$.

Let $x \in X$, $x - x = 0$ is divisible by 3, $\therefore x R x, \forall x \in X$ — 1M
 $\therefore R$ is Reflexive

Let $x, y \in R$, $\Rightarrow (x - y)$ is divisible by 3 $\Rightarrow -(x - y)$ is divisible by 3
 $\Rightarrow (y - x)$ is divisible by 3

$\therefore y R x$, R is Symmetric — 1M

Let $x, y, z \in R$, Let $x R y$ and $y R z$

Then $x - y$ is divisible by 3, $y - z$ is divisible by 3

$x - z = (x - y) + (y - z)$ is also divisible by 3.

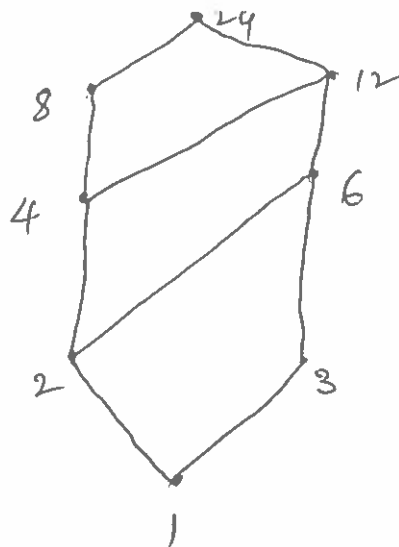
$\therefore x R z$, R is Transitive — 1M

$\therefore R$ is reflexive, symmetric, and Transitive and hence R is an equivalence relation. — 1M
 6 marks

8(b) A partial order $\leq$ on a set X ^{can be} ~~arranged~~ represented by means of a diagram is known as Hasse diagram. We represent each element by dots or small circles and if y is an immediate successor of x , we take y at higher level of x and join a pair y by a straight ~~line~~ line if there is a relation (order). A diagram formed by above is known as Hasse diagram — 2M

Divisors of 24, 1, 2, 3, 4, 6, 8, 12 and 24.

Delete loops and transitive elements. — 1M



Hasse diagram of $(D, 24)$

3M

6M

9.

(a) Define Inverse of a function:

Let $f: A \rightarrow B$, a map $g: B \rightarrow A$ is called the inverse of f if $g \circ f = I_A$ and $f \circ g = I_B$.

i.e., $g[f(x)] = x \forall x \in A$, $f[g(y)] = y \forall y \in B$.

The inverse of f is g is denoted by f^{-1} . 2M

Given $f(x) = \frac{2x+3}{5}$

$$y = \frac{2x+3}{5}$$

$$5y = 2x+3$$

$$5y-3 = 2x$$

$$\frac{5y-3}{2} = x$$

$$\therefore f^{-1}(x) = \frac{5x-3}{2} \quad 2M$$

$$f(x) = \frac{3x+4}{5}$$

$$y = \frac{3x+4}{5}$$

$$5y = 3x+4$$

$$5y-4 = 3x$$

$$\frac{5y-4}{3} = x$$

$$\therefore f^{-1}(x) = \frac{5x-4}{3} \quad 2M$$

6M

9(b)

Given $G = \{1, 2, 3, 4, 5\}$

The operation addition and multiplication modulo 6. 1M

$(G, +_6)$

$+_6$	1	2	3	4	5
1	2	3	4	5	0
2	3	4	5	0	1
3	4	5	0	1	2
4	5	0	1	2	3
5	0	1	2	3	4

Computation table of $(G, +_6)$

$\cdot_6$	1	2	3	4	5
1	1	2	3	4	5
2	2	4	0	2	4
3	3	0	3	0	3
4	4	2	0	4	2
5	5	4	3	2	1

Computation table of $(G, \cdot_6)$

From the Composition table, it is clear that all the entries in the Composition table do not belong to G , in particular $0 \notin G$. — 1M

Hence G is closed under $+$, $\cdot$. and hence $(G, +)$, $(G, \cdot)$ is not a group. 6M

10(a) Statement Euler's Theorem: If a and n are integers such that a is a prime number and $(a, n) = 1$ then $a^{\phi(n)} \equiv 1 \pmod{n}$.
 (a, n) = 1 Then $a^{\phi(n)} \equiv 1 \pmod{n}$ — 2M

Proof:

Proof. Consider $r_1, r_2, r_3, \dots, r_{\phi(n)}$ be a reduced residue system modulo n .
 Let $Z(n)$ be the Congruence class of integers are relatively prime to n .

We claim that multiplication by a is a permutation of the set $Z(n)$.
 i.e., $\{ar_1, ar_2, \dots, ar_{\phi(n)}\}$ are the ^{Just} residues modulo n of $r_1, r_2, \dots, r_{\phi(n)}$.
 Claim is true, since multiplication by a is a function of the finite set so it itself has the inverses namely $\frac{1}{a} \pmod{n}$.

Now,

$$r_1, r_2, r_3, \dots, r_{\phi(n)} \quad / \quad ar_1, ar_2, \dots, ar_{\phi(n)}$$

so the products are congruences.

$$r_1 \cdot r_2 \cdot \dots \cdot r_{\phi(n)} = ar_1 \cdot ar_2 \cdot ar_3 \cdot \dots \cdot ar_{\phi(n)}$$

$$1 \equiv a^{\phi(n)}$$

$$a^{\phi(n)} \equiv 1 \pmod{n} \quad \leftarrow \frac{4M}{6M}$$

10(b) Let $a, b, c \in \mathbb{Z}$

(i) $a|b \Rightarrow b = ka$, where $k \in \mathbb{Z}$
 $bc = c \cdot ka$

Let $bcd \Rightarrow d = cka$, $ck = m$

$d = ma \Rightarrow a|d \Rightarrow a|bc$. — 3M

(ii) $a|b \Rightarrow b = ka$ where $k \in \mathbb{Z}$

$b|c \Rightarrow c = lb$, where $l \in \mathbb{Z}$

Consider $c = lb$
 $l(ka)$

$c = lka$

$\Rightarrow a|c$

— 3M
6M

Page no: 8:

1(a) If p is a prime number, a is an integer $(a, p) > 1$ then
 $a^{p-1} \equiv 1 \pmod{p}$ — 2M

Proof We have $\gcd(a, p) = 1$.

When the numbers $a, 2a, 3a, \dots, (p-1)a$ are divided by p , the remainders are $1, 2, 3, \dots, p-1$, (not necessarily in same order)

$$\text{Let } a \equiv r_1 \pmod{p}$$

$$2a \equiv r_2 \pmod{p}$$

$$3a \equiv r_3 \pmod{p}$$

$\vdots$

$$(p-1)a \equiv r_{p-1} \pmod{p} \quad \text{--- (1)}$$

Since $r_1, r_2, \dots, r_{p-1}$ are the remainders when $a, 2a, 3a, \dots, (p-1)a$ are divided by p .

$$\therefore r_1 r_2 r_3 \dots r_{p-1} \equiv 1 \cdot 2 \cdot 3 \dots (p-1) \quad \text{--- (2)}$$

multiplying above Congruence relation

$$\cancel{a} \cdot \cancel{2a} \cdot \cancel{3a} \dots \cancel{(p-1)a} \equiv 1 \cdot 2 \cdot 3 \dots (p-1) \pmod{p}$$

$$\text{from (2) (1)} \quad a \cdot 2a \cdot 3a \dots (p-1)a \equiv r_1 r_2 r_3 \dots r_{p-1} \pmod{p} \quad \text{--- (3)}$$

$$(1 \cdot 2 \cdot 3 \dots (p-1)) a^p \equiv 1 \cdot 2 \cdot 3 \dots (p-1) \pmod{p}$$

$$(p-1)! a^p \equiv (p-1)! \pmod{p}$$

$$a^{p-1} \equiv 1 \pmod{p}$$

(or)

Proof Using mathematical induction.

$$a^p \equiv a \pmod{p}$$

Case (i), Let $a=1$, $1^p \equiv 1 \pmod{p}$, Theorem is proved for $n=1$

Case (ii) Assume $a^p \equiv a \pmod{p}$ (or) $a^{p-1} \equiv 1 \pmod{p}$

$$(a+1)^p \equiv (a+1) \pmod{p}$$

$$(a+1)^p = a^p + \binom{p}{1} a^{p-1} + \binom{p}{2} a^{p-2} + \dots + pa + 1 \equiv (a+1) \pmod{p}$$

$\vdots$

$\vdots$

$$a^p \equiv a \pmod{p} \Rightarrow a^{p-1} \equiv 1 \pmod{p}$$

--- 4M

11(b) GCD: A non-zero integer d is said to be a GCD, of a and b Page no: 9

(i) d is a Common divisor of a and b

(ii) If c is a Common divisor of a and b , then c divides d .

i.e., d is the largest of all Common divisors. — 1M

LCM: Let a and b be two non-zero integers, A non-zero integer m is said to be LCM of a and b

(i) If m is a Common multiple of a and b ($a|m$ and $b|m$)

(ii) c is a Common multiple of a and b , then c is a multiple of m
(If $a|c$ and $b|c$ then $m|c$) — 1M

(iii) If a and b are positive integers, then the smallest positive integer that is divisible by both a and b — 1M

Coprime: Coprime numbers are pairs of numbers that do not have any Common factor other than 1. Ex (4, 7), (5, 7), 18

Prime factors of 243

$$243 = 3 \times 3 \times 3 \times 3 \times 3 = 3^5$$

$$\begin{array}{r} 3 \overline{) 243} \\ \underline{3 91} \\ 3 \overline{) 27} \\ \underline{3 9} \\ 3 \end{array}$$

— 2M

$$125 = 5 \times 5 \times 5 = 5^3$$

$$\begin{array}{r} 5 \overline{) 125} \\ \underline{5 25} \\ 5 \end{array}$$

— 1M
— 3M

12. Fibonacci Recurrence Relation is $F_n = F_{n-1} + F_{n-2}$. — 1M

which is homogeneous recurrence relation of degree 2.

its characteristic equation is $x^2 - x - 1 = 0$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2 \cdot 1} = \frac{1 \pm \sqrt{1+4}}{2} = \frac{1 \pm \sqrt{5}}{2}$$

$$x_1 = \frac{1+\sqrt{5}}{2}, x_2 = \frac{1-\sqrt{5}}{2}$$

Complementary Function $F_n^{(H)} = d_1 \left(\frac{1+\sqrt{5}}{2} \right)^n + d_2 \left(\frac{1-\sqrt{5}}{2} \right)^n$ — 2M

Since it is homogeneous, its solution is $F_n = d_1 \left(\frac{1+\sqrt{5}}{2} \right)^n + d_2 \left(\frac{1-\sqrt{5}}{2} \right)^n$ — 1M

The given initial Conditions are $F_0 = 0, F_1 = 1$

$$\text{When } n=0, F_0=0 = d_1 \left(\frac{1+\sqrt{5}}{2} \right)^0 + d_2 \left(\frac{1-\sqrt{5}}{2} \right)^0 \Rightarrow d_1 + d_2 = 0 \Rightarrow d_2 = -d_1 \quad \text{— 1M}$$

$$\text{When } n=1, F_1=1 = d_1 \left(\frac{1+\sqrt{5}}{2} \right)^1 + d_2 \left(\frac{1-\sqrt{5}}{2} \right)^1 \Rightarrow d_1 \left(\frac{1+\sqrt{5}}{2} \right) - d_1 \left(\frac{1-\sqrt{5}}{2} \right) = 1 \quad \text{— 1M}$$

$$\Rightarrow d_1 \cdot \frac{2\sqrt{5}}{2} = 1 \Rightarrow d_1 = \frac{1}{\sqrt{5}} \quad \therefore d_2 = -\frac{1}{\sqrt{5}}$$

Solution of Fibonacci recurrence relation $F_n = \frac{1}{\sqrt{5}} \left(\frac{1+\sqrt{5}}{2} \right)^n - \frac{1}{\sqrt{5}} \left(\frac{1-\sqrt{5}}{2} \right)^n$ — 6M

12(b) The given recurrence relation is $a_n - 4a_{n-1} + 4a_{n-2} = 0, n \geq 2$

which is homogeneous recurrence relation of degree 2. — 1M

Its characteristic equation is $x^2 - 4x + 4 = 0$

$$(x-2)^2 = 0, x = 2, 2 \quad \text{— 1M}$$

Complementary solution is $a_n^{(H)} = (d_1 + n d_2) (2)^n$ — 1M

Since it is homogeneous, soln is $a_n = (d_1 + n d_2) 2^n$.

The given initial conditions are $a_0 = 6$, and $a_1 = 8$.

When $n=0$, $a_0 = (d_1 + 0 \cdot d_2) 2^0 \Rightarrow 6 = d_1$ — 1M

When $n=1$, $a_1 = (d_1 + 1 \cdot d_2) 2^1 \Rightarrow 8 = (d_1 + d_2) \cdot 2 \Rightarrow d_1 + d_2 = 4$

$$6 + d_2 = 4$$

$$d_2 = -2. \quad \text{— 1M}$$

$\therefore$ The solution is $a_n = (6 - 2n)(2)^n$.

1M

6M

13(a). The given recurrence relation is $a_n - 6a_{n-1} + 11a_{n-2} - 6a_{n-3} = 0$.

which is homogeneous recurrence relation of degree 3 — 1M

its characteristic equation is $x^3 - 6x^2 + 11x - 6 = 0$. — 1M

$$\Rightarrow (x-1)(x-2)(x-3) = 0$$

$$\Rightarrow x_1 = 1, x_2 = 2, x_3 = 3$$

Complementary solution is $a_n^{(H)} = d_1(1)^n + d_2(2)^n + d_3(3)^n$ — 1M

Since it is homogeneous $a_n = d_1(1)^n + d_2(2)^n + d_3(3)^n$

The given initial conditions are $a_0 = 2, a_1 = 5, a_2 = 15$.

When $n=0$, $a_0 = d_1 + d_2 + d_3 \Rightarrow d_1 + d_2 + d_3 = 2$ — 1M

When $n=1$, $a_1 = d_1 + 2d_2 + 3d_3 \Rightarrow d_1 + 2d_2 + 3d_3 = 5$

When $n=2$, $a_2 = d_1 + 4d_2 + 9d_3 \Rightarrow d_1 + 4d_2 + 9d_3 = 15$

solving, we get $d_1 = 1, d_2 = -1, d_3 = 2$ — 1M

solution is $a_n = (1)^n - (2)^n + 2(3)^n$ — 1M

6M

13(b) The given recurrence relation is $a_n = 2a_{n-1} + 2b^n$

$$a_n - 2a_{n-1} = 2b^n$$

which is non-homogeneous d.e. of degree 1.

Ch. eqn is $x - 2 = 0 \Rightarrow x = 2$ is a root, C.P. $a_n^{(H)} = d_1(2)^n$.

Assume particular solution is $a_n^{(P)} = A b^n$,

$$a_{n-1}^{(P)} = A(n-1)^2$$

$$a_n - 2a_{n-1} = 2n^2$$

$$An^2 - 2A(n-1)^2 = 2n^2 \Rightarrow An^2 - 2An^2 + 4An - 2A = 2n^2$$

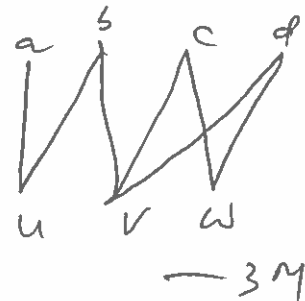
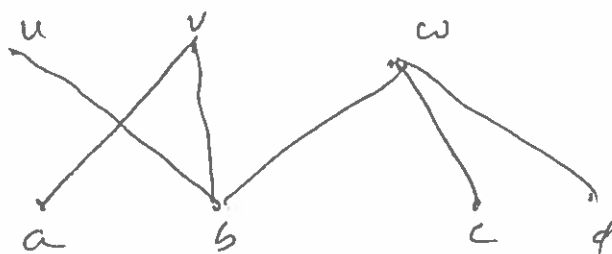
$$-An^2 + 4An - 2A = 2n^2$$

equating corresponding coefficient $A = -2$.

$$\therefore a_n^{(p)} = -2 \cdot \frac{1}{n^2}$$

Complete solution is $a_n = a_n^{(H)} + a_n^{(p)} = d(2)^n - 2n^2$.

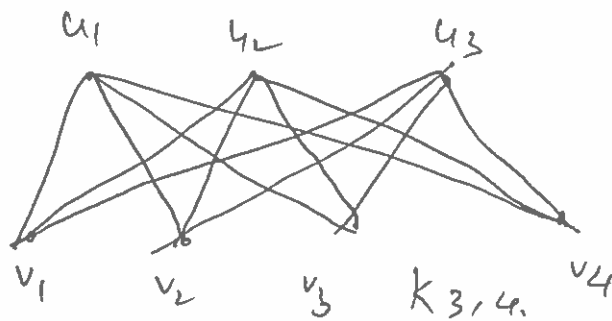
14(a). Bipartite graph: A graph $G = (V, E)$ is said to be bipartite if the vertex set V can be partitioned (divided) into two disjoint subsets V_1 and V_2 such that every edge in E connects a vertex in V_1 and a vertex in V_2 so that no edge in G connects either two vertices in V_1 or two vertices in V_2 . (V_1, V_2) is called a bipartition of G .



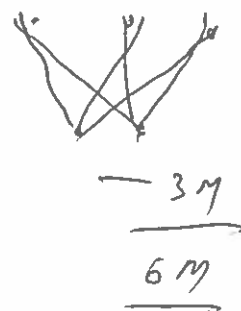
Complete bipartite graph of m and n vertices, is denoted by $K_{m,n}$ is a graph whose vertex set is divided into two disjoint sets V_1 and V_2 with m and n vertices, in which there is an edge between each pair of vertices of V_1 and V_2 .

(or)

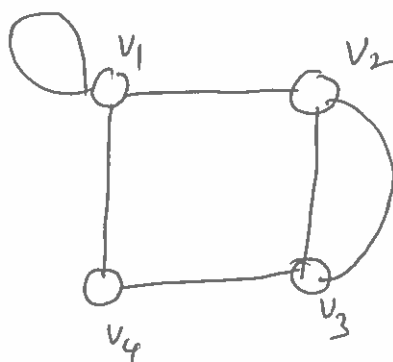
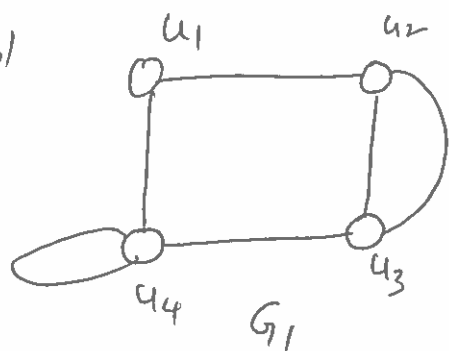
A bipartite graph G is called Complete bipartite graph if there is an edge between every vertex in V_1 and every vertex in V_2 where V_1 and V_2 are partitioned subsets of the vertex set V of G .



$K_{2,3}$



14 (b)



The vertices in $G_1 = \{u_1, u_2, u_3, u_4\}$, $|V_1| = 4$

The edges in $G_1 = \{(u_1, u_2), (u_2, u_3), (u_3, u_2), (u_3, u_4), (u_4, u_1), (u_4, u_3)\}$
 $|E_1| = 6$ loop — 1M

The vertices in $G_2 = \{v_1, v_2, v_3, v_4\}$, $|V_2| = 4$

The edges in $G_2 = \{(v_1, v_1), (v_1, v_2), (v_2, v_3), (v_3, v_2), (v_3, v_4), (v_4, v_1)\}$
 $|E_2| = 6$ loop — 1M

The degree sequence of $G_1 = 2, 3, 3, 4$

$G_2 = 2, 3, 3, 4$ — 1M

Define mapping $f: G_1 \rightarrow G_2$

$$f(u_1) = v_2$$

$$f(u_2) = v_3$$

$$f(u_3) = v_4$$

$$f(u_4) = v_1$$

$$(u_1, u_2) \in E(G_1) \Rightarrow (f(u_1), f(u_2)) = (v_2, v_3) \in E(G_2)$$

$$(u_2, u_3) \in E(G_1) \Rightarrow (f(u_2), f(u_3)) = (v_3, v_4) \in E(G_2)$$

$$(u_3, u_4) \in E(G_1) \Rightarrow (f(u_3), f(u_4)) = (v_4, v_1) \in E(G_2)$$

$$(u_4, u_1) \in E(G_1) \Rightarrow (f(u_4), f(u_1)) = (v_1, v_2) \in E(G_2)$$
 — 1M

Adjacency matrix of G_1

	1	2	3	4
1	0	1	0	1
2	1	0	1	0
3	0	1	0	1
4	1	0	1	1

Adjacency matrix of G_2

	2	3	4	1
2	0	1	0	1
3	1	0	1	0
4	0	1	0	1
1	1	0	1	1

$G_1 \cong G_2$
 G_1 and G_2 are Isomorphic — 1M

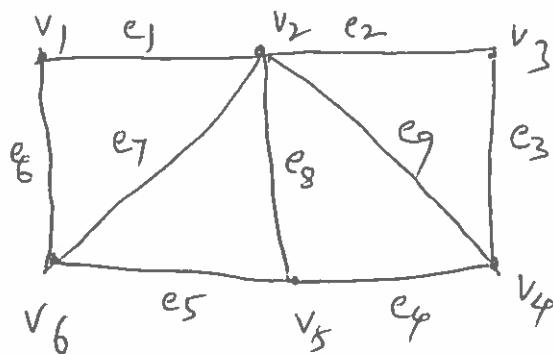
15(a) Hamiltonian graph:

Page no: 13.

A circuit in a graph G is called a Hamiltonian circuit if it contains each vertex in G exactly once, except for the starting and ending vertex that appears twice.

A graph G is called a Hamiltonian graph if it contains a Hamiltonian circuit.

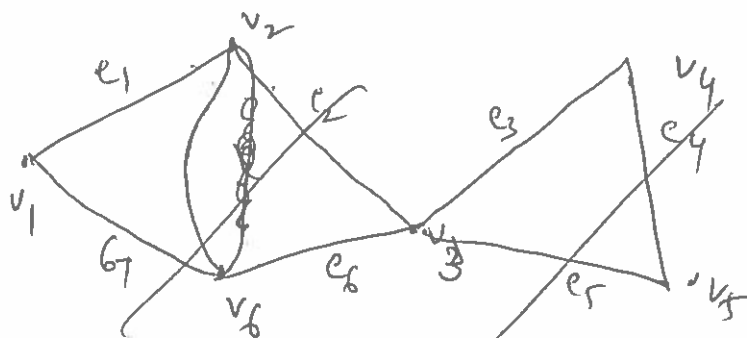
A Hamiltonian path is a path that contains all vertices of G where the endpoints may be distinct.



$v_1 e_1 v_2 e_2 v_3 e_3 v_4 e_4 v_5 e_5 v_6 e_6 v_1$ is ~~an~~ a Hamiltonian circuit.

Since the degree of vertex is not even and hence is not Eulerian.
degree of v_2 is 5

(or)



$v_1 e_1 v_2 e_2 v_3 e_3 v_4 e_4 v_5 e_5 v_6 e_6 v_1$

Eulerian.

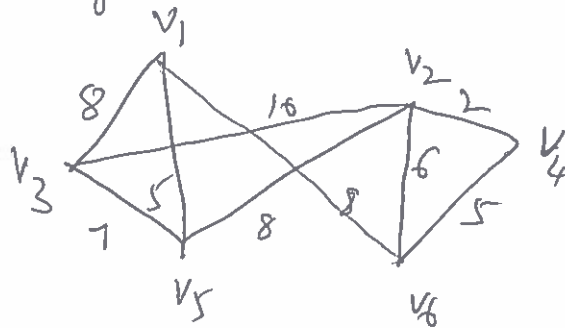
15 (5). Kruskal's algorithm: Page no: 14

Step 1. Arrange the edges in increasing order of weights and

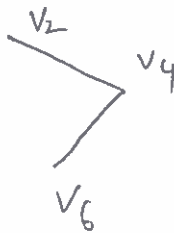
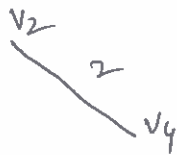
Select the edge with minimum weight.

Step 2. Proceed sequentially, add each edge which does not form a cycle until $n-1$ edges are selected.

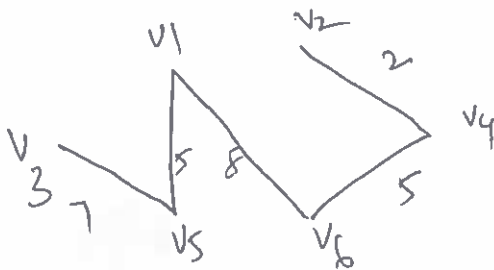
Ex:



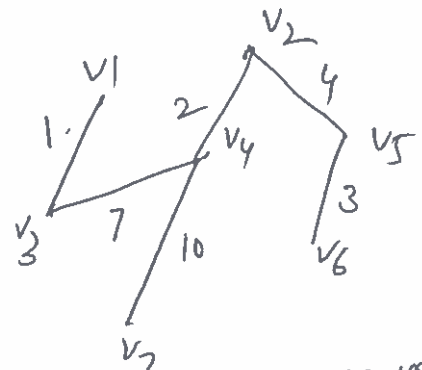
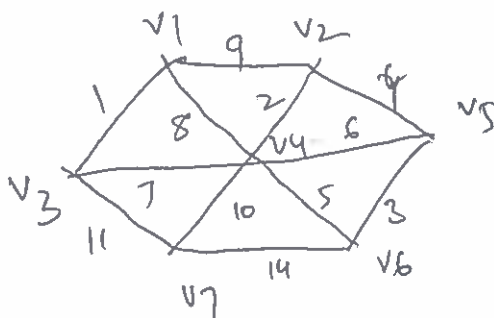
$v_2 v_4$	$v_1 v_5$	$v_2 v_6$	$v_3 v_5$	$v_1 v_3$	$v_2 v_5$	$v_1 v_6$	$v_2 v_3$
2	5	6	7	8	8	8	10



$$7 + 5 + 8 + 5 + 2 = 27$$



(or)



$$\text{Cost: } 1 + 2 + 3 + 4 + 5 + 7 + 10 = 27.$$

~~Prepared by~~

Dr P Hari Krishna

14/02/2022
94419 69937

Semester End Examination, February 2022

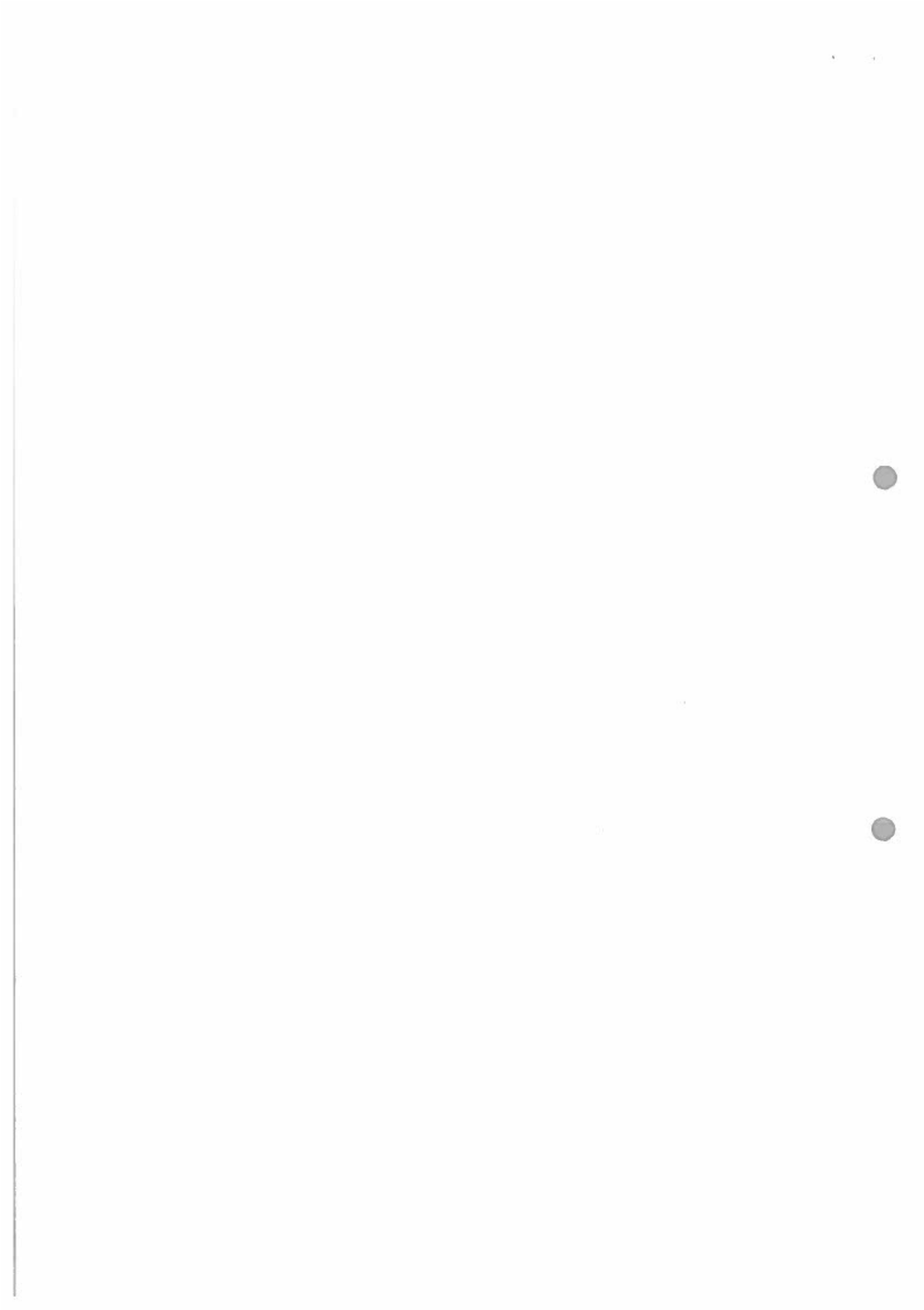
Degree	B. Tech. (U. G.)	Program	CSE, CSE (AI & ML) & CSE (DS)	Academic Year	2021 - 2022
Course Code	20CS304	Test Duration	3 Hrs.	Max. Marks	70
Course	Object Oriented Programming through C++			Semester	III

Part A (Short Answer Questions 5 x 2 = 10 Marks)

No.	Questions (1 through 5)	Learning Outcome (s)	DoK
1	Write C++ code that reads two numbers from user and prints their sum.	20CS304.1	L2
2	Define an anonymous object.	20CS304.2	L1
3	List any two advantages of inheritance.	20CS304.3	L1
4	What is a pure virtual function?	20CS304.4	L1
5	What is a list in C++ STL?	20CS304.5	L1

Part B (Long Answer Questions 5 x 12 = 60 Marks)

No.	Questions (6 through 15)	Marks	Learning Outcome (s)	DoK
6 (a)	Explain the key concepts of Object-Oriented Programming.	6M	20CS304.1	L1
6 (b)	Define recursion. Explain with a suitable example.	6M	20CS304.1	L2
OR				
7 (a)	Write a C++ program demonstrating the viability of new and delete operators for a single variable as well as an array.	7M	20CS304.1	L1
7 (b)	Explain the manipulators with suitable examples.	5M	20CS304.1	L2
8 (a)	How to define a class in C++? How to declare objects for the class? Give an example.	6M	20CS304.2	L2
8 (b)	Write a program for calculating the total marks and Grade of the 60 students in a class.	6M	20CS304.2	L1
OR				
9 (a)	Explain the visibility of base class members for the access specifiers: private, protected, and public while creating the derived class and explain the syntax for creating derived class.	6M	20CS304.2	L2
9 (b)	Write a C++ program to exchange the values between two classes using friend functions.	6M	20CS304.2	L3
10 (a)	Explain about default and parameterized constructors with suitable examples.	6M	20CS304.3	L1
10 (b)	Write a C++ program to define three overloaded functions to swap two integers, swap two floats and swap two doubles.	6M	20CS304.3	L3
OR				
11 (a)	State any two forms of inheritance. Give an example of each.	6M	20CS304.3	L1
11 (b)	Explain the syntax for passing arguments to base class constructors in multiple inheritances.	6M	20CS304.3	L3
12 (a)	How this pointer declared in C++?	4M	20CS304.4	L1
12 (b)	Explain Static Binding and Dynamic Binding.	8M	20CS304.4	L2
OR				
13 (a)	What is a Virtual Function? Explain with an example.	6M	20CS304.4	L1
13 (b)	Explain the Exception Handling mechanism.	6M	20CS304.4	L2
14 (a)	Write the difference between Templates and Macros.	6M	20CS304.5	L2
14 (b)	Write a program to swap two variables using a function template.	6M	20CS304.5	L3
OR				
15	Explain briefly the three foundational items of standard template library.	12M	20CS304.5	L2





N S RAJU INSTITUTE OF TECHNOLOGY

(AUTONOMOUS)

SONTYAM , ANANDAPURAM, VISAKHAPATNAM – 531 173

ANSWER KEY AND SCHEME OF EVALUATION

Object Oriented Programming Through C++ (20CS304)

No.	Part –A Questions (1 through 5) 5*2=10 Marks	Marks
1	Write C++ code that reads two numbers from user and prints their sum. <pre>#include<iostream> using namespace std; int main() { int num1, num2, add; cout<<"Enter Two Numbers: "; cin>>num1>>num2; add = num1+num2; cout<<"\nResult = "<<add; cout<<endl; return 0; }</pre> Output: Enter Two Numbers:50 60 Result=110	2M
2	Define an anonymous object? An anonymous object is essentially a value that has no name. Because they have no name, there's no way to refer to them beyond the point where they are created. Consequently, they have "expression scope", meaning they are created, evaluated, and destroyed all within a single expression.	2M
3	List any two advantages of inheritance I. Reusability- Inheritance helps the code to be reused in many situations. II. Saves Time and Effort- Since the main code written can be reused in various situations as needed.	1M 1M
4	What is a pure virtual function? A pure virtual function is a virtual function in C++ for which we need not to write any function definition and only we have to declare it. It is declared by assigning 0 in the declaration	2M
5	What is a list in C++ STL? Lists are sequence containers that allow non-contiguous memory allocation. As compared to vector, the list has slow traversal, but once a position has been found, insertion and deletion are quick	2M
No.	Part-B Questions (6 through 15) 5*12=60 marks	

1.5 marks for each Explanation	The key concepts of OOP are: I. Encapsulation II. Abstraction III. Inheritance IV. Polymorphism 1. Encapsulation The word, "encapsulate," means to enclose something. Just like a pill "encapsulates" or contains the medication inside of its coating, the principle of encapsulation works in a similar way in OOP: by forming a protective barrier around the information contained within a class from the rest of the code. In OOP, we encapsulate by binding the data and functions which operate on that data into a single unit, the class. By doing so, we can hide private details of a class from the outside world and only expose functionality that is important for interfacing with it. When a class does not allow calling code access to its private data directly, we say that it is well encapsulated. Example: Elaborating on the person class example from earlier, we might have private data in the class, such as "socialSecurityNumber," that should not be exposed to other objects in the program. By encapsulating this data member as a private variable in the class, outside code would not have direct access to it, and it would remain safe within that person's object. If a method is written in the person class to perform, say, a bank transaction called "bankTransaction()", that function could then access the "socialSecurityNumber" variable as necessary. The person's private data would be well encapsulated in such a class. 2. Abstraction Often, it's easier to reason and design a program when you can separate the interface of a class from its implementation, and focus on the interface. This is akin to treating a system as a "black box," where it's not important to understand the gory inner workings in order to reap the benefits of using it. This process is called "abstraction" in OOP, because we are abstracting away the gory implementation details of a class and only presenting a clean and easy-to-use interface via the class' member functions. Carefully used, abstraction helps isolate the impact of changes made to the code, so that if something goes wrong, the change will only affect the implementation details of a class and not the outside code. Example: Think of a stereo system as an object with a complex logic board on the inside. It has buttons on the outside to allow for interaction with the object. When you press any of the buttons, you're not thinking about what happens on the inside because you can't see it. Even though you can't see the logic board completing these functions as a result of pressing a button, it's still performing them, albeit hidden to you. This is the concept of abstraction, which is incredibly useful in all areas of engineering and also applied to great effect in object-oriented programming. Example: In OOP, we might have a class defined to represent the human body. One might define some functions as part of its publicly facing interface such as "walk()" or "eatFood()". Calling code could call these functions and remain completely oblivious to the complex inner workings of the human body and its necessary functions to perform the act of walking or eating. These details are completely hidden in the implementation of the walk() and eatFood() body functions and are, therefore, us abstracted away from the end user. In these cases, it's not important for calling code to understand how the brain coordinates walking or how the stomach manages digesting the food, but rather simply that a human walked or ate. 3. Inheritance Object-oriented languages that support classes almost always support the notion of "inheritance." Classes can be organized into hierarchies, where a class might have one or more parent or child classes. If a class has a parent class, we say it is derived or inherited from the parent class and it represents an "IS-A" type relationship. That is to say, the child class "IS-A" type of the parent class.

6 (a)

Explain the key concepts of Object-Oriented Programming (6)

Therefore, if a class inherits from another class, it automatically obtains a lot of the same functionality and properties from that class and can be extended to contain separate code and data. A nice feature of inheritance is that it often leads to good code reuse since a parent class' functions don't need to be re-defined in any of its child classes.

Consider two classes: one being the superclass—or parent—and the other being the subclass—or child. The child class will inherit the properties of the parent class, possibly modifying or extending its behavior. Programmers applying the technique of inheritance arrange these classes into what is called an "IS-A" type of relationship.

Example: For instance, in the animal world, an insect is a superclass. All insects share common properties, such as having a skeleton. Subclasses might be defined for grasshoppers and ants. Because they inherit or are derived from the Insect class, they automatically share all insect properties.

4. Polymorphism

In OOP, polymorphism allows for the uniform treatment of classes in a hierarchy. Therefore, calling code only needs to be written to handle objects from the root of the hierarchy, and any object instantiated by any child class will be handled properly.

Because derived objects can be treated as if they were the superclass, code can call any function in that class's interface. At run-time, the appropriate function will be called depending on the type of object passed leading to possibly different behaviors.

Example: Suppose we have a class called, "Animal" and two child classes, "Cat," and "Dog." If the Animal class has a method to make a noise, called, "makeNoise," then, we can override the "makeNoise" function that is inherited by the sub-classes, "Cat" and "Dog," to be "meow" and "bark," respectively. Another function can, then, be written that accepts any Animal object as a parameter and invokes its "makeNoise" function. The behavior will be different: either a "meow" or a "bark" depending on the type of animal object.

Define Recursion

Recursion is a technique in which a function calls itself within the body of the function.

of the function.

#include <iostream>
using namespace std;

```
fibonacci(int x) {
    if (x == 0 || x == 1)
        return(x);
    else {
        return(fibonacci(x-1)+fibonacci(x-2));
    }
}
```

6 (b)

```
int main() {
    cout << "Enter the number of terms : ";
    cin >> x;
    cout << "Fibonacci Series : ";
    while(i < x) {
        cout << " " << fibonacci(i);
        i++;
    }
    return 0;
}
```

Output

Enter the number of terms of series : 15
Fibonacci Series : 0 1 1 2 3 5 8 13 21 34 55 89 144 233 377

1M

5M

OR

Write a C++ program demonstrating the viability of new and delete operators for a single variable as well as an array. (7)

New operator

7 (a)

The new operator denotes a request for memory allocation on the Free Store. It is efficient memory is available, new operator initializes the memory and returns the address of the new memory allocated and initialized memory to the pointer variable.
Syntax to use new operator: To allocate memory of any data type, the syntax is:

pointer-variable = new data-type;
example: int *p = new int;

Here, pointer-variable is the pointer of type data-type. Data-type could be any basic or derived data type including array or any user defined data types including structure and class

delete operator
Since it is programmer's responsibility to deallocate dynamically allocated memory, programmers are provided delete operator by C++ language.
Syntax:

// Release memory pointed by pointer-variable

delete pointer-variable;
Here, pointer-variable is the pointer that points to the data object created by new.

delete p;

delete q;

// C++ program to illustrate dynamic allocation
// and deallocation of memory using new and delete
#include <iostream>
using namespace std;
int main ()

{
// Pointer initialization to null
int* p = NULL;

// Request memory for the variable
// using new operator
p = new(nothrow) int;

if (!p)
cout << "allocation of memory failed\n";
else

{
// Store value at allocated address
*p = 29;
cout << "Value of p: " << *p << endl;

}
// Request block of memory
// using new operator

float r = new float(75.25);
cout << "Value of r: " << *r << endl;
// Request block of memory of size n

int n = 5;
int* q = new(nothrow) int[n];
if (!q)

cout << "allocation of memory failed\n";

2M

2M

3 <i>setw</i> Manipulator: - This is used after <i>setw</i> manipulator. - If a value does not entirely fill a field, then the character specified in the <i>setw</i> argument of the manipulator is used for filling the fields. Example: <pre>#include <iostream> #include <omanip> void main() { cout << setw(15) << setw(15) << endl; } Output: *****9997</pre>	8 (a) How to define a class in c++? How to declare objects for the class? Give an example (6) Class: A class in C++ is the building block that leads to Object-Oriented programming. It is a user-defined data type, which holds its own data members and member functions, which can be accessed and used by creating an instance of that class. A C++ class is like a blueprint for an object. For Example: Consider the Class of Cars. There may be many cars with different names and brand but all of them will share some common properties like all of them will have 4 wheels, Speed Limit, Mileage range etc. So here, Car is the class and wheels, speed limits, mileage are their properties. - A Class is a user defined data-type which has data members and member functions. - Data members are the data variables and member functions are the functions used to manipulate these variables and together these data members and member functions defines the properties and behavior of the objects in a Class. - In the above example of class Car, the data member will be <i>speed limit</i>, <i>mileage</i> etc and member functions can be <i>apply brakes</i>, <i>increase speed</i> etc. An Object is an instance of a Class. When a class is defined, no memory is allocated but when it is instantiated (i.e. an object is created) memory is allocated. Defining Class and Declaring Objects A class is defined in C++ using keyword <i>class</i> followed by the name of class. The body of class is defined inside the curly brackets and terminated by a semicolon at the end. keyword user-defined name <pre>class ClassName { Access specifier: // can be private, public or protected Data members; // Variables to be used Member Functions() { } // Methods to access data members }; // Class name ends with a semicolon</pre> Declaring Objects: When a class is defined, only the specification for the object is defined; no memory or storage is allocated. To use the data and access functions defined in the class, you need to create objects.
Class definition - 2M Object definition - 2M Declaring objects - 2M	end.

	Syntax: ClassName ObjectName; Accessing data members and member functions: The data members and member functions of class can be accessed using the dot('.') operator with the object. For example if the name of object is <i>obj</i> and you want to access the member function with the name <i>printName()</i> then you will have to write <i>obj.printName()</i> . Accessing Data Members The public data members are also accessed in the same way given however the private data members are not allowed to be accessed directly by the object. Accessing a data member depends solely on the access control of that data member. <pre>// C++ program to demonstrate // accessing of data members #include <bits/stdc++.h> using namespace std; class Sample{ // Access specifier public: // Data Members string geekname; // Member Functions() void printname() { cout << "My name is: " << geekname; } }; int main() { // Declare an object of class geeks Sample obj1; // accessing data member obj1.geekname = "Abhi"; // accessing member function obj1.printname(); return 0; }</pre> Output: Geekname is: Abhi	Example - 2M
8 (b)	Write a program for calculating the total marks and Grade of the 60 students in a class (6) <pre>#include<iostream> using namespace std; int main(){ int m1, m2, m3; int i=1; while(i<=60){ int total=0; cout<<"Marks of Student "<<" "<<i<<" in three subjects."; cin>>m1>>m2>>m3; total= m1+m2+m3; cout<<"Total Marks:"<<" "<<total<<endl; if(total>=280) { cout<<"Grade: A+"<<endl; }</pre>	Program - 6M

9 (a)	Explain the visibility of base class members for the access specifiers: private, protected and public while creating the derived class and explain the syntax for creating derived class. (6) C++ access specifiers are used for determining or setting the boundary for the availability of class members (data members and member functions) beyond that class. For example, the class members are grouped into sections, private, protected and public. These keywords are called access specifiers which define the accessibility or visibility level of class members. By default the class members are private. So if the visibility labels are missing then by default all the class members are private. In inheritance, it is important to know when a member function in the base class can be used by the objects of the derived class. This is called accessibility and the access specifiers are used to determine this. implementing inheritance in C++: For creating a sub-class which is inherited from the base class we have to follow the below syntax. Syntax: <pre> class subclass_name : access_mode base_class_name { // body of subclass }; </pre>
	OR <pre> } else if (total >= 260) { cout << "Grade: A" << endl; } else if (total >= 240) { cout << "Grade: B+" << endl; } else if (total >= 220) { cout << "Grade: B" << endl; } else if (total >= 200) { cout << "Grade: C+" << endl; } else if (total >= 180) { cout << "Grade: C" << endl; } else if (total >= 160) { cout << "Grade: D+" << endl; } else if (total >= 140) { cout << "Grade: D" << endl; } else if (total >= 120) { cout << "Grade: E" << endl; } else { cout << "Enter the valid Marks!" << endl; } i++; } return 0; } </pre>

- 2M

Explanation

	Here, subclass_name is the name of the sub class, access_mode is the mode in which you want to inherit this sub class for example: public, private etc. and base_class_name is the name of the base class from which you want to inherit the sub class. Note: A derived class doesn't inherit access to private data members. However, it does inherit a full parent object, which contains any private members which that class declares. //Implementation of Single Inheritance using public visibility mode <pre>#include <iostream> using namespace std; class Shape { private: int count; protected: int width; int height; public: void setWidth(int w) { width = w; } void setHeight(int h) { height = h; } }; class Rectangle: public Shape { public: int getArea() { return (width * height); } }; int main() { Rectangle Rect; Rect.setWidth(5); Rect.setHeight(7); // Print the area of the object. cout<< "Total area: "<<Rect.getArea() <<endl; return 0; }</pre> Output Total area: 35	Syntax - 2M Program - 2M
	Write a c++ program to exchange the values between two classes using friend functions. (6) program is demonstrating how a common friend function can be used to exchange the private values of two classes using call-by-reference method. <pre>#include<iostream></pre>	Program

9 (b)

```
using namespace std;
class Base
{
protected:
    int num1;
public:
    Base()
    {
        num1=10;
    }
    void show()
    {
        cout<<"Value of Number 1 : "<<num1;
    }
    friend void swap(Base *num1, Derived *num2);
};

class Derived
{
protected:
    int num2;
public:
    Derived()
    {
        num2=20;
    }
    void show()
    {
        cout<<"Value of Number 2 : "<<num2;
    }
    friend void swap(Base *num1, Derived *num2);
};

int no3;
no3=no1->num1;
no1->num1=no2->num2;
no2->num2=no3;

}
int main()
{
    Base b;
    Derived d;
    swap(&b, &d);
    b.show();
    d.show();
    return 0;
}

Output:
Value of Number 1: 20
Value of Number 2: 10
```

10 (a)	Explain about default and parameterized constructors with suitable examples. (6) A constructor is a special type of member function of a class which initializes objects of a class. In C++, Constructor is automatically called when object(instance of class) create. It is special member function of the class because it does not have any return type.	Constructor definition - 1M
	1. Default Constructors: Default constructor is the constructor which doesn't take any argument. It has no parameters. <pre>// Cpp program to illustrate the // concept of Constructors #include <iostream> using namespace std; class construct { public: int a, b; // Default Constructor construct() { a = 10; b = 20; } }; int main() { // Default constructor called automatically // when the object is created construct c; cout << "a: " << c.a << endl << "b: " << c.b; return 1; }</pre> Output: a: 10 b:20 Note: Even if we do not define any constructor explicitly, the compiler will automatically provide a default constructor implicitly. 2. Parameterized Constructors: It is possible to pass arguments to constructors. Typically these arguments help initialize an object when it is created. To create a parameterized constructor, simply add parameters to it the way you would to any other function. When you define the constructor's body, use the parameters to initialize the object. <pre>// CPP program to illustrate // parameterized constructors #include <iostream> using namespace std; class Point { private: int x, y; public: // Parameterized Constructor Point(int x1, int y1) { x = x1; } }</pre>	Explanation- 3M Example-3M

	10 (b) Write a c++ program to define three overloaded functions to swap two integers, swap two floats and swap two doubles(6) Program -5M Output-1M	<pre> #include<iostream.h> #include<conio.h> void swap(int a,int b) { int temp; temp=a; a=b; b=temp; cout<<"Integer values after swapping are:"<<a<<" "<<b; } void swap(float x,float y) { float temp1=x; x=y; y=temp1; cout<<"Float values after swapping are:"<<x<<" "<<y; } void swap(double x,double y) { double temp1=x; x=y; y=temp1; cout<<"Double values after swapping are:"<<x<<" "<<y; } </pre>

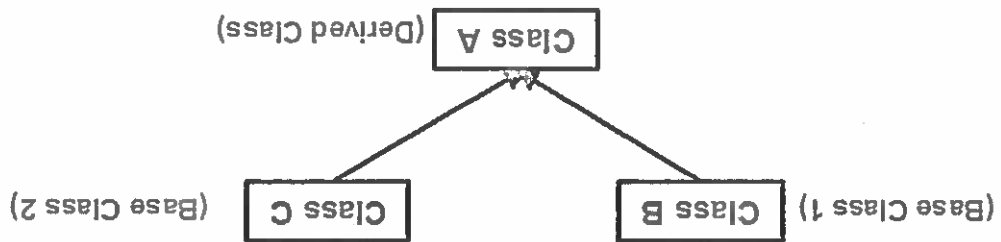
	<pre> void main() { clrscr(); swap(10,20); swap(10.15f,20.25f); swap(10.10,20.3); getch(); } </pre>	
	OR	
11 (a)	State any two forms of inheritance .give an example of each (6) Different Forms of Inheritance in Object Oriented Programming - o Single Inheritance - o Multiple Inheritance - o Multi-level Inheritance - o Hierarchical Inheritance - o Hybrid Inheritance the capability of a class to derive properties and characteristics from another class is called Inheritance. Inheritance is one of the most important feature of Object Oriented Programming. Sub Class: The class that inherits properties from another class is called Sub class or Derived Class. Super Class: The class whose properties are inherited by sub class is called Base Class or Super class. <u>Single Inheritance</u> <pre> // C++ program to explain // Single inheritance #include<iostream> using namespace std; // base class class Vehicle { public: Vehicle() { cout << "This is a Vehicle\n"; } }; // sub class derived from a single base classes class Car : public Vehicle { }; // main function int main() { // Creating object of sub class will // invoke the constructor of base classes </pre>	Definition-1M List-1M Each Form with Example-2M+2M

```
Car obj;
return 0;
}
```

Output

This is a Vehicle

2. Multiple Inheritance: Multiple Inheritance is a feature of C++ where a class can inherit from more than one classes. i.e one sub class is inherited from more than one base classes.



Syntax:

```
class subclass_name : access_mode base_class1, access_mode base_class2, ...
{
    // body of subclass
};
```

Here, the number of base classes will be separated by a comma (',') and access mode for every base class must be specified.

```
// C++ program to explain
// multiple inheritance
#include<iostream>
using namespace std;
// first base class
class Vehicle {
public:
    Vehicle()
    {
        cout << "This is a Vehicle\n";
    }
};
```

```
// second base class
class FourWheeler {
public:
    FourWheeler()
    {
        cout << "This is a 4 wheeler Vehicle\n";
    }
};
```

```
// sub class derived from two base classes
class Car : public Vehicle, public FourWheeler {
};

// main function
int main()
```

```

{
    // Creating object of sub class will
    // invoke the constructor of base classes.
    Car obj;
    return 0;
}

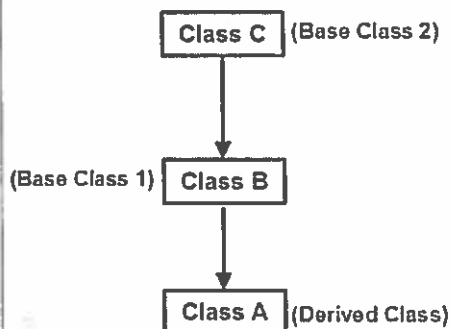
```

Output

This is a Vehicle

This is a 4 wheeler Vehicle

3. Multilevel Inheritance: In this type of inheritance, a derived class is created from another derived class.



```

// C++ program to implement
// Multilevel Inheritance
#include<iostream>
using namespace std;

// base class
class Vehicle
{
public:
    Vehicle()
    {
        cout << "This is a Vehicle\n";
    }
};

// first sub_class derived from class vehicle
class fourWheeler: public Vehicle
{
public:
    fourWheeler()
    {
        cout << "Objects with 4 wheels are vehicles\n";
    }
};

// sub class derived from the derived base class fourWheeler
class Car: public fourWheeler {
public:
    Car()

```

Explanation-3m Example-3m	Output <pre> { cout << "Car has 4 Wheels\n"; } // main function int main() { // Creating object of sub class will // invoke the constructor of base classes. Car obj; return 0; } </pre> This is a Vehicle Objects with 4 wheels are vehicles Car has 4 Wheels 11 (b) Explain the syntax for passing arguments to base class constructors in multiple inheritances. (6) Algorithm steps: step 1: Declare the base class namely BaseClass1. It consists the private member data and a constructor. Step 2: Declare the base class namely BaseClass2. It consists the private member data and a constructor. Step 3: create the derived class namely DerivedClass from BaseClass1 and BaseClass2. Step 4: Declare constructor in DerivedClass with three arguments and also base the arguments to base class constructor. Step 5: Create the object for DerivedClass in main (). Step 6: The object automatically call the Derived class constructor. Now Derived class constructor automatically call the baseclass1 & baseclass2 constructor with arguments. Step 7: Display the values. C++ Program To Implement Multiple Inheritance and also Passing Parameters To Base Class Constructors <pre> #include <iostream> using namespace std; class BaseClass1 { int a; public: BaseClass1(int x) { a = x; } int geta() { return a; } } </pre>

	<pre> }; class BaseClass2 { int b; public: BaseClass2(int x) { b = x; } int getb() { return b; } }; class DerivedClass : public BaseClass1, public BaseClass2 { int c; public: DerivedClass(int x, int y, int z) : BaseClass1(z), BaseClass2(y){ c = x; } void show() { cout << geta() << ' ' << getb() << ' '; cout << c << '\n'; } }; int main() { DerivedClass object(1, 2, 3); object.show(); return 0; } </pre> SAMPLE INPUT AND OUTPUT: 3 2 1	
12 (a)	How this pointer declared in c++? (4) C++ this Pointer In C++ programming, this is a keyword that refers to the current instance of the class. There can be 3 main usage of this keyword in C++. It can be used to pass current object as a parameter to another method. It can be used to refer current class instance variable. It can be used to declare indexers. C++ this Pointer Example <pre> #include <iostream> using namespace std; class Employee { public: int id; //data member (also instance variable) string name; //data member(also instance variable) float salary; Employee(int id, string name, float salary) { this->id = id; </pre>	Description -1M Example- 3m

	<pre> this->name = name; this->salary = salary; } void display() { cout<<id<<" "<<name<<" "<<salary<<endl; } }; int main(void) { Employee e1 = Employee(101, "Sonoo", 890000); //creating an object of Employee Employee e2 = Employee(102, "Naku", 59000); //creating an object of Employee e1.display(); e2.display(); return 0; } Output: 101 Sonoo 890000 102 Naku 59000 </pre> 12 (b) Explain Static Binding and Dynamic Binding. (8) The term "Polymorphism" is the combination of "poly" + "morphs" which means many forms. It is a greek word. In object-oriented programming, we use 3 main concepts: inheritance, encapsulation, and polymorphism. Real Life Example Of Polymorphism Let's consider a real-life example of polymorphism. A lady behaves like a teacher in a classroom, mother or daughter in a home and customer in a market. Here, a single person is behaving differently according to the situations. There are two types of polymorphism in C++: <pre> graph BT Polymorphism[Polymorphism] --> CompileTime([Compile time Polymorphism]) Polymorphism --> RunTime([Run time Polymorphism]) CompileTime --> FunctionOverloading[Function Overloading] CompileTime --> OperatorOverloading[Operator Overloading] RunTime --> VirtualFunction[Virtual Function] </pre> Compile time polymorphism: The overloaded functions are invoked by matching the type and number of arguments. This information is available at the compile time and, therefore, compiler selects the appropriate function at the compile time. It is achieved by function overloading and operator overloading which is also known as static binding or early binding. Now, let's consider the case where function name and prototype is same <pre> class A { int a; public: void display() { cout<<"Class A "; } } // base class declaration. </pre>
Explanation static binding-4M and Dynamic binding-4M	

```
};
class B : public A           // derived class declaration.
{
    int b;
    public:
    void display()
    {
        cout<<"Class B";
    }
};
```

In the above case, the prototype of display() function is the same in both the base and derived class. Therefore, the static binding cannot be applied. It would be great if the appropriate function is selected at the run time. This is known as run time polymorphism.

Run time polymorphism: Run time polymorphism is achieved when the object's method is invoked at the run time instead of compile time. It is achieved by method overriding which is also known as dynamic binding or late binding.

C++ Runtime Polymorphism Example

```
#include <iostream>
using namespace std;
class Animal {
    public:
    void eat(){
        cout<<"Eating...";
    }
};
class Dog: public Animal
{
    public:
    void eat()
    {
        cout<<"Eating bread...";
    }
};
int main(void) {
    Dog d = Dog();
    d.eat();
    return 0;
}
```

Output:
Eating bread...

OR

13 (a) What is a virtual Function? Explain with an example.(6)

C++ virtual function

- A C++ virtual function is a member function in the base class that you redefine in a derived class. It is declared using the virtual keyword.
- It is used to tell the compiler to perform dynamic linkage or late binding on the function.
- There is a necessity to use the single pointer to refer to all the objects of the different classes. So, we create the pointer to the base class that refers to all the derived objects. But, when base class pointer contains the address of the derived class object, always executes the base class function. This issue can only be resolved by using the 'virtual' function.
- A 'virtual' is a keyword preceding the normal declaration of a function.

Explanation-
4m
Program-2M

- When the function is made virtual, C++ determines which function is to be invoked at the runtime based on the type of the object pointed by the base class pointer.

```
#include <iostream>
{
public:
virtual void display()
{
cout << "Base class is invoked"<<endl;
}
}
class B:public A
{
public:
void display()
{
cout << "Derived Class is invoked"<<endl;
}
}
int main()
{
A* a; //pointer of base class
B b; //object of derived class
a = &b;
a->display(); //Late Binding occurs
}
```

Output:

Derived Class is invoked

13 (b)

Explain the Exception Handling mechanism.(6)**C++ Exception Handling**

Exception Handling in C++ is a process to handle runtime errors. We perform exception handling so the normal flow of the application can be maintained even after runtime errors.

In C++, exception is an event or object which is thrown at runtime. All exceptions are derived from `std::exception` class. It is a runtime error which can be handled. If we don't handle the exception, it prints exception message and terminates the program.

Advantage

It maintains the normal flow of the application. In such case, rest of the code is executed even after exception.

All the exception classes in C++ are derived from `std::exception` class. Let's see the list of C++ common exception classes.

Exception	Description
<code>std::exception</code>	It is an exception and parent class of all standard C++ exceptions.
<code>std::logic_failure</code>	It is an exception that can be detected by reading a code.
<code>std::runtime_error</code>	It is an exception that cannot be detected by reading a code.
<code>std::bad_exception</code>	It is used to handle the unexpected exceptions in a c++ program.
<code>std::bad_cast</code>	This exception is generally be thrown by <code>dynamic_cast</code> .
<code>std::bad_typeid</code>	This exception is generally be thrown by <code>typeid</code> .
<code>std::bad_alloc</code>	This exception is generally be thrown by <code>new</code> .

In C++, we use 3 keywords to perform exception handling:

- try
- catch, and
- throw

```
#include <iostream>
using namespace std;
float division(int x, int y) {
    if( y == 0 ) {
        throw "Attempted to divide by zero!";
    }
    return (x/y);
}
int main () {
    int i = 25;
    int j = 0;
    float k = 0;
    try {
        k = division(i, j);
        cout << k << endl;
    } catch (const char* e) {
        cerr << e << endl;
    }
}
```

Description-3M**Syntax -1M****Example-2M**

	<pre> return 0; } Output: Attempted to divide by zero! </pre>	
	14 (a) Write the difference between Templates and Macros.(6) When used as nouns, macro means a comparatively human-friendly abbreviation of complex input to a computer program, whereas template means a physical object whose shape is used as a guide to make other objects. Macro are used in C and C++ for replacement of numbers, small inline functions etc. Template is only available in C++ language. It is used to write small macro like functions etc. <i>Data type check in macro</i> Macro cannot check the data type of arguments. This below code compiles fine. <pre> int main () { int i = 10; float j = 12.2; swap(i, j); } Template checks the data type #include <iostream> template <typename T> T swap(T &a, T &b) { T c; </pre>	

Any 3 differences with example-6M

	<pre> c = b; b = a; a = c; } int main () { int i = 10; float j = 12.2; swap(i, j); } </pre> One parameter is integer and another in float Operators used in macro If parameters of any macro are given with operators ++ and -- with it and the macro used this argument more than one times then the increment or decrement of the original variable will that much times. Compilation error in macro Macros are expanded by the preprocessor and then compilation takes place. Compiler will refer error messages in expanded macro or the line where macro has been called.	
14 (b)	Write a program to swap two variables using a function template.(6) <pre> // C++ program to implement // function templates #include <iostream> using namespace std; // Function template to swap // two numbers template <class T> int swap_numbers(T& x, T& y) { T t; t = x; x = y; y = t; return 0; } // Driver code int main() { int a, b; a = 10, b = 20; // Invoking the swap() swap_numbers(a, b); cout << a << " " << b << endl; return 0; } </pre> Output: 20 10	Description about function template-2M Program-3M Output-1M

OR

Explain briefly the three foundational items of standard template library (STL) (12) Standard Template Library (STL) of C++ is a collection of template classes that provide data structures such as arrays, vectors, queue, etc. STL is a library consisting of containers, algorithms, and iterators. As STL consists of a collection of template classes, it's a generalized library that is independent of data types. Components Of STL STL mainly consists of the following components which are mentioned below: #(1) Containers A container is a collection of objects of a particular type of data structure. In STL, we have various types of container classes like Array, vector, queue, deque, list, map, set, etc. These containers are generic in nature and are implemented as class templates. Containers are dynamic in nature and can be used to hold various types of objects. #(2) Algorithms Algorithms are the methods or functions that act on containers. By using algorithms provided by STL, we can have methods to search, sort, modify, transform or initialize the contents of container class objects. Algorithms provided by STL have built-in functions that can directly operate on complex data structure instead of having to write the algorithms ourselves. For Example, reverse() function in STL can be used to reverse the linked list. #(3) Iterators Iterators are the very important and distinguishing feature of STL. Iterators are the constructs that are used to traverse through the container objects. Similar to indexes that we use to step through the arrays, iterators act on container class objects and can be used to step through the data. Containers store objects and data. They are basically template-based generic classes. Containers in STL are divided into the following types: #(1) Sequential Containers Containers the can be accessed in a sequential or linear manner are said to be sequential containers. Arrays, Vectors, Lists, Deques are the STL containers that store data linearly and can be accessed in a sequential manner. #(2) Associative Containers Associative containers are containers that implement sorted data structures. These containers are fast to search. Some of the <u>Examples</u> of associative containers are Map, Set, Multimap, Multiset, etc. These containers are usually implemented in a key/value pair fashion. #(3) Container Adapters	15 Description-1M List of component-2M Each component-3M Atleast 3 functions-9M

Container adopters are sequential containers, however, they are implemented by providing a different interface. Thus containers like a queue, deque, stack, and priority-queue are all classified as container adopters.

Iterators

Iterators are constructs that we use to traverse or step through containers in STL. Iterators are very important in STL as they act as a bridge between algorithms and containers. Iterators always point to containers and in fact algorithms actually, operate on iterators and never directly on containers.

Iterators are of following types:

Input Iterators: Simplest and is used mostly in single-pass algorithms.

Output Iterators: Same as input iterators but not used for traversing.

Bidirectional Iterators: These iterators can move in both directions.

Forward Iterators: Can be used only in the forward direction, one step at a time.

Random Access Iterators: Same as pointers. Can be used to access any element randomly.

Algorithms

Algorithms are a set of functions or methods provided by STL that act on containers. These are built-in functions and can be used directly with the STL containers and iterators instead of writing our own algorithms.

STL supports the following types of algorithms:

Searching algorithms

Sorting algorithms

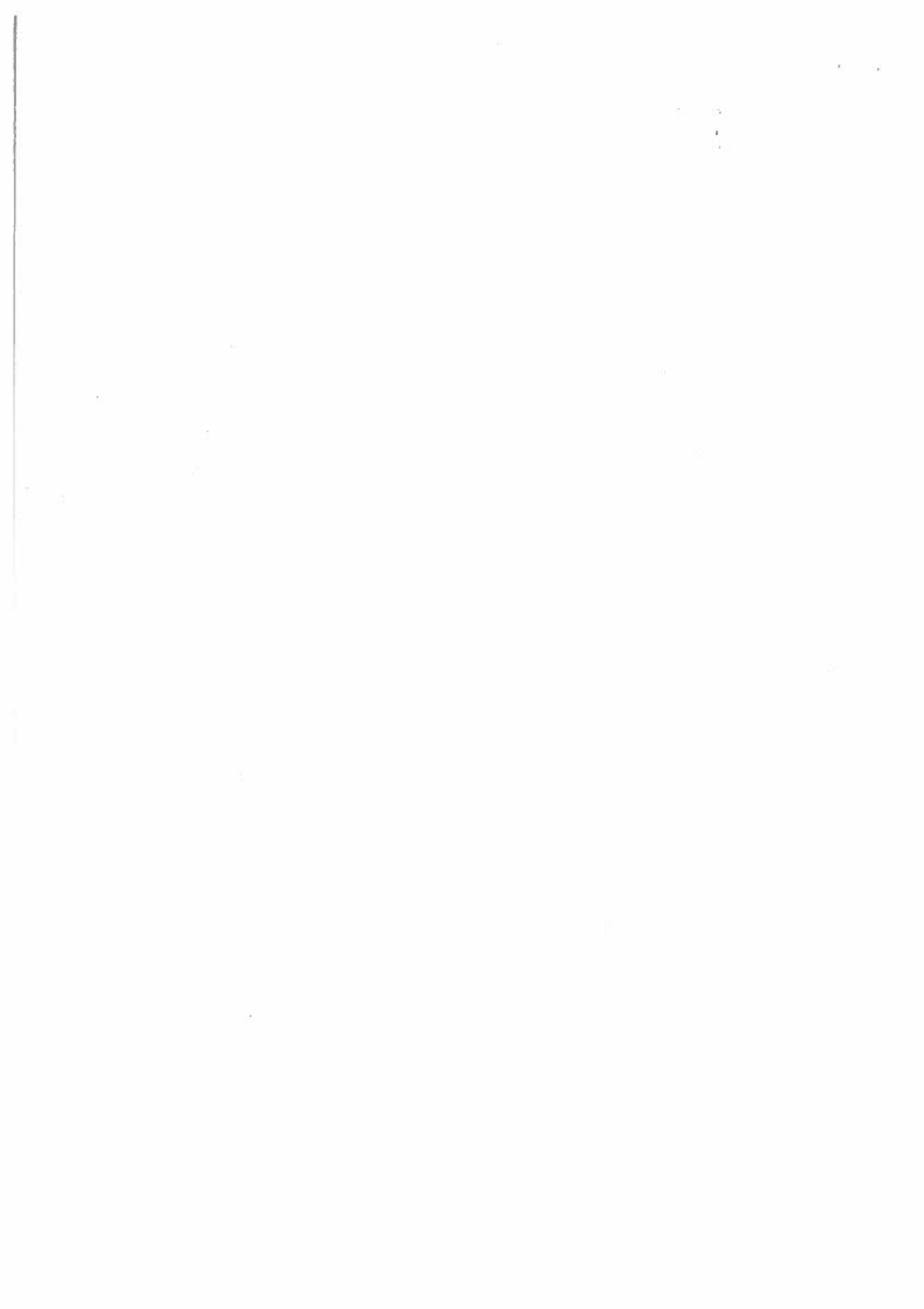
Modifying or manipulating algorithms

Non-modifying algorithms

Numeric algorithms

Min/Max algorithms

As each of the algorithm types suggests, these algorithms can be used to achieve different functionality in STL containers like searching, sorting, transforming the data in the containers, finding min/max value, etc.



Semester End Examination, February, 2022

Degree	B. Tech. (U. G.)	Program	CSE, CSM & CSD	Academic Year	2021 – 2022
Course Code	20CS303	Test Duration	3 Hrs.	Max. Marks	70
Course	Database Management Systems	Semester	III		

Part A (Short Answer Questions 5 x 2 = 10 Marks)

No.	Questions (1 through 5)	Learning Outcome (s)	DoK
1	What is a database Management System? Give any two examples.	20CS303.1	L1
2	List out any two aggregate operators	20CS303.2	L1
3	Write the difference between Drop, Delete and Truncate statements in SQL Server.	20CS303.3	L1
4	What is ISAM?	20CS303.4	L1
5	State two-phase locking protocol	20CS303.5	L1

Part B (Long Answer Questions 5 x 12 = 60 Marks)

No.	Questions (6 through 15)	Marks	Learning Outcome (s)	DoK
6 (a)	Compare database systems and file systems.	6M	20CS303.1	L2
6 (b)	Explain the different roles performed by the data base administrator.	6M	20CS303.1	L2
OR				
7 (a)	Explain the E-R diagram components and notations with their extended features.	6M	20CS303.1	L2
7 (b)	Define Data Abstraction and discuss levels of Abstraction.	6M	20CS303.1	L2
8	Explain various operations in relational algebra with example.	12M	20CS303.2	L2
OR				
9 (a)	Write about views and updates on views.	6M	20CS303.2	L2
9 (b)	What are the various integrity constraint relations of relational model?	6M	20CS303.2	L2
10 (a)	What are Null values? Discuss about disallowing Null values.	6M	20CS303.3	L3
10 (b)	Define a nested query with an example.	6M	20CS303.3	L3
OR				
11 (a)	Illustrate trigger with example.	6M	20CS303.3	L3
11 (b)	Discuss different types of aggregate functions with examples in SQL.	6M	20CS303.3	L2
12 (a)	Define functional dependencies. How are primary keys related to FD's?	6M	20CS303.4	L2
12 (b)	Explain Dense and Sparse indices.	6M	20CS303.4	L2
OR				
13	Define normalization, why do we need Normalization? Explain 1NF, 2NF, 3NF Normal forms	12M	20CS303.4	L2
14 (a)	Explain ACID properties and Illustrate them through examples.	6M	20CS303.5	L2
14 (b)	Discuss about conflict Serializability with an example.	6M	20CS303.5	L3
OR				
15 (a)	What is ARIES and list the phases of ARIES.	6M	20CS303.5	L3
15 (b)	Discuss about various database recovery approaches.	6M	20CS303.5	L2



N S RAJU INSTITUTE OF TECHNOLOGY
(AUTONOMOUS)
SONTYAM , ANANDAPURAM, VISAKHAPATNAM – 531 173

ANSWER KEY AND SCHEME OF EVALUATION
Semester End Examination, February, 2022

Degree	B. Tech. (U. G.)	Program	CSE, CSM &CSD			Academic Year	2021 – 2022
Course Code	20CS303	Test Duration	3 Hrs.	Max. Marks	70	Semester	III
Course	Database Management Systems						

PART –A(Short answer questions 5X2=10 M)

1. What is data base management system ? give two examples

Ans: Definition – 1M

Examples – 1M

Database is a collection of related data and data is a collection of facts and figures that can be processed to produce information.

Ex : Telecom,Industry,Banking System, Sales, Airlines and Education sector

2. List out two aggregate operators

Ans)

The aggregation operators perform mathematical operations like Average, Aggregate, Count, Max, Min and Sum, on the numeric property of the elements in the collection. Any two of the following

Method	Description
Aggregate	Performs a custom aggregation operation on the values in the collection.
Average	calculates the average of the numeric items in the collection.
Count	Counts the elements in a collection.
LongCount	Counts the elements in a collection.
Max	Finds the largest value in the collection.
Min	Finds the smallest value in the collection.
Sum	Calculates sum of the values in the collection.

3. What is the difference between drop delete and truncate in SQL

Ans) What is the difference between DELETE DROP and TRUNCATE in SQL?

DROP and TRUNCATE are DDL commands, whereas DELETE is a DML command. DELETE operations can be rolled back (undone), while DROP and TRUNCATE operations **cannot be rolled back**.

4. What is ISAM ?

ISAM (an acronym for **Indexed sequential access method**) is a method for creating, maintaining, and manipulating computer files of data so that records can be retrieved sequentially or randomly by one or more keys.

5. State two phase locking protocols

Every **transaction will lock and unlock the data item** in two different phases. Growing Phase – All the locks are issued in this phase. No locks are released, after all changes to data-items are committed and then the second phase (shrinking phase) starts.

A transaction is said to follow the **Two-Phase Locking protocol** if Locking and Unlocking can be done in **two phases**.

Part –B (long questions 5 x 12 =60 Marks)

6.(a) Compare database system and file system 6M

Ans)

DBMS	File System
DBMS is a collection of data. In DBMS, the user is not required to write the procedures.	File system is a collection of data. In this system, the user has to write the procedures for managing the database.
DBMS gives an abstract view of data that hides the details.	File system provides the detail of the data representation and storage of data.
DBMS provides a crash recovery mechanism, i.e., DBMS protects the user from the system failure.	File system doesn't have a crash mechanism, i.e., if the system crashes while entering some data, then the content of the file will lost.
DBMS provides a good protection mechanism.	It is very difficult to protect a file under the file system.
DBMS contains a wide variety of sophisticated techniques to store and retrieve the data.	File system can't efficiently store and retrieve the data.
DBMS takes care of Concurrent access of data using some form of locking.	In the File system, concurrent access has many problems like redirecting the file while other deleting some information or updating some information.

6 b)

Explain the difference roles performed by the database administrator 6 M

Ans) A DBA has many responsibilities. A good-performing database is in the hands of DBA.

- **Installing and upgrading the DBMS Servers:** – DBA is responsible for installing a new DBMS server for the new projects. He is also responsible for upgrading these servers as there are new versions that come into the market or requirement. If there is any failure in the up-gradation of the existing servers, he should be able to revert the new changes back to the older version, thus maintaining the DBMS working. He is also responsible for updating the service packs/ hotfixes/ patches to the DBMS servers.
- **Design and implementation:** – Designing the database and implementing is also DBA's responsibility. He should be able to decide on proper memory management, file organizations, error handling, log maintenance, etc for the database.
- **Performance tuning:** – Since the database is huge and it will have lots of tables, data, constraints, and indices, there will be variations in the performance from time to time. Also, because of some designing issues or data growth, the database will not work as expected. It is the responsibility of the DBA to tune the database performance. He is responsible to make sure all the queries and programs work in a fraction of seconds.
- **Migrate database servers:** – Sometimes, users using oracle would like to shift to SQL server or Netezza. It is the responsibility of DBA to make sure that migration happens without any failure, and there is no data loss.
- **Backup and Recovery:** – Proper backup and recovery programs needs to be developed by DBA and has to be maintained him. This is one of the main responsibilities of DBA. Data/objects should be backed up regularly so that if there is any crash, it should be recovered without much effort and data loss.
- **Security:** – DBA is responsible for creating various database users and roles, and giving them different levels of access rights.
- **Documentation:** – DBA should be properly documenting all his activities so that if he quits or any new DBA comes in, he should be able to understand the database without any effort. He should basically maintain all his installation, backup, recovery, security methods. He should keep various reports about database performance.

Or

7(a) Explain E-R diagram components and notation with their extended features 6M

Ans) An ER diagram shows the relationship among entity sets. An entity set is a group of similar entities and these entities can have attributes. In terms of DBMS, an entity is a table or attribute of a table in database, so by showing relationship among tables and their attributes, ER diagram shows the complete logical structure of a database.

Rectangle: Represents Entity sets.

Ellipses: Attributes

Diamonds: Relationship Set

Lines: They link attributes to Entity Sets and Entity sets to Relationship Set

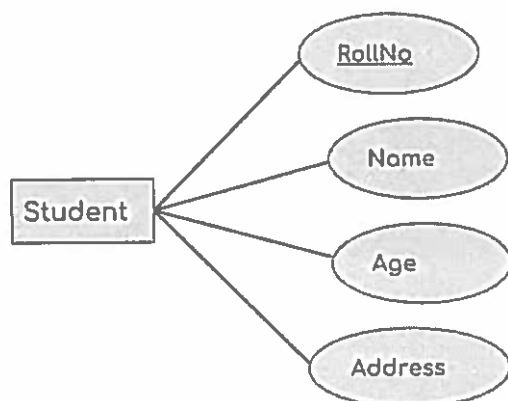
Double Ellipses: Multivalued Attributes

Dashed Ellipses: Derived Attributes

Double Rectangles: Weak Entity Sets

Double Lines: Total participation of an entity in a relationship set

Example of an ER Diagram for student entity



7(b) Define data abstraction and levels of data abstraction

Ans) **Data Abstraction** is a process of hiding unwanted or irrelevant details from the end user. It provides a different view and helps in achieving data independence which is used to enhance the security of data.

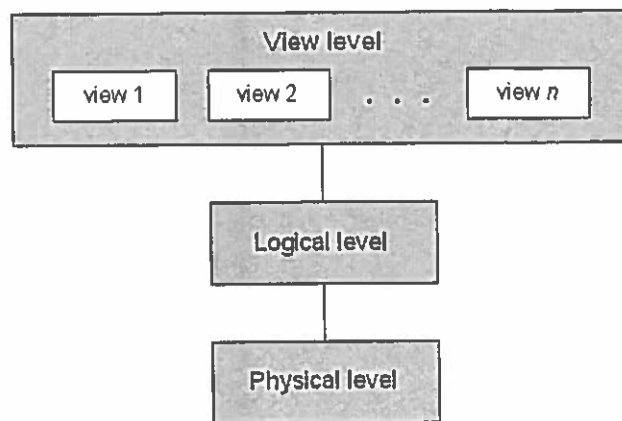
The database systems consist of complicated data structures and relations. For users to access the data easily, these complications are kept hidden, and only the relevant part of the database is made accessible to the users through data abstraction.

Levels of abstraction for DBMS

Database systems include complex data-structures. In terms of retrieval of data, reduce complexity in terms of usability of users and in order to make the system efficient, developers use levels of abstraction that hide irrelevant details from the users. Levels of abstraction simplify database design.

Mainly there are three levels of abstraction for DBMS, which are as follows –

- Physical or Internal Level
- Logical or Conceptual Level
- View or External Level



Physical or Internal Level

It is the lowest level of abstraction for DBMS which defines how the data is actually stored, it defines data-structures to store data and access methods used by the database. Actually, it is decided by developers or database application programmers how to store the data in the database.

So, overall, the entire database is described in this level that is physical or internal level. It is a very complex level to understand. For example, customer's information is stored in tables and data is stored in the form of blocks of storage such as bytes, gigabytes etc.

Logical or Conceptual Level

Logical level is the intermediate level or next higher level. It describes what data is stored in the database and what relationship exists among those data. It tries to describe the entire or whole data because it describes what tables to be created and what are the links among those tables that are created.

It is less complex than the physical level. Logical level is used by developers or database administrators (DBA). So, overall, the logical level contains tables (fields and attributes) and relationships among table attributes.

View or External Level

It is the highest level. In view level, there are different levels of views and every view only defines a part of the entire data. It also simplifies interaction with the user and it provides many views or multiple views of the same database.

View level can be used by all users (all levels' users). This level is the least complex and easy to understand.

For example, a user can interact with a system using GUI that is view level and can enter details at GUI or screen and the user does not know how data is stored and what data is stored, this detail is hidden from the user.

8) explain various operations in relational algebra with examples - 12 M

Ans) **RELATIONAL ALGEBRA** is a widely used procedural query language. It collects instances of relations as input and gives occurrences of relations as output.

Types – 2 M

Explanation of any 5 of the following with example – 10 M

1. Relational Algebra
2. SELECT(σ)
3. Projection(π)
4. Rename (ρ)
5. Union operation ($\cup$)
6. Set Difference ($-$)
7. Intersection
8. Cartesian product($\times$)
9. Join Operations
10. Inner Join:
11. Theta Join:
12. EQUI join:
13. NATURAL JOIN ($\bowtie$)
14. OUTER JOIN
15. Left Outer Join($A \ltimes B$)
16. Right Outer Join: ($A \ltimes B$)
17. Full Outer Join: ($A \ltimes B$)

SELECT (σ)

The SELECT operation is used for selecting a subset of the tuples according to a given selection condition. Sigma(σ) Symbol denotes it. It is used as an expression to choose tuples which meet the selection condition. Select operator selects tuples that satisfy a given predicate.

$\sigma_p(r)$

σ is the predicate

r stands for relation which is the name of the table

p is propositional logic

Example 1

$\sigma_{\text{topic} = \text{'Database'}}(\text{Tutorials})$

Output – Selects tuples from Tutorials where topic = 'Database'.

Projection(π)

The projection eliminates all attributes of the input relation but those mentioned in the projection list. The projection method defines a relation that contains a vertical subset of Relation.

This helps to extract the values of specified attributes to eliminate duplicate values. (π) symbol is used to choose attributes from a relation. This operator helps you to keep specific columns from a relation and discards the other columns.

Ex : $\pi_{\text{CustomerName, Status}}(\text{Customers})$

Rename (ρ)

Rename is a unary operation used for renaming attributes of a relation.

$\rho_{(a/b)}R$ will rename the attribute 'b' of relation by 'a'.

ex: The result $\leftarrow A \cup B$

For a union operation to be valid, the following conditions must hold –

- R and S must be the same number of attributes.
- Attribute domains need to be compatible.
- Duplicate tuples should be automatically removed.

Set Difference ($-$)

– Symbol denotes it. The result of $A - B$, is a relation which includes all tuples that are in A but not in B.

- The attribute name of A has to match with the attribute name in B.
- The two-operand relations A and B should be either compatible or Union compatible.
- It should be defined relation consisting of the tuples that are in relation A, but not in B.

Example

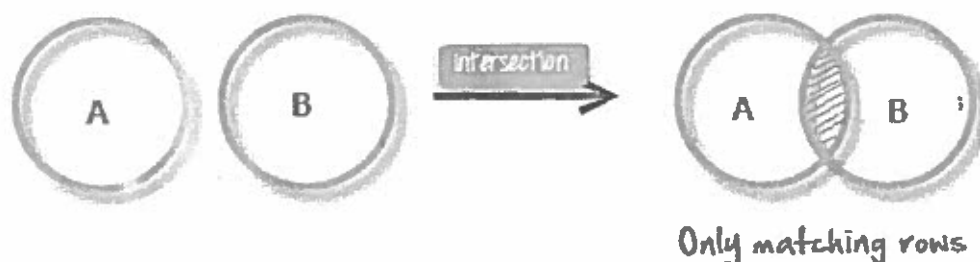
Ex : $A - B$

Intersection

An intersection is defined by the symbol $\cap$

$A \cap B$

Defines a relation consisting of a set of all tuple that are in both A and B. However, A and B must be union-compatible.



Ex: $A \cap B$

Cartesian Product(X) in DBMS

Cartesian Product in DBMS is an operation used to merge columns from two relations. Generally, a cartesian product is never a meaningful operation when it performs alone. However, it becomes meaningful when it is followed by other operations. It is also called Cross Product or Cross Join.

Example – Cartesian product

$\sigma_{\text{column 2} = '1'}(A \times B)$

9(a) write about views and updates on views – 6M

Ans : Definition with syntax and example – 4 M

Updating of view with syntax and example – 2 M

Views in SQL are considered as a **virtual table**. A view also contains rows and columns. To create the view, we can select the fields from one or more tables present in the database. A view can either have specific rows based on certain condition or all the rows of a table.

Syntax :

Student_Detail

STU_ID	NAME	ADDRESS
1	Stephan	Delhi
2	Kathrin	Noida
3	David	Ghaziabad
4	Alina	Gurugram

Student_Marks

STU_ID	NAME	MARKS	AGE

1	Stephan	97	19
2	Kathrin	86	21
3	David	74	18
4	Alina	90	20
5	John	96	18

1. Creating view

A view can be created using the **CREATE VIEW** statement. We can create a view from a single table or multiple tables.

Syntax: e Ministers of India | List of Prime Minister of India (1947-2020)

```
CREATE VIEW view_name AS
SELECT column1, column2,....
FROM table_name
WHERE condition;
```

2. Creating View from a single table

In this example, we create a View named DetailsView from the table Student_Detail.

Query:

```
CREATE VIEW DetailsView AS
SELECT NAME, ADDRESS
FROM Student_Details
WHERE STU_ID < 4;
```

Just like table query, we can query the view to view the data.

```
SELECT * FROM DetailsView;
```

Output:

NAME	ADDRESS
Stephan	Delhi
Kathrin	Noida
David	Ghaziabad

Updates on views

4. Deleting View

A view can be deleted using the Drop View statement.

Syntax

DROP VIEW view_name;

Example:

If we want to delete the View **MarksView**, we can do this as:

DROP VIEW MarksView;

9(b) what are the various integrity constraint relations of relational model ? – 6M

Definition and types 4 M

Example of any one following constraint – 2M

Database integrity refers to the validity and consistency of stored data. Integrity is usually expressed in terms of constraints, which are consistency rules that the database is not permitted to violate. Constraints may apply to each attribute or they may apply to relationships between tables.

TYPES OF INTEGRITY CONSTRAINTS

Various types of integrity constraints are-

1. Domain Integrity
2. Entity Integrity Constraint
3. Referential Integrity Constraint
4. Key Constraints

Domain Integrity-

Domain integrity means the definition of a valid set of values for an attribute. You define data type, length or size, is null value allowed, is the value unique or not for an attribute, the default value, the range (values in between) and/or specific values for the attribute.

Entity Integrity Constraint-

This rule states that in any database relation value of attribute of a primary key can't be null.

EXAMPLE- Consider a relation "STUDENT" Where "Stu_id" is a primary key and it must not contain any null value whereas other attributes may contain null value e.g "Branch" in the following relation contains one null value.

Stu_id	Name	Branch
11255234	Aman	CSE
11255369	Kapil	EcE

11255324	Ajay	ME
11255237	Raman	CSE
11255678	Aastha	ECE

3.Referential Integrity Constraint-

It states that if a foreign key exists in a relation then either the foreign key value must match a primary key value of some tuple in its home relation or the foreign key value must be null.

2. Unique Constraints

A unique column constraint in a table is similar to a primary key in that the value in that column for every row of data in the table must have a unique value. Although a primary key constraint is placed on one column, you can place a unique constraint on another column even though it is not actually for use as the primary key.

```
CREATE TABLE EMPLOYEE_TBL
(EMP_ID    CHAR(9)    NOT NULL    PRIMARY KEY,
EMP_NAME   VARCHAR (40) NOT NULL,
EMP_ST_ADDR VARCHAR (20) NOT NULL,
EMP_CITY   VARCHAR (15) NOT NULL,
EMP_ST     CHAR(2)    NOT NULL,
EMP_ZIP    INTEGER(5) NOT NULL,
EMP_PHONE  INTEGER(10) NULL    UNIQUE,
EMP_PAGER  INTEGER(10) NULL)
```

3. Foreign Key Constraints

A foreign key is a column in a child table that references a primary key in the parent table. A foreign key constraint is the main mechanism used to enforce referential integrity between tables in a relational database. A column defined as a foreign key is used to reference a column defined as a primary key in another table.

```
CREATE TABLE EMPLOYEE_PAY_TBL
(EMP_ID    CHAR(9)    NOT NULL,
POSITION   VARCHAR2(15) NOT NULL,
DATE_HIRE  DATE       NULL,
PAY_RATE   NUMBER(4,2) NOT NULL,
DATE_LAST_RAISE DATE   NULL,
```

4. NOT NULL Constraints

Previous examples use the keywords NULL and NOT NULL listed on the same line as each column and after the data type. NOT NULL is a constraint that you can place on a table's column. This constraint disallows the entrance of

NULL values into a column; in other words, data is required in a NOT NULL column for each row of data in the table. NULL is generally the default for a column if NOT NULL is not specified, allowing NULL values in a column.

5. Check Constraints

Check (CHK) constraints can be utilized to check the validity of data entered into particular table columns. Check constraints are used to provide back-end database edits, although edits are commonly found in the front-end application as well. General edits restrict values that can be entered into columns or objects, whether within the database itself or on a front-end application. The check constraint is a way of providing another protective layer for the data.

```
CREATE TABLE EMPLOYEE_TBL
(EMP_ID    CHAR(9)    NOT NULL,
EMP_NAME   VARCHAR2(40) NOT NULL,
EMP_ST_ADDR VARCHAR2(20) NOT NULL,
EMP_CITY   VARCHAR2(15) NOT NULL,
EMP_ST     CHAR(2)    NOT NULL,
EMP_ZIP    NUMBER(5)  NOT NULL,
EMP_PHONE  NUMBER(10) NULL,
EMP_PAGER  NUMBER(10) NULL),
PRIMARY KEY (EMP_ID),
CONSTRAINT CHK_EMP_ZIP CHECK ( EMP_ZIP = '46234');
```

10(a) What are null values ? discuss about disallowing null values - 6 M

Ans :

Explanation for null values with example : 4 M

Disallowing Null Values : 2M

The fields can take on null values if they are not declared as not null. We can restrict the insertion of null values for the field declaring that field as not null. It means the field cannot take null values. For primary key constraint, the field which is declared as primary key is also declared as not null. It is done implicitly in DBMS.

```
EX: CREATE TABLE student ( sid NOT NULL ,
                             Snamechar(10) NOT NULL ,
                             Project varchar(15),
                             Class int,
                             Primary key (sid));
```

In this declaration , creation of student table , sid is the primary key hence it must be unique and it should be not null. We can restrict entering null values in the field.

b) Define a nested query with example – 6 M

definition : 2 M

syntax with Example – 4M

A Nested query or Subquery or Inner query is a query within another SQL query and embedded within the WHERE clause.

Nested Query –

In Nested Query, Inner query runs first, and only once. Outer query is executed with result from Inner query. Hence, Inner query is used in execution of Outer query.

Syntax :

```
SELECT column_name [, column_name ]  
FROM table1 [, table2 ]  
WHERE column_name OPERATOR  
      (SELECT column_name [, column_name ]  
       FROM table1 [, table2 ]  
       [WHERE])
```

Example –

Orders (OrderID, CustomerID, OrderDate);

Customers (CustomerID, CustomerName, ContactName, Country);

Query : Find details of customers who have ordered.

```
SELECT * FROM Customers WHERE  
CustomerID IN (SELECT CustomerID FROM Orders);
```

11 (a) illustrate Trigger with example – 6 M

Definition – 2M

Explanation with syntax and example – 4 M

Ans) **TRIGGERS**

Trigger is invoked by Oracle engine automatically whenever a specified event occurs. Trigger is stored into database and invoked repeatedly, when specific condition match.

Triggers are stored programs, which are automatically executed or fired when some event occurs.

Triggers are written to be executed in response to any of the following events.

- A database manipulation (DML) statement (DELETE, INSERT, or UPDATE).
- A database definition (DDL) statement (CREATE, ALTER, or DROP).
- A database operation (SERVERERROR, LOGON, LOGOFF, STARTUP, or SHUTDOWN).

Triggers could be defined on the table, view, schema, or database with which the event is associated.

Creating a trigger:

Syntax for creating trigger:

```
CREATE [OR REPLACE] TRIGGER trigger_name
{BEFORE | AFTER | INSTEAD OF }
{INSERT [OR] | UPDATE [OR] | DELETE}
[OF col_name]
ON table_name
[REFERENCING OLD AS o NEW AS n]
[FOR EACH ROW]
WHEN (condition)
DECLARE
    Declaration-statements
BEGIN
    Executable-statements
EXCEPTION
    Exception-handling-statements
END;
```

Let's take a simple example to demonstrate the trigger. In this example, we are using the following employees table:

Create table and have records:

ID	NAME	AGE	ADDRESS	SALARY
1	Ramesh	23	Allahabad	20000
2	Suresh	22	Kanpur	22000
3	Mahesh	24	Ghaziabad	24000
4	Chandan	25	Noida	26000
5	Alex	21	Paris	28000
6	Sunita	20	Delhi	30000

Create trigger:

Let's take a program to create a row level trigger for the CUSTOMERS table that would fire for INSERT or UPDATE or DELETE operations performed on the CUSTOMERS table. This trigger will display the salary difference between the old values and new values:

```
CREATE OR REPLACE TRIGGER display_salary_changes
BEFORE DELETE OR INSERT OR UPDATE ON customers
FOR EACH ROW
WHEN (NEW.ID > 0)
DECLARE
    sal_diff number;
```

BEGIN

```
sal_diff := :NEW.salary - :OLD.salary;  
dbms_output.put_line('Old salary: ' || :OLD.salary);  
dbms_output.put_line('New salary: ' || :NEW.salary);  
dbms_output.put_line('Salary difference: ' || sal_diff);
```

END;

/

After the execution of the above code at SQL Prompt, it produces the following result.

Trigger created.

Check the salary difference by procedure:

UPDATE customers

SET salary = salary + 5000;

Output:

Old salary: 20000
New salary: 25000

11(a) Discuss different types of aggregate functions with examples in SQL.

Ans: Types – 1 M

Examples to each one – 5M

The following are the most commonly used SQL aggregate functions:

AVG – calculates the average of a set of values.

COUNT – counts rows in a specified table or view.

MIN – gets the minimum value in a set of values.

MAX – gets the maximum value in a set of values.

SUM – calculates the sum of values.

PRODUCT	COMPANY	QTY	RATE	COST
Item1	Com1	2	10	20
Item2	Com2	3	25	75
Item3	Com1	2	30	60
Item4	Com3	5	10	50
Item5	Com2	2	20	40
Item6	Cpm1	3	25	75
Item7	Com1	5	30	150

Item8	Com1	3	10	30
Item9	Com2	2	25	50
Item10	Com3	4	30	120

COUNT FUNCTION

- COUNT function is used to Count the number of rows in a database table. It can work on both numeric and non-numeric data types.
- COUNT function uses the COUNT(*) that returns the count of all the rows in a specified table. COUNT(*) considers duplicate and Null.

Syntax

1. COUNT()
2. or
3. COUNT([ALL|DISTINCT] expression)

Example: COUNT()

Skip Ad

1. SELECT COUNT(*)
2. FROM PRODUCT_MAST;

Output:

102. SUM Function

Sum function is used to calculate the sum of all selected columns. It works on numeric fields only.

Syntax

1. SUM()
2. or
3. SUM([ALL|DISTINCT] expression)

Example: SUM()

1. SELECT SUM(COST)
2. FROM PRODUCT_MAST;

Output:

3. AVG function

The AVG function is used to calculate the average value of the numeric type. AVG function returns the average of all non-Null values.

Syntax

1. AVG()
2. or
3. AVG([ALL|DISTINCT] expression)

Example:

1. SELECT AVG(COST)
2. FROM PRODUCT_MAST;

Output:

67.00

4. MAX Function

MAX function is used to find the maximum value of a certain column. This function determines the largest value of all selected values of a column.

Syntax

1. MAX()
2. or
3. MAX([ALL|DISTINCT] expression)

Example:

1. SELECT MAX(RATE)
2. FROM PRODUCT_MAST;

80

5. MIN Function

MIN function is used to find the minimum value of a certain column. This function determines the smallest value of all selected values of a column.

Syntax

1. MIN()
2. or
3. MIN([ALL|DISTINCT] expression)

Example:

1. SELECT MIN(RATE)
2. FROM PRODUCT_MAST;

Output:

10

11(a)

Define functional dependencies. How are primary keys related to FD's? - 6 M

Ans) Definition - 3 M

Explanation - 3M

Functional Dependency (FD) is a constraint that determines the relation of one attribute to another attribute in a Database Management System (DBMS). Functional Dependency helps to maintain the quality of data in the database. It plays a vital role to find the difference between good and bad database design.

A functional dependency is denoted by an arrow " $\rightarrow$ ". The functional dependency of X on Y is represented by $X \rightarrow Y$. Let's understand Functional Dependency in DBMS with example.

Example:

Employee number	Employee Name	Salary	City
1	Dana	50000	San Francisco
2	Francis	38000	London
3	Andrew	25000	Tokyo

In this example, if we know the value of Employee number, we can obtain Employee Name, city, salary, etc. By this, we can say that the city, Employee Name, and salary are functionally depended on Employee number.

How are primary keys related to FD's

A table's primary key is **used to establish relationships with other tables**. This is achieved by including the primary key of one table as the Foreign Key of another. The general rule of thumb is to take the primary key of the one side of the relationship and include it in the table of the many side of the relationship.

Or

Here is by definition a functional dependency between **all superkeys** (not just the primary key) of a relation and all attributes in that relation (not just non-key attributes). In a table that is at least in 1st normal form, the primary key determines every attribute in the table.

12 (b) Explain Dense and Sparse indices.

Ans :

Definition : 2M

Explanation : 4 M

Definition : Dense Index: It has index entries for every search key value (and hence every record) in the database file. The dense index can be built on order as well as unordered fields of the database files.

Sparse Index: It has index entries for only some of the search key values/records in the database file.

Ordered Indexing is of two types -

- Dense Index
- Sparse Index

Dense Index

In dense index, there is an index record for every search key value in the database. This makes searching faster but requires more space to store index records itself. Index records contain search key value and a pointer to the actual record on the disk.



Sparse Index

In sparse index, index records are not created for every search key. An index record here contains a search key and an actual pointer to the data on the disk. To search a record, we first proceed by index record and reach at the actual location of the data. If the data we are looking for is not where we directly reach by following the index, then the system starts sequential search until the desired data is found.



13) Define normalization, why do we need Normalization? Explain 1NF, 2NF, 3NF Normal forms – 12 M

Ans :

Definition : 2 M

Explanation with example: 4 M

SCHEMA REFINEMENT / NORMALIZATION: schema refinement is intended to use a refinement approach based on decomposition. By we can reduce data redundancy.

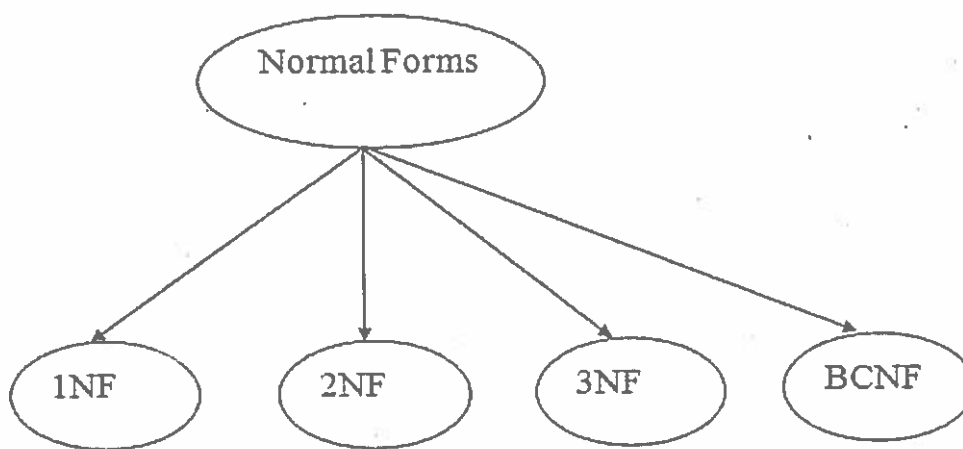
Normalization

- Normalization is the process of organizing the data in the database.

- Normalization is used to minimize the redundancy from a relation or set of relations. It is also used to eliminate the undesirable characteristics like Insertion, Update and Deletion Anomalies.
- Normalization divides the larger table into the smaller table and links them using relationship.
- The normal form is used to reduce redundancy from the database table.

Types of Normal Forms

There are the four types of normal forms:



Normal Form	Description
<u>1NF</u>	A relation is in 1NF if it contains an atomic value.
<u>2NF</u>	A relation will be in 2NF if it is in 1NF and all non-key attributes are fully functional dependent on the primary key.
<u>3NF</u>	A relation will be in 3NF if it is in 2NF and no transitive dependency exists.

First Normal Form (1NF)

- A relation will be 1NF if it contains an atomic value.
- It states that an attribute of a table cannot hold multiple values. It must hold only single-valued attribute.

- First normal form disallows the multi-valued attribute, composite attribute, and their combinations.

Example: Relation EMPLOYEE is not in 1NF because of multi-valued attribute EMP_PHONE.

EMPLOYEE table:

EMP_ID	EMP_NAME	EMP_PHONE	EMP_STATE
14	John	7272826385, 9064738238	UP
20	Harry	8574783832	Bihar
12	Sam	7390372389, 8589830302	Punjab

The decomposition of the EMPLOYEE table into 1NF has been shown below:

EMP_ID	EMP_NAME	EMP_PHONE	EMP_STATE
14	John	7272826385	UP
14	John	9064738238	UP
20	Harry	8574783832	Bihar
12	Sam	7390372389	Punjab
12	Sam	8589830302	Punjab

Second Normal Form (2NF)

- In the 2NF, relational must be in 1NF.
- In the second normal form, all non-key attributes are fully functional dependent on the primary key

Example: Let's assume, a school can store the data of teachers and the subjects they teach. In a school, a teacher can teach more than one subject.

TEACHER table

TEACHER_ID	SUBJECT	TEACHER_AGE
25	Chemistry	30
25	Biology	30

47	English	35
83	Math	38
83	Computer	38

In the given table, non-prime attribute TEACHER_AGE is dependent on TEACHER_ID which is a proper subset of a candidate key. That's why it violates the rule for 2NF.

Third Normal Form (3NF)

- A relation will be in 3NF if it is in 2NF and not contain any transitive partial dependency.
- 3NF is used to reduce the data duplication. It is also used to achieve the data integrity.
- If there is no transitive dependency for non-prime attributes, then the relation must be in third normal form.

A relation is in third normal form if it holds atleast one of the following conditions for every non-trivial function dependency $X \rightarrow Y$.

1. X is a super key.
2. Y is a prime attribute, i.e., each element of Y is part of some candidate key.

Example:

EMPLOYEE_DETAIL table:

EMP_ID	EMP_NAME	EMP_ZIP	EMP_STATE	EMP_CITY
222	Harry	201010	UP	Noida
333	Stephan	02228	US	Boston
444	Lan	60007	US	Chicago
555	Katharine	06389	UK	Norwich
666	John	462007	MP	Bhopal

Super key in the table above:

1. {EMP_ID}, {EMP_ID, EMP_NAME}, {EMP_ID, EMP_NAME, EMP_ZIP}....so on

Candidate key: {EMP_ID}

Non-prime attributes: In the given table, all attributes except EMP_ID are non-prime.

Here, EMP_STATE & EMP_CITY dependent on EMP_ZIP and EMP_ZIP dependent on EMP_ID. The non-prime attributes (EMP_STATE, EMP_CITY) transitively dependent on super key(EMP_ID). It violates the rule of third normal form.

That's why we need to move the EMP_CITY and EMP_STATE to the new <EMPLOYEE_ZIP> table, with EMP_ZIP as a Primary key.

EMPLOYEE table:

EMP_ID	EMP_NAME	EMP_ZIP
222	Harry	201010
333	Stephan	02228
444	Lan	60007
555	Katharine	06389
666	John	462007

EMPLOYEE_ZIP table:

EMP_ZIP	EMP_STATE	EMP_CITY
201010	UP	Noida
02228	US	Boston
60007	US	Chicago
06389	UK	Norwich
462007	MP	Bhopal

14 (a) Explain ACID properties and Illustrate them through examples. 6M

Definition -2M

Elobaration -4 M

In the context of transaction processing, the acronym ACID refers to the four key properties of a transaction: **atomicity, consistency, isolation, and durability**. All changes to data are performed as if they are a single operation. That is, all the changes are performed, or none of them are.

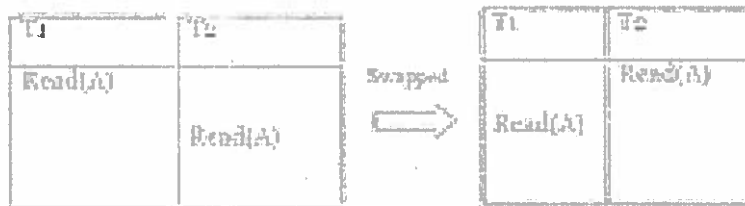
14 (b) Discuss about conflict Serializability with an example. -6M

Definition and example – 6 M

conflict serializability

Conflict Serializable: A schedule is called conflict serializable if it can be transformed into a serial schedule by swapping non-conflicting operations. **Conflicting operations:** Two operations are said to be conflicting if all conditions satisfy: They belong to different transactions. They operate on the same data item

1. T₁: Read(A) T₂: Read(A)



Schedule S₁

Schedule S₂

A schedule is called conflict serializability if after swapping of non-conflicting operations, it can transform into a serial schedule. The schedule will be a conflict serializable if it is conflict equivalent to a serial schedule.

15 a) What is ARIES and list the phases of ARIES

Definition and explanation – 6 M

The ARIES recovery procedure consists of three main steps: **analysis**, **REDO**, and **UNDO**. The analysis step identifies the dirty (updated) pages in the buffer and the set of transactions active at the time of the crash. The appropriate point in the log where the REDO operation should start is also determined.

(a)

Len	Last_Len	Tran_id	Type	Page_id	Other_information
1	0	T ₁	update	C	...
2	0	T ₂	update	B	...
3	1	T ₃	commit		...
4	begin checkpoint				
5	end checkpoint				
6	0	T ₃	update	A	...
7	2	T ₃	update	C	...
8	7	T ₃	commit		...

(b)

Transaction_id	Last_Len	Status
T ₁	3	commit
T ₃	2	in progress

Page_id	Len
C	1
B	2

(b)

Transaction_id	Last_Len	Status
T ₁	3	commit
T ₃	2	in progress

Page_id	Len
C	1
B	2

(c)

Transaction_id	Last_Len	Status
T ₁	3	commit
T ₂	8	commit
T ₃	6	in progress

Page_id	Len
C	1
B	2
A	6

15 (b) Discuss about various database recovery approaches.- 6 M

Explanation of any one method - 6 M

There are four different recovery techniques are available in the Database.

6. Mirroring.
7. Recovery using Backups.
8. Recovery using Transaction Logs.
9. Shadow Paging.

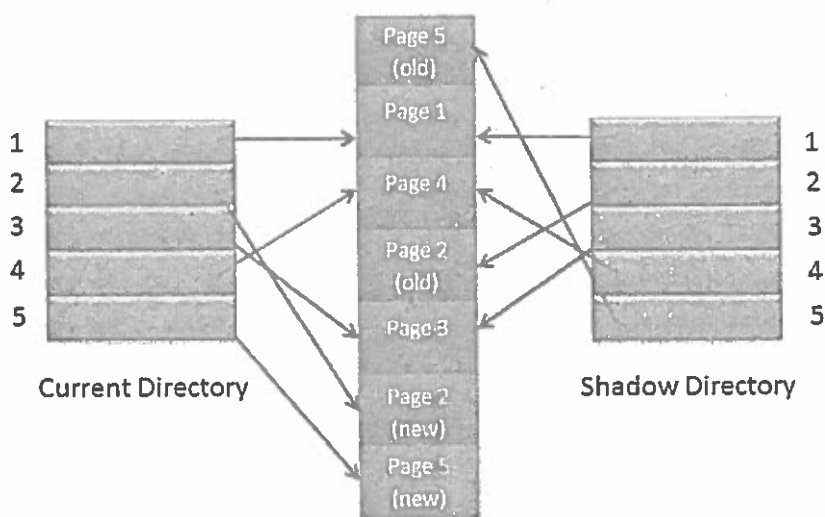


Fig: Shadow Paging Scheme

Semester End Examination, January/February 2022

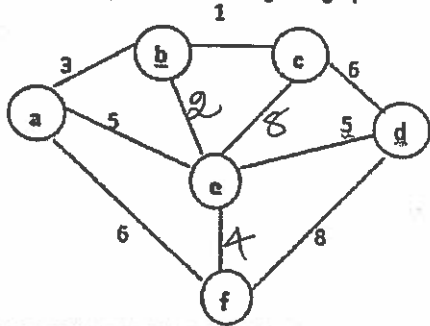
Degree	B. Tech. (U. G.)	Program	CSE	Academic Year	2021 - 2022
Course Code	20CS302	Test Duration	3 Hrs.	Max. Marks	70
Course	Design and Analysis of Algorithms	Semester	III		

Part A (Short Answer Questions 5 x 2 = 10 Marks)

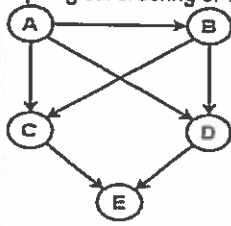
No.	Questions (1 through 5)	Learning Outcome (s)	DoK
1	Define time complexity and space complexity	20CS302.1	L1
2	Write an algorithm to find the sum of squares of first n integers	20CS302.2	L2
3	Compare divide and conquer strategy with dynamic programming.	20CS302.3	L2
4	What are NP-hard problems?	20CS302.4	L1
5	State the reason for terminating the search path at the current node in the branch and bound algorithm	20CS302.5	L2

Part B (Long Answer Questions 5 x 12 = 60 Marks)

No.	Questions (6 through 15)	Marks	Learning Outcome (s)	DoK
6 (a)	Explain asymptotic classes with examples.	6M	20CS302.2	L2
6 (b)	Write a recursive algorithm to find the factorial of a given number and analyze the time and space complexity.	6M	20CS302.2	L2
OR				
7 (a)	Write a linear search algorithm and analyze the efficiency of the algorithm	6M	20CS302.2	L2
7 (b)	Write the bubble sort algorithm and analyze its complexity	6M	20CS302.2	L2

Write the Prim's algorithm and using Prim's algorithm, determine the minimum cost spanning tree for the weighted graph shown below.				
8 (a)		12M	20CS302.4	L3

OR				
9 (a)	Find an optimal solution to the knapsack instance n=4 objects and the capacity of knapsack m=15, profits (10, 5, 7, 11) and weight are (3, 4, 3, 5).	12M	20CS302.4	L3
10 (b)	Write a quick sort algorithm and trace this algorithm for n = 8 elements 24, 12, 35, 23, 45, 34, 20, 48	12M	20CS302.4	L3

OR				
11 (a)	Write the topological sorting algorithm and apply it to find the topological ordering of vertices in the following graph	12M	20CS302.4	L3
				



N S RAJU INSTITUTE OF TECHNOLOGY

(AUTONOMOUS)

SONTYAM, ANANDAPURAM, VISAKHAPATNAM - 531 173

ANSWER KEY AND SCHEME OF EVALUATION

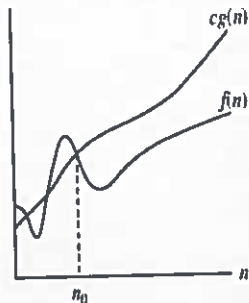
Part A

1. Define time complexity and space complexity
Time complexity is the total amount of CPU time taken to get the correct output.
Space complexity is the total amount of memory space required by the algorithm
2. Write the algorithm to find the sum of squares of first n integers
Algorithm: SumSq(n)
Input: no of integers n
Output: sum of squares of first n integers
Begin
 S=0
 For I = 1 to n do
 S = S + i*i
 Return s
3. Compare divide and conquer strategy with dynamic programming
In divide and conquer the given problem is divided into sub problems; sub problems are solved and solutions are integrated to get solution to given problem.
In dynamic programming also the given problem is divided into sub problems. But sub problems are repeated and are solved only once.
4. What is NP hard problems?
Problems that could not be solve in polynomial time are called NP hard problems;
Only few instances of the problems can be solved with polynomial time
5. State the reason for terminating the search path at the current node in the branch and bound algorithm
If the bound value of the current node is not better than the current best solution, the node is not evaluated.

Part B

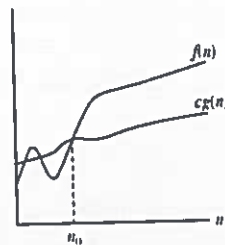
6. a. Asymptotic classes

$O(g(n)) = \{f(n) : \text{there exist positive constants } c \text{ and } n_0 \text{ such that } 0 \leq f(n) \leq cg(n) \text{ for all } n \geq n_0\}$.



(2)

$\Omega(g(n)) = \{f(n) : \text{there exist positive constants } c \text{ and } n_0 \text{ such that } 0 \leq cg(n) \leq f(n) \text{ for all } n \geq n_0\}$.



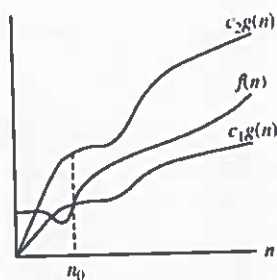
(2)

$g(n)$ is an **asymptotic upper bound** for $f(n)$.

$g(n)$ is an **asymptotic lower bound** for $f(n)$.

$\Theta(g(n)) = \{f(n) : \text{there exist positive constants } c_1, c_2, \text{ and } n_0 \text{ such that } 0 \leq c_1g(n) \leq f(n) \leq c_2g(n) \text{ for all } n \geq n_0\}$.

(2)



$g(n)$ is an **asymptotically tight bound** for $f(n)$.

b. Write the recursive algorithm to find the factorial of given number and find its time and space complexity

(2) Algorithm : RFact(n)
input : a number n
output : factorial of n
begin

if $n == 0$
return 1

else

return (RFact($n-1$) * n)

end

$$\therefore T(n) = n$$

(4) $T(n)$ = time complexity
Let $M(n)$ be no of multiplications require.

$$M(n) = M(n-1) + 1 \quad n > 0$$

$$M(0) = 0$$

$$M(n) = M(n-1) + 1$$

$$M(n-1) = M(n-2) + 1$$

$$M(n) = M(n-2) + 2$$

$$\therefore M(n) = M(n-i) + i$$

$$\text{put } i = n$$

$$M(n) = M(0) + n$$

$$M(n) = n$$

7. a Write the algorithm to perform linear search and analyze its efficiency

Algorithm: LinSearch(A, n, k) (4)
 input: array A and key k
 output: position of k if it is present
 and -1 otherwise
 begin

for i = 1 to n do
 if A[i] == k
 return i
 return -1

$$\begin{aligned} C_{\text{best}}(n) &= 1 & (2) \\ C_{\text{worst}}(n) &= n \\ C_{\text{avg}}(n) &= \frac{n(n+1)}{2} \end{aligned}$$

b. Write the bubble sort algorithm and find its complexity

Algorithm: BSort(A, n) (3)

begin
 for i = 0 to n-2 do
 for j = 0 to n-2-i do
 if A[j+1] < A[j]
 swap(A[j] and
 A[j+1])

end

$$\therefore T(n) \in \Theta(n^2)$$

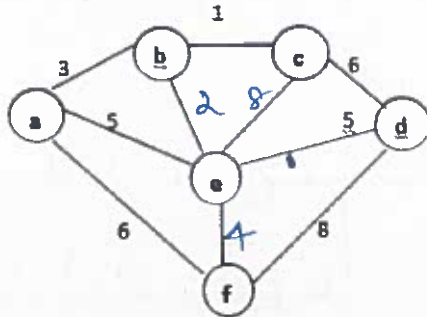
Analysis

Basic operation:
 Comparison

$C(n)$ = total no of
 comp. required

$$\begin{aligned} C(n) &= \sum_{i=0}^{n-2} \sum_{j=0}^{n-2-i} 1 \\ &= \sum_{i=0}^{n-2} (n-2-i-0+1) \\ &= \sum_{i=0}^{n-2} (n-i-1) \\ &= (n-1)(n-2)(n-3) + 1 \\ &= \frac{n(n-1)}{2} \end{aligned}$$

8. Write the Prim's algorithm and using Prim's algorithm, determine the minimum cost spanning tree for the weighted graph shown below.



ALGORITHM *Prim(G)*

//Prim's algorithm for constructing a minimum spanning tree
 //Input: A weighted connected graph $G = (V, E)$
 //Output: E_T , the set of edges composing a minimum spanning tree of G
 $V_T \leftarrow \{v_0\}$ //the set of tree vertices can be initialized with any vertex
 $E_T \leftarrow \emptyset$
 for $i \leftarrow 1$ to $|V| - 1$ do
 find a minimum-weight edge $e^* = (v^*, u^*)$ among all the edges (v, u)
 such that v is in V_T and u is in $V - V_T$
 $V_T \leftarrow V_T \cup \{u^*\}$
 $E_T \leftarrow E_T \cup \{e^*\}$
 return E_T

Tree set

a

a, b

a, b, c

a, b, c,

e

a, b, c, e, f

Fringe set

b(a, 3) c(-, ∞)

d(-, ∞) e(a, 5) f(a, 6)

c(b, 1) e(b, 2)

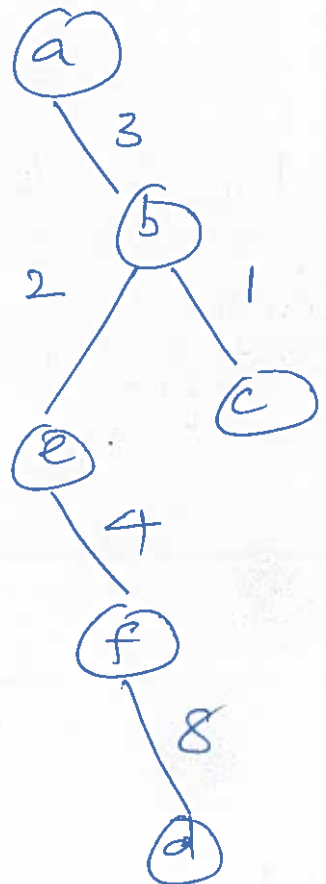
d(-, ∞) f(a, 6)

d(c, 6) e(b, 2)

f(a, 6)

d(e, 5)

f(e, 4)



Cost = 18

9. A. Find an optimal solution to the knapsack instance $n=4$ objects and the capacity of knapsack $m=15$, profits $(10, 5, 7, 11)$ and weight are $(3, 4, 3, 5)$.

$$S^0 = \{(0, 0)\} \quad S_1^0 = \{(10, 3)\} \quad \begin{matrix} (10, 3) \\ (5, 4) \\ (7, 3) \\ (11, 5) \end{matrix}$$

$$S^1 = \{(0, 0), (10, 3)\} \quad S_1^1 = \{(5, 4), (15, 7)\}$$

$$S^2 = \{(0, 0), (5, 4), (10, 3), (15, 7)\}$$

$$S_1^2 = \{(7, 3), (12, 7), (17, 6), (22, 10)\}$$

$$S^3 = \{(0, 0), \cancel{(5, 4)}, \cancel{(7, 3)}, (10, 3), \cancel{(15, 7)}, (17, 6), (22, 10)\}$$

$$S_1^3 = \{(11, 5), (21, 8), (28, 11), (33, 15)\}$$

$$S^4 = \{(0, 0), (10, 3), (11, 5), (17, 6), \cancel{(22, 10)}, (22, 10), (28, 11), (33, 15)\}$$

Profit : 33

Solution : $\begin{pmatrix} x_1 & x_2 & x_3 & x_4 \\ 1 & 1 & 1 & 1 \end{pmatrix}$

$$\begin{aligned} & (33, 15) - (11, 5) \\ & = (22, 10) \\ & (22, 10) - (7, 3) \\ & = (15, 7) \\ & (15, 7) - (5, 4) \\ & = (10, 3) \\ & (10, 3) - (10, 3) = (0, 0) \end{aligned}$$

10. Write a quick sort algorithm and trace this algorithm for n =8 elements
24,12, 35, 23, 45, 34,20, 48

pivot 24 12 35 23 45 34 20 48

↑
i

↑
j

24 12 20 23 45 34 35 48

↑ ↑

j i

23 12 20 (24) 45 34 35 48

Left right

20 12 23

12 20 23

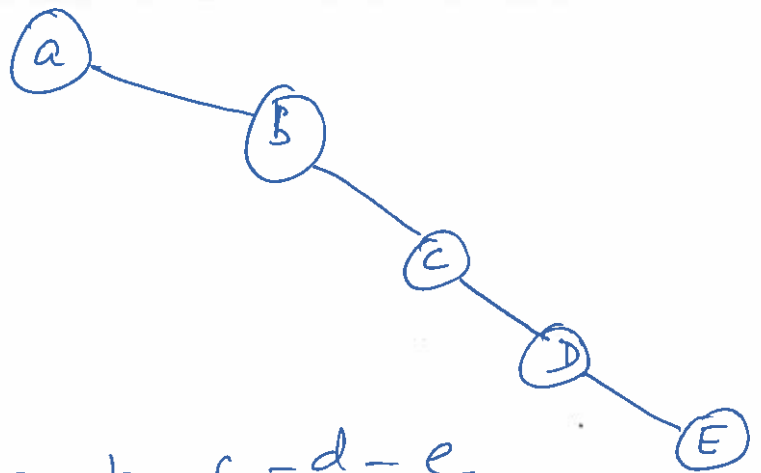
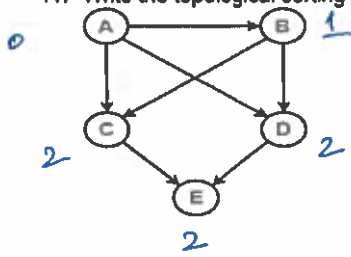
(45) 34 35 48
 ↑ ↑
 U ;

35 34 45 48

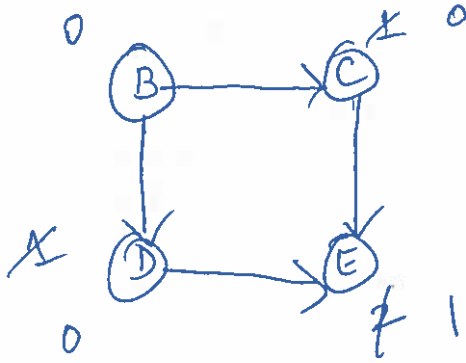
34 35 45 48

$\therefore$ 12 20 23 24 34 35 45 48

11. Write the topological sorting algorithm and apply it to find the topological ordering of vertices in the following graph



a-b-c-d-e



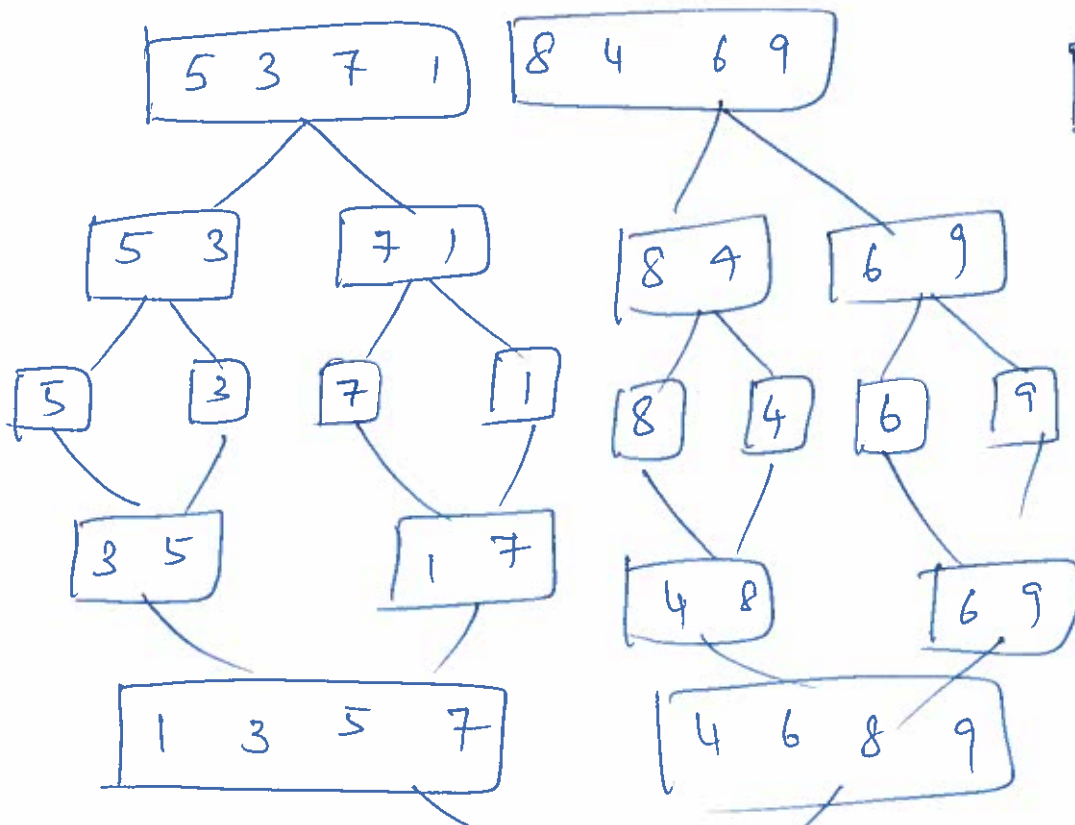
12. A. Describe NP-hard, NP-complete problems, and their characteristics. Give examples in graph problems

NP hard - non polynomial with few instances having solution

NP complete - NP hard of it having soln so as for NP complete

b. Sort the following set of numbers using merge sort algorithm

5, 3, 7, 1, 8, 4, 6, 9



1 3 4 5 6 7
8 9

13. State and explain cook's theorem.

In computational complexity theory, the Cook–Levin theorem, also known as Cook's theorem, states that the Boolean satisfiability problem is NP-complete. That is, it is in NP, and any problem in NP can be reduced in polynomial time by a deterministic Turing machine to the Boolean satisfiability problem.

satisfiability problem(SAT) is NP-complete. Stephen Cook, in 1971, published an important paper titled 'The complexity of Theorem Proving Procedures', in which he outlined the way of obtaining the proof of an NP-complete problem by reducing it to SAT. He proved Circuit-SAT and 3CNF-SAT problems are as hard as SAT. Similarly, Leonid Levin independently worked on this problem in the then Soviet Union. The proof that SAT is NP-complete was obtained due to the efforts of these two scientists. Later, Karp reduced 21 optimization problems, such as Hamiltonian tour, vertex cover, and clique, to the SAT and proved that those problems are NP-complete.

Hence, an SAT is a significant problem and can be stated as follows:

Given a boolean expression F having n variables $x_1, x_2, \dots, x_n$, and Boolean operators, is it possible to have an assignment for variables true or false such that binary expression F is true?

This problem is also known as the formula – SAT. An SAT(formula-SAT or simply SAT) takes a Boolean expression F and checks whether the given expression(or formula) is satisfiable. A Boolean expression is said to be satisfactory for some valid assignments of variables if the evaluation comes to be true. Prior to discussing the details of SAT, let us now discuss some important terminologies of a Boolean expression.

- **Boolean variable:** A variable, say x, that can have only two values, true or false, is called a boolean variable
- **Literal:** A literal can be a logical variable, say x, or the negation of it, that is x or $\bar{x}$; x is called a positive literal, and $\bar{x}$ is called the negative literal
- **Clause:** A sequence of variables($x_1, x_2, \dots, x_n$) that can be separated by a logical OR operator is called a clause. For example, $(x_1 \vee x_2 \vee x_3)$ is a clause of three literals.
- **Expressions:** One can combine all the preceding clauses using a Boolean operator to form an expression.
- **CNF form:** An expression is in CNF form(conjunctive normal form) if the set of clauses are separated by an AND ($\wedge$), operator, while the literals are connected by an OR ($\vee$) operator. The following is an example of an expression in the CNF form:

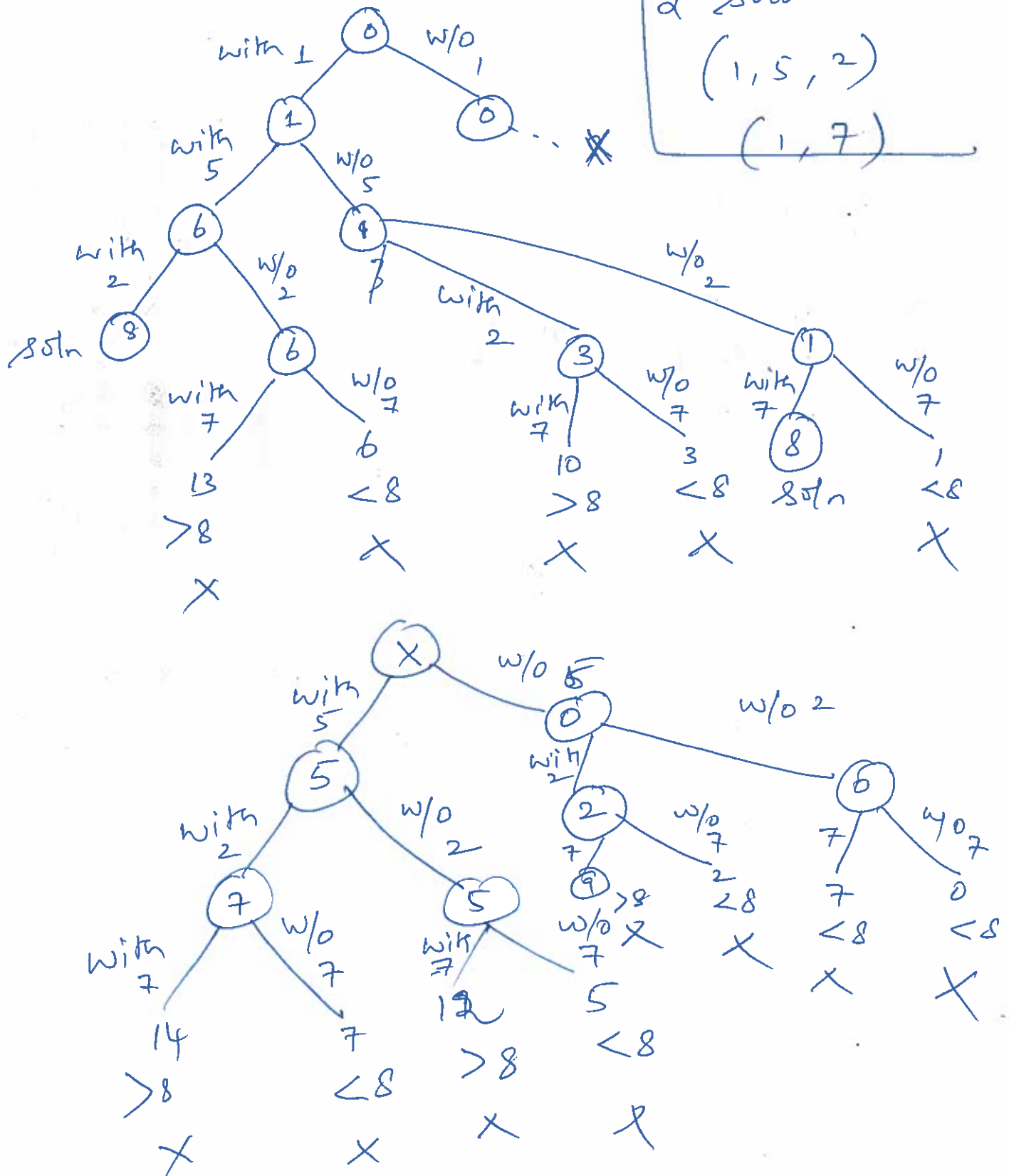
$$f = (x_1 \vee \bar{x}_2 \vee x_3) \wedge (x_1 \vee \bar{x}_3 \vee x_2)$$

- **3 - CNF:** An expression is said to be in 3-CNF if it is in the conjunctive normal form, and every clause has exact three literals.

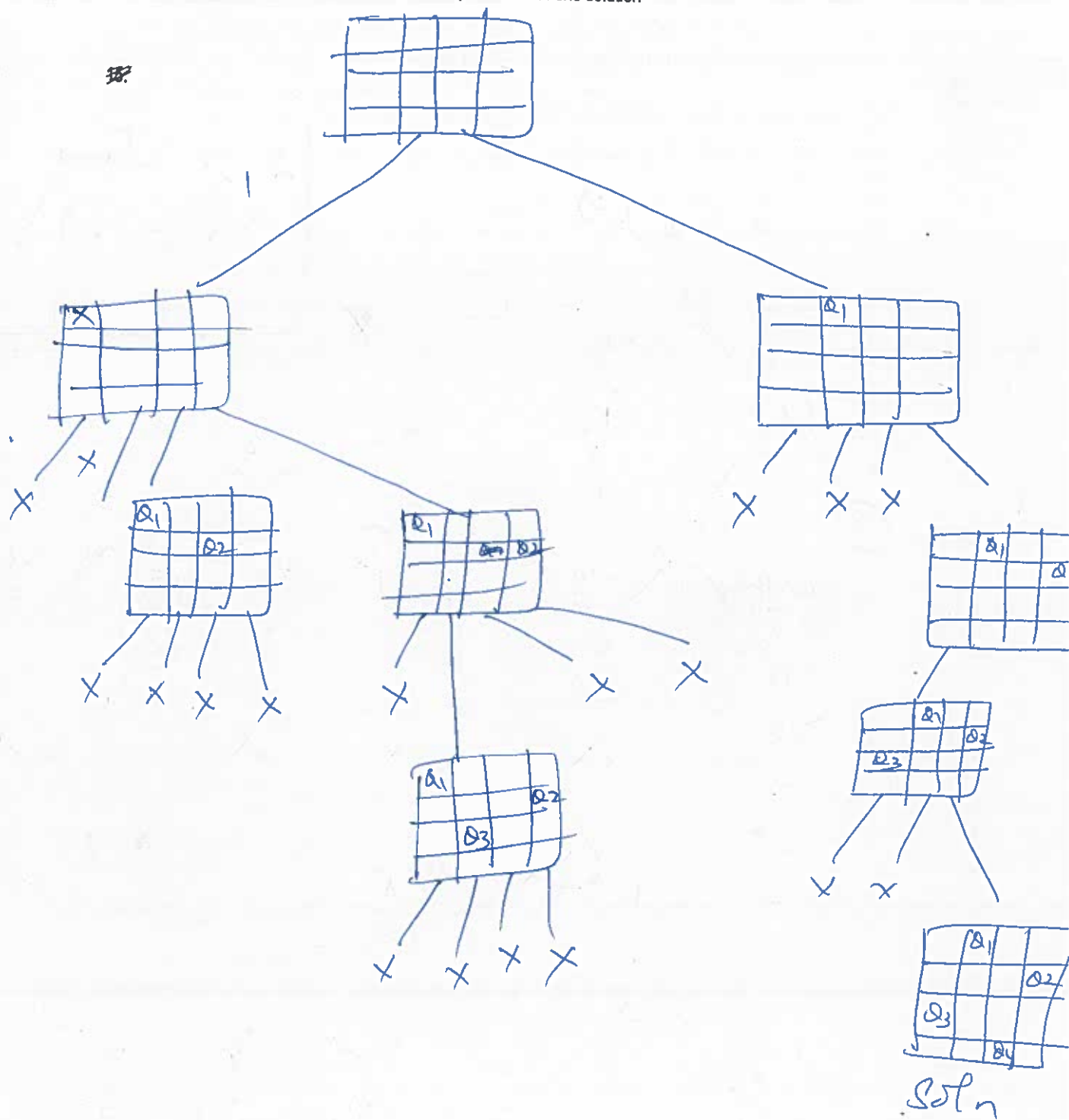
Thus, an SAT is one of the toughest problems, as there is no known algorithm other than the brute force approach.

14. A. Solve the following instance of sum of subset problem:
 $S = \{1, 5, 2, 7\}$ with $d = 8$ using backtracking

2 solutions
 $(1, 5, 2)$
 $(1, 7)$



b. Construct state space tree for 4 queen's problem with one solution



(9) Given the cost matrix, solve the TSP, by applying Branch and Bound.

	C0	C1	C2	C3	C4
C0	INF	20	30	10	11
C1	15	INF	16	4	2
C2	3	5	INF	2	4
C3	19	6	18	INF	3
C4	16	4	7	16	INF

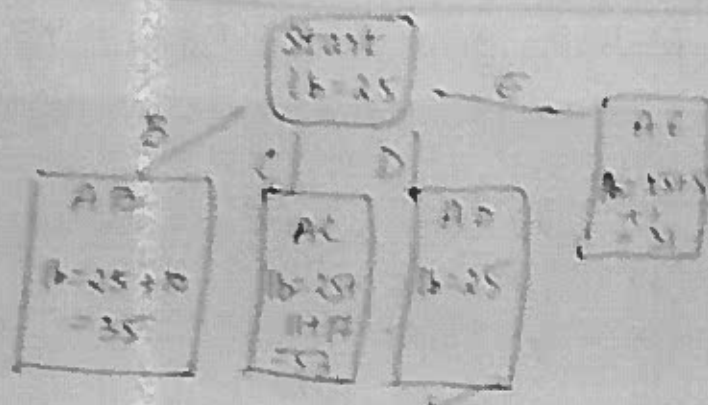
Ans:

	A	B	C	D	E	
A	∞	20	30	10	11	-10
B	15	∞	16	4	2	2
C	3	5	∞	2	4	2
D	19	6	18	∞	3	3
E	16	4	7	16	∞	4
						21

∞	10	20	0	1
13	∞	14	2	0
1	3	∞	0	2
16	3	15	∞	0
12	0	3	12	∞
1	0	3	0	0

LB = 25

∞	10	17	0	1
12	∞	11	2	0
0	3	∞	0	2
15	3	12	∞	0
11	0	0	12	∞



A-B

$$\begin{bmatrix} 2 & 2 & 2 & 2 & 2 & 2 \\ 12 & 0 & 3 & 0 & 12 & 0 \\ 0 & 3 & 0 & 0 & 0 & 0 \\ 12 & 0 & 0 & 2 & 0 & 2 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{bmatrix}$$

AC

$$\begin{bmatrix} 2 & 2 & 2 & 2 & 2 & 2 \\ 12 & 0 & 3 & 0 & 12 & 0 \\ 0 & 3 & 0 & 0 & 0 & 0 \\ 12 & 0 & 0 & 2 & 0 & 2 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 2 & 2 & 2 & 2 & 2 \\ 12 & 0 & 3 & 0 & 12 & 0 \\ 0 & 3 & 0 & 0 & 0 & 0 \\ 12 & 0 & 0 & 2 & 0 & 2 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{bmatrix}$$

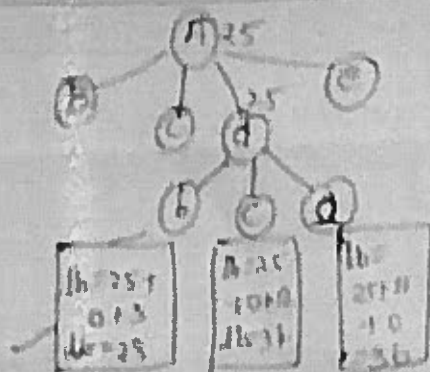
A-D

$$\begin{bmatrix} 2 & 2 & 2 & 2 & 2 & 2 \\ 12 & 0 & 3 & 0 & 12 & 0 \\ 0 & 3 & 0 & 0 & 0 & 0 \\ 12 & 0 & 0 & 2 & 0 & 2 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{bmatrix}$$

AE

$$\begin{bmatrix} 2 & 2 & 2 & 2 & 2 & 2 \\ 12 & 0 & 3 & 0 & 12 & 0 \\ 0 & 3 & 0 & 0 & 0 & 0 \\ 12 & 0 & 0 & 2 & 0 & 2 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 2 & 2 & 2 & 2 & 2 \\ 12 & 0 & 3 & 0 & 12 & 0 \\ 0 & 3 & 0 & 0 & 0 & 0 \\ 12 & 0 & 0 & 2 & 0 & 2 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{bmatrix}$$



DB

0	0	0	0	0
12	0	11	0	0
0	0	0	0	2
0	0	0	0	0
11	0	0	0	0

DC

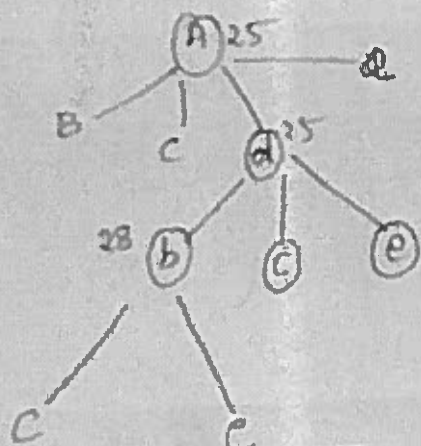
0	0	0	0	0
12	0	0	0	0
0	0	0	0	2
0	0	0	0	0
11	0	0	0	0

DE

0	0	0	0	0
12	0	11	0	0
0	0	0	0	2
0	0	0	0	0
11	0	0	0	0

↓

0	0	0	0	0
12	0	0	0	0
0	0	0	0	2
0	0	0	0	0
11	0	0	0	0



BC

0	0	0	0	0
12	0	0	0	0
0	0	0	0	2
0	0	0	0	0
11	0	0	0	0

→

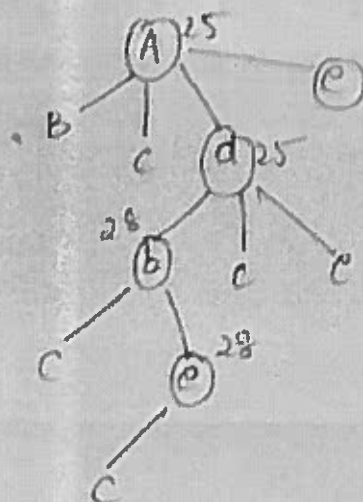
0	0	0	0	0
12	0	0	0	0
0	0	0	0	2
0	0	0	0	0
11	0	0	0	0

$$\rightarrow \begin{bmatrix} 2 & 2 & 2 & 2 & 2 \\ 2 & 2 & 2 & 2 & 2 \\ 0 & 2 & 2 & 2 & 0 \\ 2 & 2 & 2 & 2 & 0 \\ 0 & 2 & 2 & 2 & 2 \end{bmatrix}$$

13

BE

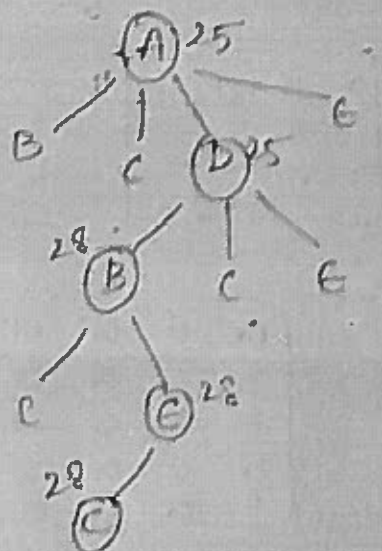
$$\begin{bmatrix} 2 & 2 & 2 & 2 & 2 \\ 2 & 2 & 2 & 2 & 2 \\ 0 & 2 & 2 & 2 & 2 \\ 2 & 2 & 2 & 2 & 2 \\ 1 & 2 & 0 & 2 & 2 \end{bmatrix}$$



$$\begin{aligned} W_b &= 28 + \\ &0 + 0 \\ &= 28 \end{aligned}$$

EC

$$\begin{bmatrix} 2 & 2 & 2 & 2 & 2 \\ 2 & 2 & 2 & 2 & 2 \\ 0 & 2 & 2 & 2 & 2 \\ 2 & 2 & 2 & 2 & 2 \\ 2 & 2 & 2 & 2 & 2 \end{bmatrix}$$



16. Apply branch bound principle to solve the following TSP

	C0	C1	C2	C3	C4
C0	INF	20	30	10	11
C1	15	INF	16	4	2
C2	3	5	INF	2	4
C3	19	6	18	INF	3
C4	16	4	7	16	INF

Semester End Examination, January/February, 2022

Degree	B. Tech. (U. G.)	Program	CSE (AI & ML)	Academic Year	2021 - 2022
Course Code	20AI302	Test Duration	3 Hrs. Max. Marks 70	Semester	III
Course	Artificial Neural Networks				

Part A (Short Answer Questions 5 x 2 = 10 Marks)

No.	Questions (1 through 5)	Learning Outcome (s)	DoK
1	What are artificial neural networks?	20AI302.1	L1
2	State Hebb's rule	20AI302.2	L1
3	List any four applications of ANN	20AI302.3	L1
4	List the two types of signals identified in the backpropagation network.	20AI302.4	L1
5	Compare auto-associative and hetero associative memories based on the input and output vectors.	20AI302.5	L2

Part B (Long Answer Questions 5 x 12 = 60 Marks)

No.	Questions (6 through 15)	Marks	Learning Outcome (s)	DoK
6 (a)	Explain the biological prototype of a neuron and any four of its characteristics of the neuron.	4M	20AI302.1	L1
6 (b)	Describe any two classifications of ANNs based on their functions.	8M	20AI302.1	L1

OR

7	Explain the following with necessary diagram (i) activation functions (ii) Types of ANN architecture (iii) Functions of single neuron	12M	20AI302.1	L1
8	Describe Hebbian architecture and its learning with diagram and example	12M	20AI302.2	L2

OR

9	Consider a feed-forward Neural Network having 2 inputs (label -1 and label -2) with fully connected layers and we have 2 hidden layers: - Hidden layer-1: Nodes labeled as 3 and 4 - Hidden layer-2: Nodes labeled as 5 and 6 A weight on the connection between nodes i and j is represented by w_{ij}, such as w_{24} is the weight on the connection between nodes 2 and 4. The following lists contain all the weights values used in the given network: $w_{13}=-2$, $w_{35}=1$, $w_{23}=3$, $w_{45}=-1$, $w_{14}=4$, $w_{36}=-1$, $w_{24}=-1$, $w_{46}=1$ Each of the nodes 3, 4, 5, and 6 use the following activation function: $\phi(v) = 1$ if $v \geq 0$ otherwise 0 where v denotes the weighted sum of a node. Each of the input nodes (1 and 2) can only receive binary values (either 0 or 1). Calculate the output of the network (y_5 and y_6) for the input pattern given by (node-1 and node-2 as 0, 0 respectively).	12M	20AI302.4	L2

10	State and Prove Perceptron Convergence theorem.	12M	20AI302.3	L2

OR

11	Draw the architecture of a single-layer perceptron (SLP) and explain its operations. Mention any three of its advantages and disadvantages.	12M	20AI302.3	L2

12	Draw the backpropagation network and explain the backpropagation algorithm	12M	20AI302.4	L2

OR

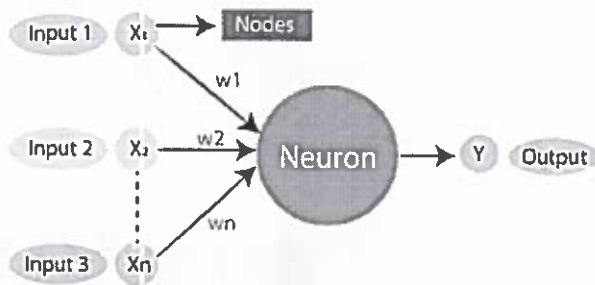
13 (a)	Draw the architecture of a Multilayer perceptron (MLP) with its operations.	6M	20AI302.4	L2

13 (b)	"Why the XOR problem cannot be solved by a single layer perceptron and how it is solved by a Multilayer Perceptron" - Justify.	6M	20AI302.4	L2
14 (a)	Compare Hetero Associative Memory and Interpolative Associative Memory.	6M	20AI302.5	L2
14 (b)	Explain why the layers in the Bidirectional Associative Memory are called x and y layers.	6M	20AI302.5	L2
OR				
15	Explain the architecture of Bidirectional Associative Memory (BAM).	12M	20AI302.5	L2

1) Artificial Neural Network

The term "Artificial neural network" refers to a biologically inspired sub-field of artificial intelligence modeled after the brain. An Artificial neural network is usually a computational network based on biological neural networks that construct the structure of the human brain. Similar to a human brain has neurons interconnected to each other, artificial neural networks also have neurons that are linked to each other in various layers of the networks. These neurons are known as nodes.

Artificial Neural Network structure



Dendrites from Biological Neural Network represent inputs in Artificial Neural Networks, cell nucleus represents Nodes, synapse represents Weights, and Axon represents Output.

Artificial neural network consists of three layers:

- **Input layer** represents dimensions of the input vector.
- **Hidden layer** represents the intermediary nodes that divide the input space into regions with (soft) boundaries. It takes in a set of weighted input and produces output through an activation function.
- **Output layer** represents the output of the neural network.

2) Hebbian Learning Rule

It identifies how to modify the weights of nodes of a network.

The training steps of the algorithm are as follows:

- Initially, the weights are set to zero, i.e. $w = 0$ for all inputs $i = 1$ to n and n is the total number of input neurons.
- Let s be the output. The activation function for inputs is generally set as an identity function.
- The activation function for output is also set to $y = t$.
- The weight adjustments and bias are adjusted to:

$$W(n) = w(o) + x * y$$

$$B(n) = b(\text{old}) + y$$

The change in weights is $\Delta w = x * y \Rightarrow w(n) = w(o) + \Delta w$

- The steps 2 to 4 are repeated for each input vector and output.

3) Applications of ANN

Facial Recognition

Facial Recognition Systems are serving as robust systems of surveillance. Recognition Systems matches the human face and compares it with the digital images. They are used in offices for selective entries. The systems thus authenticate a human face and match it up with the list of IDs that are present in its database.

Social media

Neural networks duplicate the behaviours of social media users. Post analysis of individuals' behaviours via social media networks the data can be linked to people's spending habits. Multilayer Perceptron ANN is used to mine data from social media applications.

Aerospace

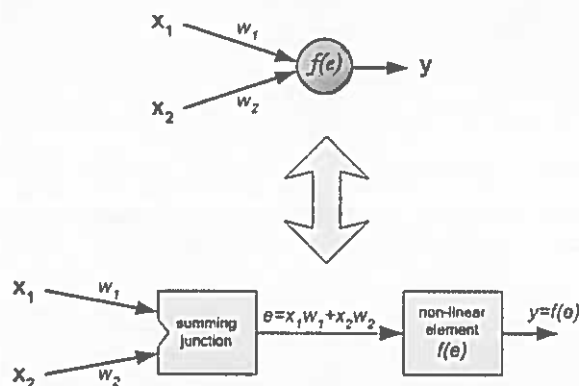
Aerospace Engineering is an expansive term that covers developments in spacecraft and aircraft. Fault diagnosis, high performance auto piloting, securing the aircraft control systems, and modeling key dynamic simulations are some of the key areas that neural networks have taken over. Time delay Neural networks can be employed for modelling non linear time dynamic systems. MLP forecasts social media trends, it uses different training methods like Mean Absolute Error (MAE), Root Mean Squared Error (RMSE), and Mean Squared Error (MSE). MLP takes into consideration several factors like user's favourite instagram pages, bookmarked choices etc. These factors are considered as inputs for training the MLP model.

Healthcare

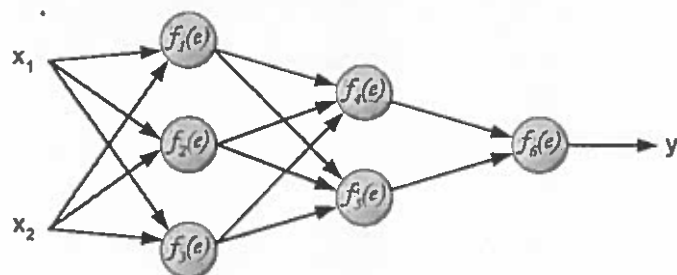
Convolutional Neural Networks are actively employed in the healthcare industry for X ray detection, CT Scan and ultrasound.

4) Signals identifies in backpropagation network:

a) Each neuron is composed of two units. First unit adds products of weights coefficients and input signals. The second unit realise nonlinear function, called neuron activation function. Signal e is adder output signal, and $y = f(e)$ is output signal of nonlinear element. Signal y is also output signal of neuron.

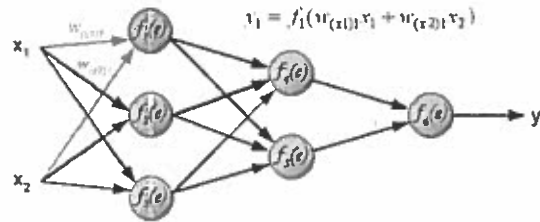


The project describes teaching process of multi-layer neural network employing *backpropagation* algorithm. To illustrate this process the three layer neural network with two inputs and one output, which is shown in the picture below, is used:

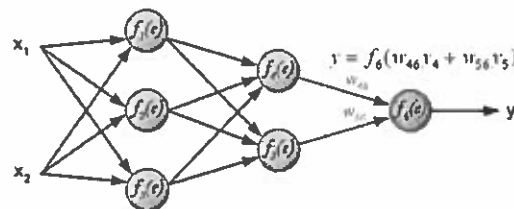


To teach the neural network we need training data set. The training data set consists of input signals (x_1 and x_2) assigned with

corresponding target (desired output) z . The network training is an iterative process. In each iteration weights coefficients of nodes are modified using new data from training data set. Modification is calculated using algorithm described below: Each teaching step starts with forcing both input signals from training set. After this stage we can determine output signals values for each neuron in each network layer. Pictures below illustrate how signal is propagating through the network, Symbols w_{mn} represent weights of connections between network input x_m and neuron n in input layer. Symbols y_n represents output signal of neuron n .



Propagation of signals through the output layer.



5. Compare auto-associative and hetero associative memories based on the input and output vectors.

Autoassociative Networks:

- Autoassociative networks are special kind of networks used to simulate associative processes.
- These are achieved through interaction of set of simple processing elements which are connected through weighted connections.
- They are capable to retrieve piece of data with the partial information and also capable for remembering from small portion of data.

Heteroassociative Networks:

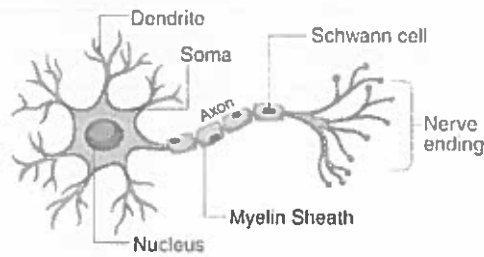
- Heteroassociative networks stores input-output pattern pairs to recall stored output pattern by receiving noisy or incomplete version.
- In each of the pairs, an input pattern should differ from an output pattern.

In this basic logical operations are used to determine associations among common and special features of reference patterns

6. (A) Explain the biological prototype of a neuron and any four of its characteristics of the neuron

Neuron: The neuron is the fundamental building block of neural networks. In the biological systems, a neuron is a cell just like any other cell of the body, which has a DNA code and is generated in the same way as the other cells. Though it might have different DNA, the function is similar in all the organisms. A neuron comprises three major parts: the cell body (also called Soma), the dendrites, and the axon. The dendrites are like fibers branched in different directions and are connected to many cells in that cluster. Dendrites receive the signals from surrounding neurons, and the axon transmits the signal to the other neurons. At the ending terminal of the axon, the contact with the dendrite is made through a synapse. Axon is a long fiber that transports the output signal as electric impulses along its length. Each neuron has one axon. Axons pass impulses from one neuron to another like a domino effect.

STRUCTURE OF NEURON



Working of a Biological Neuron

As shown in the above diagram, a typical neuron consists of the following four parts with the help of which we can explain its working -

- Dendrites - They are tree-like branches, responsible for receiving the information from other neurons it is connected to. In other sense, we can say that they are like the ears of neuron.
- Soma - It is the cell body of the neuron and is responsible for processing of information, they have received from dendrites.
- Axon - It is just like a cable through which neurons send the information.
- Synapses - It is the connection between the axon and other neuron dendrites.

Artificial neural networks were inspired by the biological neural networks of the human body. The modeling of biological neural networks was a crucial step in the development of artificial neural networks. Many scientists attempted to understand the working of the brain. Artificial neural networks today are being used for various applications, some are biologically related, and most of them are engineering related.

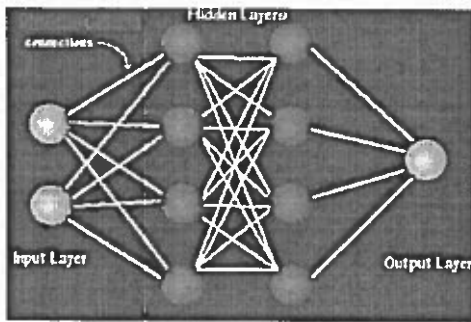
Even though biological neural networks and artificial neural networks are similar in function, they still have many differences. Many attempts have been made to understand the complex mechanism of biological neural networks. Yet, they still hold many secrets to unlock and inspire the future of artificial intelligence.

Biological Neural Network	Artificial Neural Network
Dendrites	Inputs
Cell nucleus	Nodes
Synapse	Weights
Axon	Output

6. (B) Describe any two classifications of ANNs based on their functions.

Artificial Neural Networks (ANN) is a part of Artificial Intelligence (AI) and this is the area of computer science which is related in making computers behave more intelligently. Artificial Neural Networks(ANN) process data and exhibit some intelligence and they behaves exhibiting intelligence in such a way like pattern recognition, Learning and generalization.

An artificial neural network is a programmed computational model that aims to replicate the neural structure and functioning of the human brain.



Types of Artificial Neural Networks:

There are different types of Artificial Neural Networks (ANN)– Depending upon the human brain neuron and network functions, an artificial neural network or ANN performs tasks in a similar manner. Most of the artificial neural networks will have some resemblance with more complex biological counterparts and are very effective at their intended tasks like for e.g. segmentation or classification. Types of Artificial Neural Networks

Feedback ANN – In this type of ANN, the output goes back into the network to achieve the best-evolved results internally. The feedback network feeds information back into itself and is well suited to solve optimization problems, according to the University of Massachusetts, Lowell Center for Atmospheric Research. Feedback ANNs are used by the Internal system error corrections.

Feed Forward ANN – A feed-forward network is a simple neural network consisting of an input layer, an output layer and one or more layers of neurons. Through evaluation of its output by reviewing its input, the power of the network can be noticed base on group behavior of the connected neurons and the output is decided. The main advantage of this network is that it learns to evaluate and recognize input patterns.

Classification-Prediction ANN –It is the subset of feed-forward ANN and the classification-prediction ANN is applied to data-mining scenarios. The network is trained to identify particular patterns and classify them into specific groups and then further classify them into "novel patterns" which are new to the network.

An artificial neural network is a computational simulation of a biological neural network. These possess the behavior of neurons and the electrical signals in which they communicate between input such as from the eyes or nerve endings in the hand to the output of the brain such as reacting to light, touch or heat.

Scientists were researching in the designing of artificial neural networks and the creation of artificial intelligence about the way neurons semantically communicate.

7. Explain the following with necessary diagram

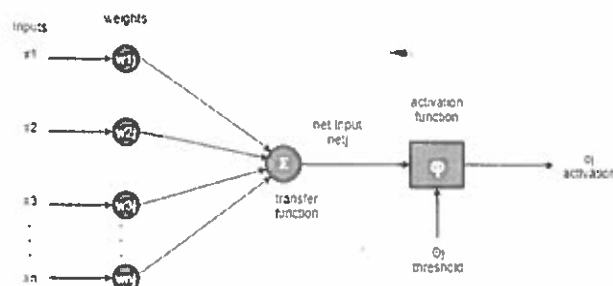
(i) activation functions

(ii) Types of ANN architecture

(iii) Functions of single neuron

(i) **Definition of activation function:-** Activation function decides, whether a neuron should be activated or not by calculating weighted sum and further adding bias with it. The purpose of the activation function is to introduce non-linearity into the output of a neuron.

To put in simple terms, an artificial neuron calculates the 'weighted sum' of its inputs and adds a bias, as shown in the figure below by the net input.



Types of Activation Functions –

Several different types of activation functions are used in Deep Learning. Some of them are explained below:

1. Step Function:

Step Function is one of the simplest kind of activation functions. In this, we consider a threshold value and if the value of net input say y is greater than the threshold then the neuron is activated.

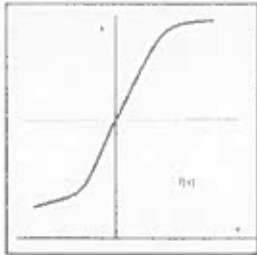
Mathematically, Given below is the graphical representation of step function.



2. Sigmoid Function:

Sigmoid function is a widely used activation function. It is defined as:

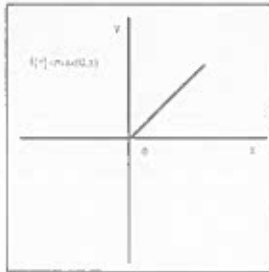
Graphically,



This is a smooth function and is continuously differentiable. The biggest advantage that it has over step and linear function is that it is non-linear. This is an incredibly cool feature of the sigmoid function. This essentially means that when I have multiple neurons having sigmoid function as their activation function – the output is non linear as well. The function ranges from 0-1 having an S shape.

3. ReLU:

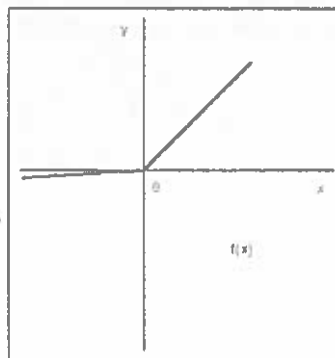
The ReLU function is the Rectified linear unit. It is the most widely used activation function. It is defined as: Graphically,



The main advantage of using the ReLU function over other activation functions is that it does not activate all the neurons at the same time. What does this mean? If you look at the ReLU function if the input is negative it will convert it to zero and the neuron does not get activated.

4. Leaky ReLU:

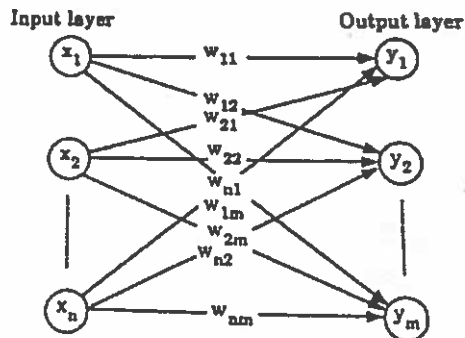
Leaky ReLU function is nothing but an improved version of the ReLU function. Instead of defining the Relu function as 0 for x less than 0, we define it as a small linear component of x . It can be defined as: Graphically,



(II) The different types of neural network architectures are -

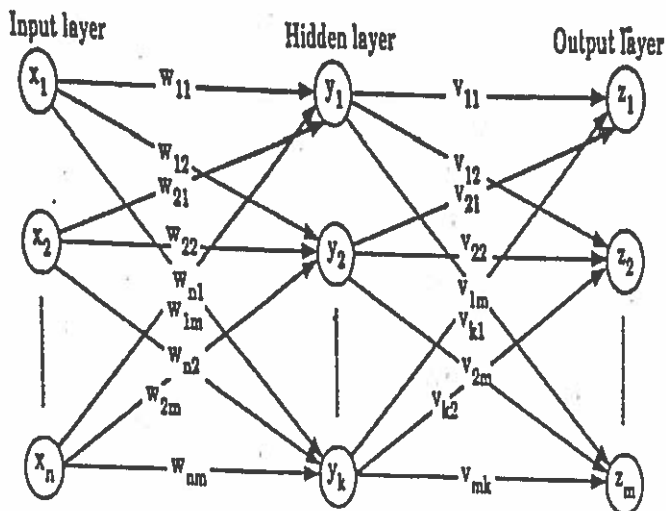
Single Layer Feed Forward Network

1. In this type of network, we have only two layers, i.e. input layer and output layer but the input layer does not count because no computation is performed in this layer.
2. Output Layer is formed when different weights are applied on input nodes and the cumulative effect per node is taken.
3. After this, the neurons collectively give the output layer to compute the output signals.



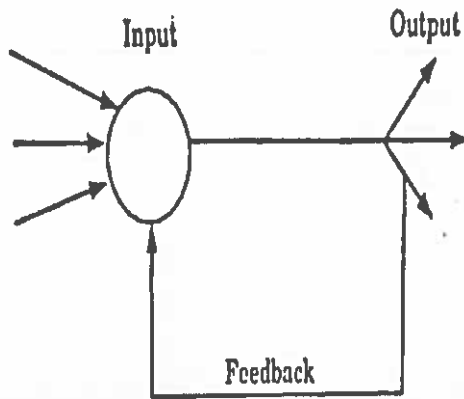
Multilayer Feed Forward Network

1. This network has a hidden layer that is internal to the network and has no direct contact with the external layer.
2. The existence of one or more hidden layers enables the network to be computationally stronger.
3. There are no feedback connections in which outputs of the model are fed back into itself.



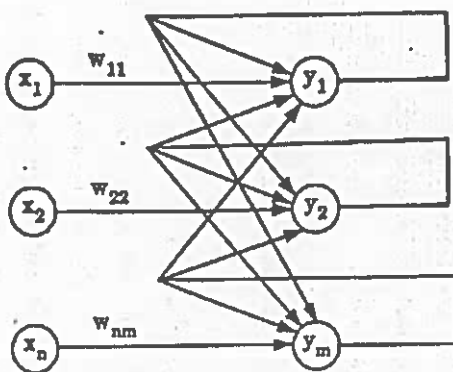
Single node with its own feedback

1. When outputs can be directed back as inputs to the same layer or proceeding layer nodes, then it results in feedback networks.
2. Recurrent networks are feedback networks with closed loop. The figure below shows a single recurrent having a single neuron with feedback to itself.



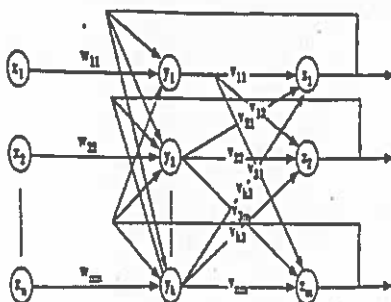
Single Layer Recurrent Network

1. This network is a single-layer network with a feedback connection in which the processing element's output can be directed back to itself or to other processing elements or both.
2. A recurrent neural network is a class of artificial neural network where the connection between nodes forms a directed graph along a sequence.
3. This allows it to exhibit dynamic temporal behavior for a time sequence. Unlike feedforward neural networks, RNNs can use their internal state (memory) to process sequences of inputs.



Multilayer Recurrent Network

1. In this type of network, processing element output can be directed to the processing element in the same layer and in the preceding layer forming a multilayer recurrent network.
2. They perform the same task for every element of the sequence, with the output being dependent on the previous computations. Inputs are not needed at each time step.
3. The main feature of a multilayer recurrent network is its hidden state, which captures information about a sequence.



(III) Functions of single neuron:

A single neuron transforms given input into some output. Depending on the given input and weights assigned to each input, decide whether the neuron fired or not. The function performed by a single neuron is very limited, mostly a binary output. To perform complex tasks, we need a network of interconnected neurons or perceptrons

8 Describe Hebbian architecture and its learning with diagram and example

Hebbian Learning Rule, also known as Hebb Learning Rule, was proposed by Donald O Hebb. It is one of the first and also easiest learning rules in the neural network. It is used for pattern classification. It is a single layer neural network, i.e. it has one input layer and one output layer. The input layer can have many units, say n . The output layer only has one unit. Hebbian rule works by updating the weights between neurons in the neural network for each training sample.

Hebbian Learning Rule Algorithm :

1. Set all weights to zero, $w_i = 0$ for $i=1$ to n , and bias to zero.
2. For each input vector, $S(\text{input vector}) : t(\text{target output pair})$, repeat steps 3-5.
3. Set activations for input units with the input vector $X_i = S_i$ for $i = 1$ to n .
4. Set the corresponding output value to the output neuron, i.e. $y = t$.
5. Update weight and bias by applying Hebb rule for all $i = 1$ to n :

$$w_i(\text{new}) = w_i(\text{old}) + x_i y$$

$$b(\text{new}) = b(\text{old}) + y$$

Implementing AND Gate :

INPUT				TARGET	
	x_1	x_2	b		y
X_1	-1	-1	1	Y_1	-1
X_2	-1	1	1	Y_2	-1
X_3	1	-1	1	Y_3	-1
X_4	1	1	1	Y_4	1

Truth Table of AND Gate using bipolar sigmoidal function

There are 4 training samples, so there will be 4 iterations. Also, the activation function used here is Bipolar Sigmoidal Function so the range is $[-1, 1]$.

Step 1 :

Set weight and bias to zero, $w = [0\ 0\ 0]^T$ and $b = 0$.

Step 2 :

Set input vector $X_i = S_i$ for $i = 1$ to 4.

$$X_1 = [-1\ -1\ 1]^T$$

$$X_2 = [-1\ 1\ 1]^T$$

$$X_3 = [1\ -1\ 1]^T$$

$$X_4 = [1\ 1\ 1]^T$$

Step 3 :

Output value is set to $y = t$.

Step 4 :

Modifying weights using Hebbian Rule:

First iteration –

$$w(\text{new}) = w(\text{old}) + x_i y = [0\ 0\ 0]^T + [-1\ -1\ 1]^T \cdot [-1] = [1\ 1\ -1]^T$$

For the second iteration, the final weight of the first one will be used and so on.

Second iteration –

$$w(\text{new}) = [1 \ 1 \ -1]^T + [-1 \ 1 \ 1]^T \cdot [-1] = [2 \ 0 \ -2]^T$$

Third iteration –

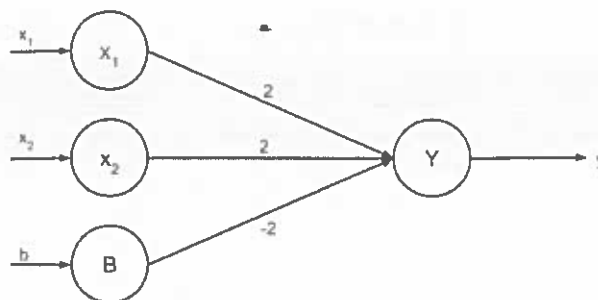
$$w(\text{new}) = [2 \ 0 \ -2]^T + [1 \ -1 \ 1]^T \cdot [-1] = [1 \ 1 \ -3]^T$$

Fourth iteration –

$$w(\text{new}) = [1 \ 1 \ -3]^T + [1 \ 1 \ 1]^T \cdot [1] = [2 \ 2 \ -2]^T$$

So, the final weight matrix is $[2 \ 2 \ -2]^T$

Testing the network :



The network with the final weights

$$\text{For } x_1 = -1, x_2 = -1, b = 1, Y = (-1)(2) + (-1)(2) + (1)(-2) = -6$$

$$\text{For } x_1 = -1, x_2 = 1, b = 1, Y = (-1)(2) + (1)(2) + (1)(-2) = -2$$

$$\text{For } x_1 = 1, x_2 = -1, b = 1, Y = (1)(2) + (-1)(2) + (1)(-2) = -2$$

$$\text{For } x_1 = 1, x_2 = 1, b = 1, Y = (1)(2) + (1)(2) + (1)(-2) = 2$$

The results are all compatible with the original table.

Decision Boundary :

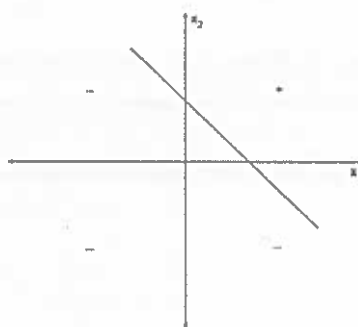
$$2x_1 + 2x_2 - 2b = y$$

$$\text{Replacing } y \text{ with } 0, 2x_1 + 2x_2 - 2b = 0$$

$$\text{Since bias, } b = 1, \text{ so } 2x_1 + 2x_2 - 2(1) = 0$$

$$2(x_1 + x_2) = 2$$

$$\text{The final equation, } x_2 = -x_1 + 1$$



Decision Boundary of AND Function

9. Consider a feed-forward Neural Network having 2 inputs (label -1 and label -2) with fully connected layers and we have 2 hidden layers:

- Hidden layer-1: Nodes labeled as 3 and 4
- Hidden layer-2: Nodes labeled as 5 and 6

A weight on the connection between nodes i and j is represented by w_{ij} , such as w_{24} is the weight on the connection between nodes 2 and 4. The following lists contain all the weights values used in the given network: $w_{13}=-2$, $w_{35}=1$, $w_{23}=3$, $w_{45}=-1$, $w_{14}=4$, $w_{36}=-1$, $w_{24}=-1$, $w_{46}=1$

Each of the nodes 3, 4, 5, and 6 use the following activation function:
 $\phi(v) = 1$ if $v \geq 0$ otherwise 0

where v denotes the weighted sum of a node. Each of the input nodes (1 and 2) can only receive binary values (either 0 or 1).

Calculate the output of the network (y_5 and y_6) for the input pattern given by (node-1 and node-2 as 0, 0 respectively).

To find the output of the network it is necessary to calculate weighted sums of hidden nodes 3 and 4:

$$v_3 = w_{13}x_1 + w_{23}x_2, \quad v_4 = w_{14}x_1 + w_{24}x_2$$

Then find the outputs from hidden nodes using activation function ϕ :

$$y_3 = \phi(v_3), \quad y_4 = \phi(v_4).$$

Use the outputs of the hidden nodes y_3 and y_4 as the input values to the output layer (nodes 5 and 6), and find weighted sums of output nodes 5 and 6:

$$v_5 = w_{35}y_3 + w_{45}y_4, \quad v_6 = w_{36}y_3 + w_{46}y_4.$$

Finally, find the outputs from nodes 5 and 6 (also using ϕ):

$$y_5 = \phi(v_5), \quad y_6 = \phi(v_6).$$

The output pattern will be (y_5, y_6) .

Perform this calculation for the given input – Input pattern (0, 0)

$$v_3 = -2 \cdot 0 + 3 \cdot 0 = 0, \quad y_3 = \phi(0) = 1$$

$$v_4 = 4 \cdot 0 - 1 \cdot 0 = 0, \quad y_4 = \phi(0) = 1$$

$$v_5 = 1 \cdot 1 - 1 \cdot 1 = 0, \quad y_5 = \phi(0) = 1$$

$$v_6 = -1 \cdot 1 + 1 \cdot 1 = 0, \quad y_6 = \phi(0) = 1$$

Therefore, the output of the network for a given input pattern is (1, 1).

10. State and Prove Perceptron Convergence theorem.

Theorem: If all of the above holds, then the Perceptron algorithm makes at most $1/\gamma^2$ mistakes.

Proof:

Keeping what we defined above, consider the effect of an update (w becomes $w + yx$) on the two terms $w^T w^*$ and $w^T w$. We will use two facts:

- $y(x^T w) \leq 0$: This holds because x is misclassified by w - otherwise we wouldn't make the update.
- $y(x^T w^*) > 0$: This holds because w^* is a separating hyper-plane and classifies all points correctly.

1. Consider the effect of an update on $w^T w^*$:

$$(w + yx)^T w^* = w^T w^* + y(x^T w^*) \geq w^T w^* + \gamma$$

The inequality follows from the fact that, for w^* , the distance from the hyperplane defined by w^* to x must be at least γ (i.e. $y(x^T w^*) = |x^T w^*| \geq \gamma$).

This means that for each update, $w^{TW} * w^{TW}$ grows by at least γ .

2. Consider the effect of an update on $w_{TW}w_{TW}$:

$$(w+yx)^T(w+yx) = w^T w + 2y(w^T x) + y^2(x^T x) \leq w^T w + 2y(w^T x) + y^2(x^T x) = (w+yx)^T(w+yx)$$

The inequality follows from the fact that

- $2y(wTx) < 0.2y(wTx) < 0$ as we had to make an update, meaning xx was misclassified
- $0 \leq y_2(xTx) \leq 10 \leq y_2(xTx) \leq 1$ as $y_2 = 1, y_2 = 1$ and all $xTx \leq 1, xTx \leq 1$ (because $\|x\| \leq 1, \|x\| \leq 1$).

This means that for each update, $w^T w^T w$ grows by at most 1.

3. Now we know that after MM updates the following two inequalities must hold:

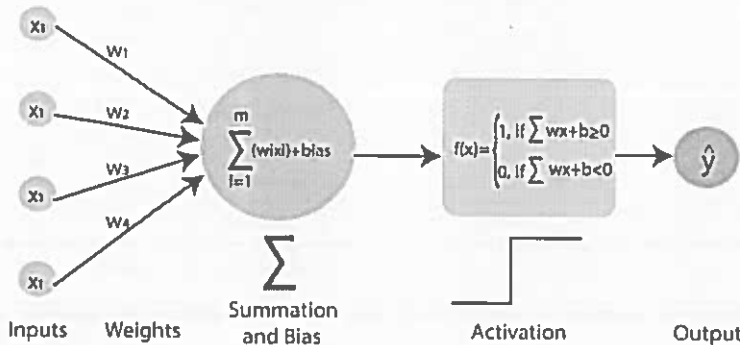
- (1) $WTW \geq MWTW \geq MY$
- (2) $WTW \leq MWTW \leq M$.

We can then complete the proof:

$M\gamma \leq \langle w, w^* \rangle = \|w\| \cos(\theta) \leq \|w\| = \sqrt{M} \gamma \implies \cos(\theta) \leq \sqrt{M} \implies M \geq \frac{1}{\cos^2(\theta)}$ (1) by definition of inner-product, where θ is the angle between w and w^* . by definition of $\cos$, we must have $\cos(\theta) \leq 1$. by definition of $\|w\|$ (2) And hence, the number of updates M is bounded from above by a constant.

11. Draw the architecture of a single-layer perceptron (SLP) and explain its operations. Mention any three of its advantages and disadvantages.

- In a single layer perceptron, the weights to each input node are assigned randomly since there is no a priori knowledge associated with the nodes.
- Also, a threshold value is assigned randomly
- Now SLP sums all the weights which are inputted and if the sums are above the threshold then the network is activated.
- If the calculated value is matched with the desired value, then the model is successful
- If it is not, then since there is no back-propagation technique involved in this the error needs to be calculated using the below formula and the weights need to be adjusted again.



Advantages-

Less computation

- Single layer neural networks are easy to set up and train them as there is absence of hidden layers
- It has explicit links to statistical models
- Less computation

Disadvantages:

- It can work better only for linearly separable data.
- Single layer neural network has low accuracy as compared to multi-layer neural network.

12. Draw the backpropagation network and explain the backpropagation algorithm

Neural network A neural network is a collection of connected units. Each connection has a weight associated with it. This system helps in building predictive models based on huge data sets. It works like a human nervous system and helps in understanding images, learning like a human, and synthesizing speech, among many others.

What is backpropagation?

We can define the backpropagation algorithm as an algorithm that trains some given feed-forward Neural Network for a given input pattern where the classifications are known to us. At the point when every passage of the example set is exhibited to the network, the network looks at its yield reaction to the example input pattern. After that, the comparison done between output response and expected output with the error value is measured. Later, we adjust the connection weight based upon the error value measured.

Before we deep dive into backpropagation, we should be aware about who introduced this concept and when. It was first introduced in the 1960s and 30 years later it was popularized by David Rumelhart, Geoffrey Hinton, and Ronald Williams in the famous 1986 paper. In this paper, they spoke about the various neural networks. Today, backpropagation is doing good. Neural network training happens through backpropagation. By this approach, we fine-tune the weights of a neural net based on the error rate obtained in the previous run. The right manner of applying this technique reduces error rates and makes the model more reliable.

Backpropagation is used to train the neural network of the chain rule method. In simple terms, after each feed-forward passes through a network, this algorithm does the backward pass to adjust the model's parameters based on weights and biases. A typical supervised learning algorithm attempts to find a function that maps input data to the right output. Backpropagation works with a multi-layered neural network and learns internal representations of input to output mapping.

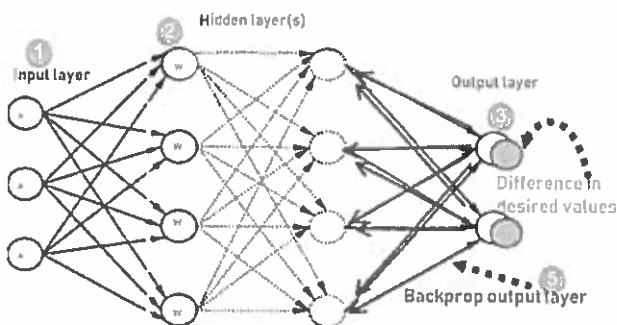
How does backpropagation work?

Let us take a look at how backpropagation works. It has four layers: input layer, hidden layer, hidden layer II and final output layer.

So, the main three layers are:

1. Input layer
2. Hidden layer
3. Output layer

Each layer has its own way of working and its own way to take action such that we are able to get the desired results and correlate these scenarios to our conditions. Let us discuss other details needed to help summarizing this algorithm.



This image summarizes the functioning of the backpropagation approach.

1. Input layer receives x
2. Input is modeled using weights w
3. Each hidden layer calculates the output and data is ready at the output layer
4. Difference between actual output and desired output is known as the error

5. Go back to the hidden layers and adjust the weights so that this error is reduced in future runs. This process is repeated till we get the desired output. The training phase is done with supervision. Once the model is stable, it is used in production.

Loss function

One or more variables are mapped to real numbers, which represent some price related to those values. Intended for backpropagation, the loss function calculates the difference between the network output and its probable output.

Why do we need backpropagation?

Backpropagation has many advantages, some of the important ones are listed below-

- Backpropagation is fast, simple and easy to implement
- There are no parameters to be tuned
- Prior knowledge about the network is not needed thus becoming a flexible method
- This approach works very well in most cases
- The model need not learn the features of the function
- Types of backpropagation

There are two types of backpropagation networks.

- Static backpropagation
- Recurrent backpropagation

Static backpropagation

In this network, mapping of a static input generates static output. Static classification problems like optical character recognition will be a suitable domain for static backpropagation.

Recurrent backpropagation

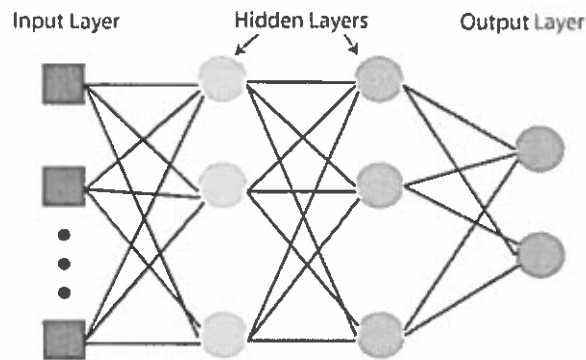
Recurrent backpropagation is conducted until a certain threshold is met. After the threshold, the error is calculated and propagated backward.

The difference between these two approaches is that static backpropagation is as fast as the mapping is static.

13 (A) Draw the architecture of a Multilayer perceptron (MLP) with its operations.

Multi-Layer perceptron defines the most complex architecture of artificial neural networks. It is substantially formed from multiple layers of the perceptron. TensorFlow is a very popular deep learning framework released by, and this notebook will guide to build a neural network with this library. If we want to understand what is a Multi-layer perceptron, we have to develop a multi-layer perceptron from scratch using Numpy.

The pictorial representation of multi-layer perceptron learning is as shown below-



MLP networks are used for supervised learning format. A typical learning algorithm for MLP networks is also called back propagation's algorithm.

A multilayer perceptron (MLP) is a feed forward artificial neural network that generates a set of outputs from a set of inputs. An MLP is characterized by several layers of input nodes connected as a directed graph between the input nodes connected as a directed graph between the input and output layers. MLP uses backpropagation for training the network. MLP is a deep learning method.

The Basic Structure

A multi-layered perceptron consists of interconnected *neurons* transferring information to each other, much like the human brain. Each neuron is assigned a value. The network can be divided into three main layers.

Input Layer

This is the initial layer of the network which takes in an input which will be used to produce an output.

Hidden Layer(s)

The network needs to have at least one hidden layer. The hidden layer(s) perform computations and operations on the input data to produce something meaningful.

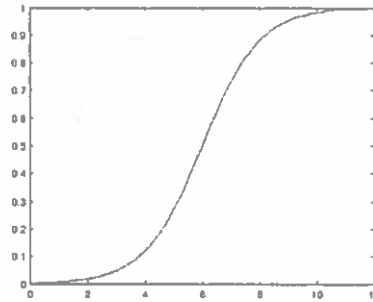
Output Layer

The neurons in this layer display a meaningful output.

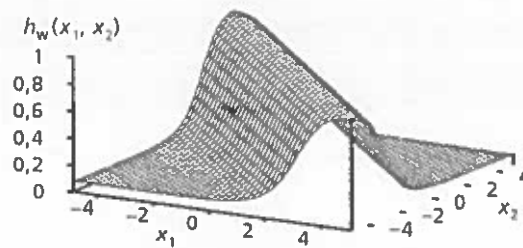
13. (b) "Why the XOR problem cannot be solved by a single layer perceptron and how it is solved by a Multilayer Perceptron" - Justify.

A single neuron represents a single function which can divide function space into two parts, whereas the XOR problem has to be divided into two parts.

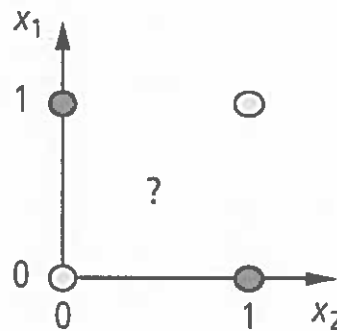
So for our XOR problem we need a hidden layer with at least two hidden neurons. If we only have one hidden neuron, this will still produce a faulty representation. This looks like this, if we have a logistic activation function, which separates the space into two subspaces.



But if we use two neurons inside our hidden layer, we can combine two of these sigmoid curves to produce a ridge of this form.



Which resembles the function space of the XOR truth table like in this picture?



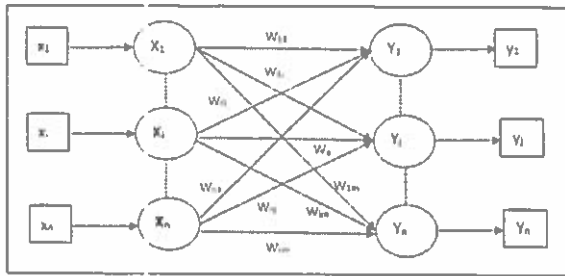
14.a Compare Hetero Associative Memory and Interpolative Associative Memory.

Interpolative Associative Memory

This is a single layer neural network in which the input training vector and the output target vectors are the same. The weights are determined so that the network stores a set of patterns.

Architecture

As shown in the following figure, the architecture of Auto Associative memory network has 'n' number of input training vectors and similar 'n' number of output target vectors.



Training Algorithm

Initialize all the weights to zero as $w_{ij} = 0$ $i=1 \text{ to } n, j=1 \text{ to } m$

Activate each input unit as follows -

$$x_i = \begin{cases} 1 & \text{if } i \text{ is active} \\ 0 & \text{otherwise} \end{cases}$$

Activate each output unit as follows -

$$y_j = \begin{cases} 1 & \text{if } j \text{ is active} \\ 0 & \text{otherwise} \end{cases}$$

Adjust the weights as follows -

$$w_{ij}(\text{new}) = w_{ij}(\text{old}) + x_i y_j$$

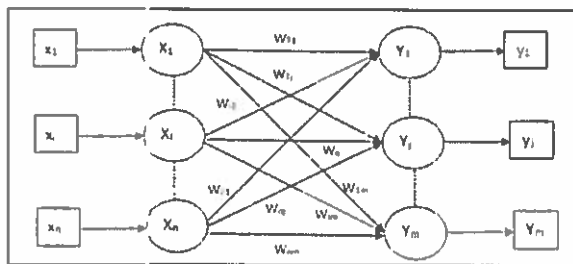
$$y_j = f(\text{net}_j) = \begin{cases} 1 & \text{if } \text{net}_j > 0 \\ 0 & \text{otherwise} \end{cases}$$

Hetero Associative memory

Similar to Auto Associative Memory network, this is also a single layer neural network. However, in this network the input training vector and the output target vectors are not the same. The weights are determined so that the network stores a set of patterns. Hetero associative network is static in nature, hence, there would be no non-linear and delay operations.

Architecture

As shown in the following figure, the architecture of Hetero Associative Memory network has 'n' number of input training vectors and 'm' number of output target vectors.



Training Algorithm

For training, this network is using the Hebb or Delta learning rule.

Initialize all the weights to zero as $w_{ij} = 0$ $i=1 \text{ to } n, j=1 \text{ to } m$

Activate each input unit as follows -

$$x_i = \begin{cases} 1 & \text{if } i \text{ is active} \\ 0 & \text{otherwise} \end{cases}$$

Activate each output unit as follows -

$$y_j = \begin{cases} 1 & \text{if } j \text{ is active} \\ 0 & \text{otherwise} \end{cases}$$

Adjust the weights as follows -

$$w_{ij}(\text{new}) = w_{ij}(\text{old}) + x_i y_j$$

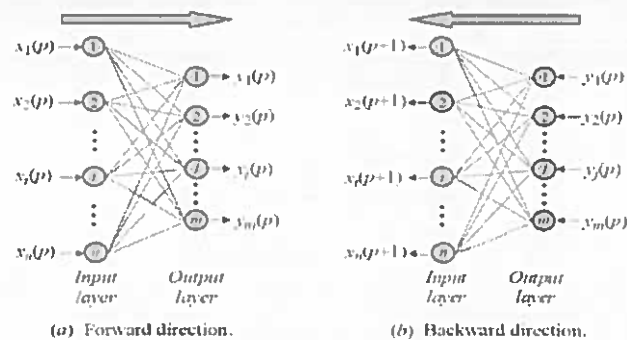
15. Explain the architecture of Bidirectional Associative Memory (BAM).

Bidirectional Associative Memory (BAM) is a supervised learning model in Artificial Neural Network. This is hetero-associative memory, for an input pattern, it returns another pattern which is potentially of a different size. This phenomenon is very similar to the human brain. Human memory is necessarily associative. It uses a chain of mental associations to recover a lost memory like associations of faces with names, in exam questions with answers, etc.

In such memory associations for one type of object with another, a Recurrent Neural Network (RNN) is needed to receive a pattern of one set of neurons as an input and generate a related, but different, output pattern of another set of neurons.

BAM Architecture:

When BAM accepts an input of n -dimensional vector X from set A then the model recalls m -dimensional vector Y from set B . Similarly when Y is treated as input, the BAM recalls X .



Algorithm:

1. **Storage (Learning):** In this learning step of BAM, weight matrix is calculated between M pairs of patterns (fundamental memories) are stored in the synaptic weights of the network
2. **Testing:** We have to check that the BAM recalls perfectly for corresponding and recalls Using. All pairs should be recalled accordingly.
3. **Retrieval:** For an unknown vector X (a corrupted or incomplete version of a pattern from set A or B) to the BAM and retrieve a previously stored association:

- **Initialize the BAM:**
- **Calculate the BAM output at iteration :**
- **Update the input vector :**
- **Repeat** the iteration until convergence, when input and output remain unchanged.

Limitations of BAM:

- **Storage capacity of the BAM:** In the BAM, stored number of associations should not be exceeded the number of neurons in the smaller layer.
- **Incorrect convergence:** Always the closest association may not be produced by BAM.

Semester End Examination, January/February, 2022

Degree	B. Tech. (U. G.)	Program	CSE (DS)			Academic Year	2021 - 2022
Course Code	20DS302	Test Duration	3 Hrs.	Max. Marks	70	Semester	III
Course	Foundations of Data Science						

Part A (Short Answer Questions 5 x 2 = 10 Marks)

No.	Questions (1 through 5)	Learning Outcome (s)	DoK
1	Define Data Science.	20DS302.1	L1
2	What are the differences between excel and knime for doing data science problem?	20DS302.2	L1
3	Distinguish data mining and data science.	20DS302.3	L2
4	How to make maps from spatial data?	20DS302.4	L1
5	List out the four concepts of data science in journalism.	20DS302.5	L1

Part B (Long Answer Questions 5 x 12 = 60 Marks)

No.	Questions (6 through 15)	Marks	Learning Outcome (s)	DoK
6 (a)	Distinguish Business intelligence and Data Science.	6M	20DS302.1	L2
6 (b)	Explain the conversion of raw data into actionable insights with data analytics.	6M	20DS302.1	L2
OR				
7 (a)	Analyze the Pieces of Data Science puzzle.	6M	20DS302.1	L2
7 (b)	Illustrate Hadoop Architecture with neat example.	6M	20DS302.1	L2
8 (a)	List out the four Python Data types. Explain it with neat code.	6M	20DS302.2	L2
8 (b)	Explain any two python data visualization packages.	6M	20DS302.2	L2
OR				
9 (a)	What are the differences between R and SQL in data science? Explain it.	6M	20DS302.2	L2
9 (b)	List out the two applications of Knime with neat example.	6M	20DS302.2	L1
10 (a)	Summarize Artificial Neural Network Model with example.	6M	20DS302.3	L2
10 (b)	Define Linear Regression Model. How it is modeled? Explain it.	6M	20DS302.3	L1
OR				
11 (a)	Explain K-Means algorithm for clustering food and drinks with K=2.	6M	20DS302.3	L2
11 (b)	Distinguish Classification and Clustering with neat example.	6M	20DS302.3	L2
12	Examine D3.js for Datweb based applications for visual design.	12M	20DS302.4	L3
OR				
13	Relate and explain any two open source applications for data visualization.	12M	20DS302.4	L2
14	Explain how data science can be useful in Environmental data science.	12M	20DS302.5	L3
OR				
15	Explain the process of crime analysis and E-Commerce in data science	12M	20DS302.5	L3

Foundations of Data Science
(20DS302)



N S RAJU INSTITUTE OF TECHNOLOGY
(AUTONOMOUS)

SONTYAM, ANANDAPURAM, VISAKHAPATNAM – 531 173

ANSWER KEY AND SCHEME OF EVALUATION

Short Answer Questions:

1. Define Data Science.

Data science is a multidisciplinary approach to extract actionable insights from the large and ever-increasing volumes of data collected and created by today's organizations.

2. What are the differences between excel and knime for doing data science problem?

Use KNIME Analytics Platform as an Excel alternative, so you can build visual workflows - from simple to complex - that perform any task on your data in a reproducible and scalable way. There are no limits to the number of data sources, rows or columns, or steps in the process. Once the data is in KNIME, you can manipulate and enhance it using state-of-the-art advanced analysis methods. Results can be visualized directly in KNIME or sent to a database, reporting tool, or written into a new file.

Answer is explained in general terms students can explain in their own way.

3. Distinguish data mining and data science.

Data science is a broad field that includes the processes of capturing of data, analyzing, and deriving insights from it. On the other hand, data mining is mainly about finding useful information in a dataset and utilizing that information to uncover hidden patterns.

4. How to make maps from spatial data?

1. Open a new worksheet.
2. In the Data pane, under Measures, double-click the Geometry field.
3. The Geometry field is added to Detail on the Marks card, and the Latitude (generated) and Longitude (generated) fields are added to the Columns and Rows shelves. A map view is created.

Answer is explained in Tableau, however student can choose any programming language

4. List out the four concepts of data science in journalism

Who Is the Audience?

What: Getting Directly to the Point

When Did It Happen?

Where Does the Story Matter?

Any four concepts related to data journalism

Part B (Long Answer Questions 5 x 12 = 60 Marks)

6 (a) Distinguish Business intelligence and Data Science. 6M

Parameters	Business Intelligence	Business-Centric Data Science
Data Sources	Uses only structured data from relational databases	Use structured and unstructured data
Outputs	Products include reports, data tables, and decision-support dashboards	Products involve either dashboard analytics or another type of advanced data visualization, but rarely tabular data reports
Technology	Relational databases, data warehouses, OLAP, and ETL technologies	Hadoop, MapReduce, Massively Parallel Processing
Expertise	Relies on IT and business technology expertise	Relies on expertise in statistics, math, programming, and business

6 (b) Explain the conversion of raw data into actionable insights with data analytics. 6M

Types of analytics

Common challenges in analytics

Data wrangling

7 (a) Analyze the Pieces of Data Science puzzle. 6M

Collecting, querying, and consuming data

Applying mathematical modeling to data science tasks

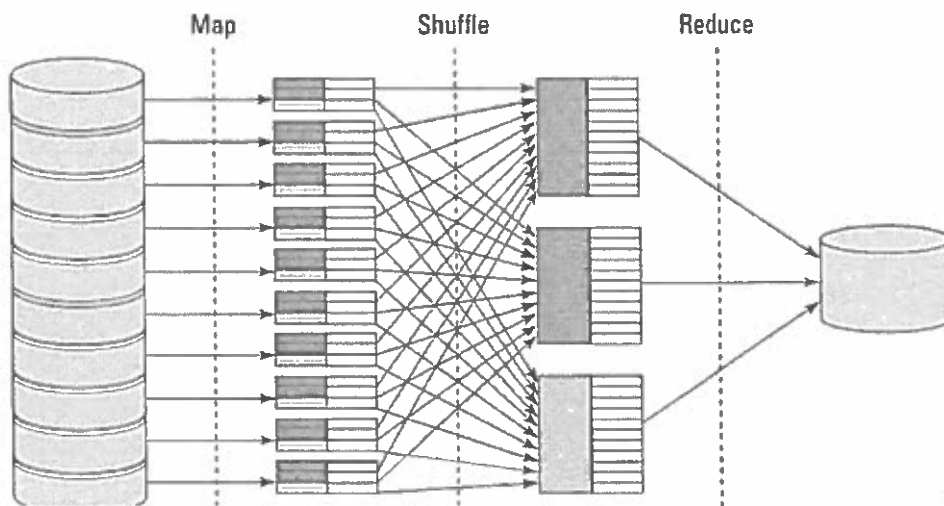
Deriving insights from statistical methods

Applying data science to a subject area

Communicating data insights

7 (b) Illustrate Hadoop Architecture with neat example. 6M

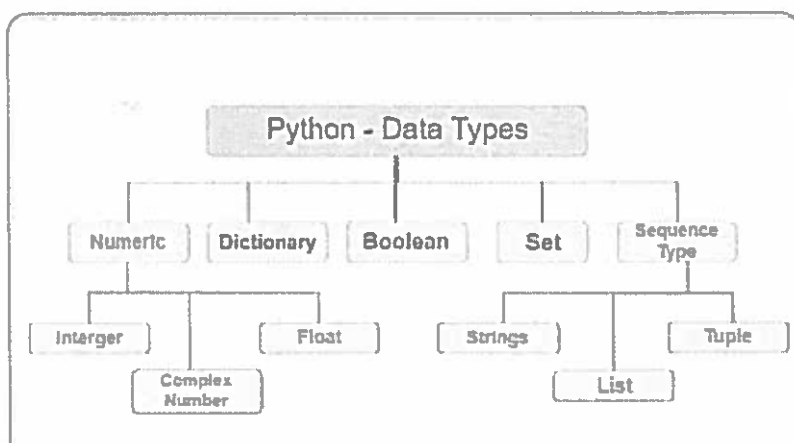
Digging into Map Reduce



Storing data on the Hadoop distributed file system (HDFS)

8 (a) List out the four Python Data types. Explain it with neat code. 6M

Students can explain any four data types



8 (b) Explain any two python data visualization packages. 6M

1. Matplotlib
2. Plotly
3. Seaborn
4. GGplot

9 (a) What are the differences between R and SQL in data science? Explain it. 6M

R Programming language 3 Marks

Sql Programming language 3 marks

9 (b) List out the two applications of Ktime with neat example.

Students can list down any examples related to ktime following are some of the applications where Ktime can be useful.

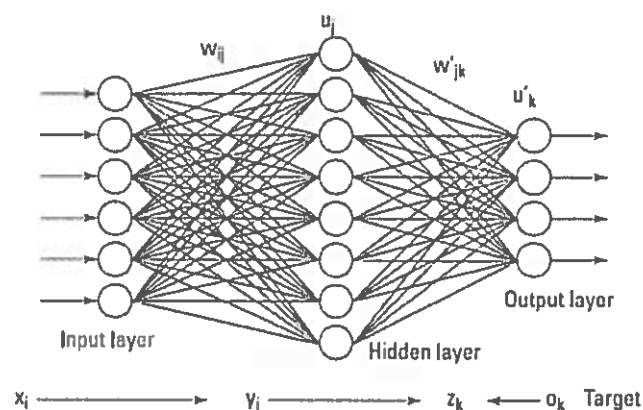
Up sell and cross-sell: Build cross-selling and upselling models that enable you to increase sales by making optimal recommendations of other products that customers are likely to be interested in purchasing as well.

Churn reduction: Mine customer data and identify which customers you're most likely to lose and why

Sentiment and network analysis: Analyze the sentiment of people and organizations in your social networks, to help identify which areas of your business are performing well and which ones may need some work.

Energy usage prediction and auditing: Perform time series analyses and build regression models from energy usage data.

10 (a) Summarize Artificial Neural Network Model with example. 6M



Students should be able to explain each layer.

10 (b) Define Linear Regression Model. How it is modeled? Explain it. 6M

Linear regression is a machine learning method you can use to describe and quantify the relationship between your target variable, y — the predictant, in statistics lingo — and the dataset features you've chosen to use as predictor variables (commonly designated as dataset X in machine learning). When you use just one variable as your predictor, linear regression is as simple as the middle school algebra formula $y=mx+b$. But you can also use linear regression to quantify correlations between several variables in a dataset — called multiple linear regressions.

The representation is a linear equation that combines a specific set of input values (x) the solution to which is the predicted output for that set of input values (y). As such, both the input values (x) and the output value are numeric.

The linear equation assigns one scale factor to each input value or column, called a coefficient and represented by the capital Greek letter Beta (B). One additional coefficient is also added, giving the line an additional degree of freedom (e.g. moving up and down on a two-dimensional plot) and is often called the intercept or the bias coefficient.

For example, in a simple regression problem (a single x and a single y), the form of the model would be:

$$y = B_0 + B_1 * x$$

In higher dimensions when we have more than one input (x), the line is called a plane or a hyper-plane. The representation therefore is the form of the equation and the specific values used for the coefficients (e.g. B0 and B1 in the above example).

11 (a) Explain K-Means algorithm for clustering food and drinks with K=2. 6M

K-Means clustering is an unsupervised learning algorithm. There is no labeled data for this clustering, unlike in supervised learning. K-Means performs the division of objects into clusters that share similarities and are dissimilar to the objects belonging to another cluster.

The term 'K' is a number. You need to tell the system how many clusters you need to create. For example, K = 2 refers to two clusters. There is a way of finding out what is the best or optimum value of K for a given data.

For a better understanding of k-means, let's take an example from cricket. Imagine you received data on a lot of cricket players from all over the world, which gives information on the runs scored by the player and the wickets taken by them in the last ten matches. Based on this information, we need to group the data into two clusters, namely batsman and bowlers.

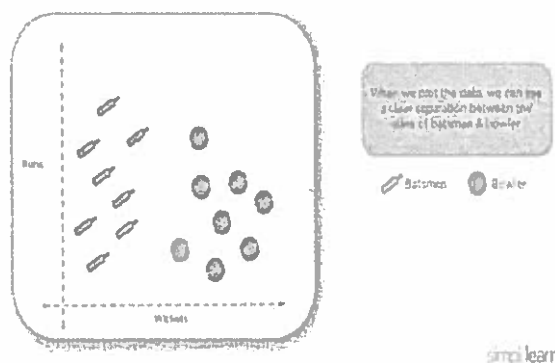
Let's take a look at the steps to create these clusters.

Solution:

Assign data points

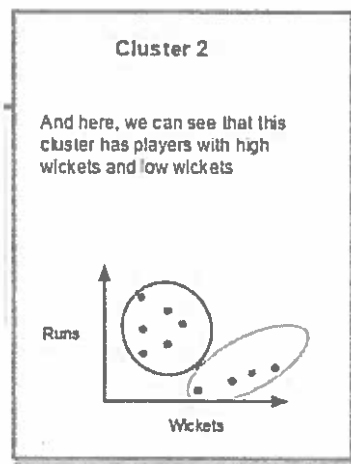
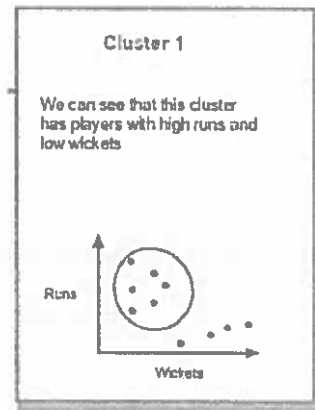
Here, we have our data set plotted on 'x' and 'y' coordinates. The information on the y-axis is about the runs scored, and on the x-axis about the wickets taken by the players.

If we plot the data, this is how it would look:



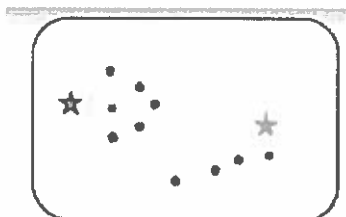
Perform Clustering

We need to create the clusters, as shown below:



Considering the same data set, let us solve the problem using K-Means clustering (taking $K = 2$).

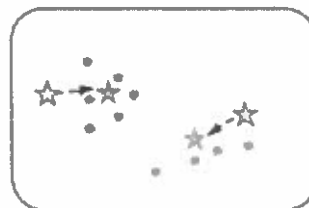
The first step in k-means clustering is the allocation of two centroids randomly (as $K=2$). Two points are assigned as centroids. Note that the points can be anywhere, as they are random points. They are called centroids, but initially, they are not the central point of a given data set.



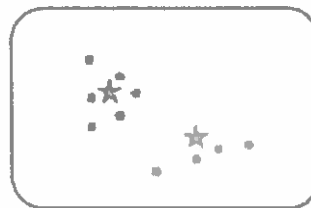
The next step is to determine the distance between each of the randomly assigned centroids' data points. For every point, the distance is measured from both the centroids, and whichever distance is less, that point is assigned to that centroid. You can see the data points attached to the centroids and represented here in blue and yellow.



The next step is to determine the actual centroid for these two clusters. The original randomly allocated centroid is to be repositioned to the actual centroid of the clusters.



This process of calculating the distance and repositioning the centroid continues until we obtain our final cluster. Then the centroid repositioning stops.



As seen above, the centroid doesn't need anymore repositioning, and it means the algorithm has converged, and we have the two clusters with a centroid.

11 (b) Distinguish Classification and Clustering with neat example. 6M

Parameter	CLASSIFICATION	CLUSTERING
Type	used for supervised learning	used for unsupervised learning
Basic	process of classifying the input instances based on their corresponding class labels	grouping the instances based on their similarity without the help of class labels

Parameter	CLASSIFICATION	CLUSTERING
Need	it has labels so there is need of training and testing dataset for verifying the model created	there is no need of training and testing dataset
Complexity	more complex as compared to clustering	less complex as compared to classification
Example Algorithms	Logistic regression, Naive Bayes classifier, Support vector machines, etc.	k-means clustering algorithm, Fuzzy c-means clustering algorithm, Gaussian (EM) clustering algorithm, etc.

12 Examine D3.js for web based applications for visual design. 12M

The main purpose of D3.js in the data visualization world is to make creative, interactive, web-based visualizations using only a small bit of code. Because D3.js is so powerful and open-ended, it's also more complex than some other JavaScript libraries, web applications, and software packages. D3.js has the same basic syntax as JavaScript, so you need a basic working knowledge of JavaScript in order to code in D3.js. Beyond using a bit of JavaScript, you need to know a whole vocabulary and programming. (Description of D3.JS 3 Marks)

Bringing in the HTML and DOM - 2 Marks

Bringing in the JavaScript and SVG - 2 Marks

Bringing in the Cascading Style Sheets (CSS) - 2 Marks

Bringing in the web servers and PHP - 3 Marks

13 Relate and explain any two open source applications for data visualization. 12M

GGPlot 6 Marks

Plotly 6 Marks

Students can explain any open source applications not only limited to the above

14 Explain how data science can be useful in Environmental data science. 12M

1. Modeling Environmental-Human Interactions with Environmental Intelligence 4 Marks
2. Modeling Natural Resources in the Raw 4 Marks
3. Using Spatial Statistics to Predict for Environmental Variation across Space 4 Marks

15 Explain the process of crime analysis and E-Commerce in data science 12M

The process of crime analysis

1. Temporal Analysis for Crime Prevention and Monitoring 2 Marks
2. Spatial Crime Prediction and Monitoring 2 Marks
3. Probing the Problems with Data Science for Crime Analysis 2 Marks

E-Commerce

1. Making Sense of Data for E-Commerce Growth 3 Marks
2. Optimizing E-Commerce Business Systems 3 Marks



Semester End Examination, February, 2022

Degree	B. Tech. (U. G.)	Program	CSE, CSE (AI & ML) & CS (DS)	Academic Year	2021 - 2022
Course Code	20CS305	Test Duration	3 Hrs.	Max. Marks	70
Course	COMPUTER ORGANIZATION	Semester	III		

Part A (Short Answer Questions 5 x 2 = 10 Marks)

No.	Questions (1 through 5)	Learning Outcome (s)	DoK
1	Find (1001101 - 10101001) using 2's complement?	20CS305.1	L2
2	List any three Memory reference Instructions.	20CS305.2	L1
3	What are the functions of control memory?	20CS305.3	L1
4	What is Divide over flow?	20CS305.4	L1
5	List any two advantages of Virtual memory	20CS305.5	L1

Part B (Long Answer Questions 5 x 12 = 60 Marks)

No.	Questions (6 through 15)	Marks	Learning Outcome (s)	DoK
6 (a)	Explain operations on unsigned binary numbers and perform the subtraction with the following unsigned binary numbers using 2's complement. i) 11010-10000 ii) 11010-1101 iii) 100-110000	6M	20CS305.1	L2
6 (b)	Draw the circuit diagram of a 4-bit binary counter with parallel load and explain its working with its function table.	6M	20CS305.1	L1
OR				
7 (a)	Represent the decimal number 7453 in (i) BCD (ii) Excess-3 (iii) 2421 (iv) Binary number	6M	20CS305.1	L2
7 (b)	Explain the fixed point representation with an example.	6M	20CS305.1	L2
8 (a)	Explain about arithmetic micro operations in detail.	6M	20CS305.2	L2
8 (b)	Explain memory-reference instructions. Draw the flow chart for memory-reference instructions.	6M	20CS305.2	L2
OR				
9 (a)	Explain about the 4-bit binary adder-subtractor with neat sketches	6M	20CS305.2	L2
9 (b)	Draw the diagram of 4-bit arithmetic circuit. Explain the function of circuit with the help of function table.	6M	20CS305.2	L2
10 (a)	What are different types of instruction formats? Explain with examples.	8M	20CS305.3	L2
10 (b)	Explain about the format of microinstruction.	4M	20CS305.3	L2
OR				
11 (a)	Explain five addressing modes with examples in detail.	5M	20CS305.3	L2
11 (b)	Explain about various components in the organization of a micro programmed control unit.	7M	20CS305.3	L2
12 (a)	What are the steps involved in the addition of 2's complement notation. Explain with an example	6M	20CS305.4	L2
12 (b)	Explain Booth multiplication algorithm with an example	6M	20CS305.4	L2
OR				
13 (a)	What is divide overflow. Explain about division of two signed-magnitude numbers with a neat flow chart.	6M	20CS305.4	L2
13 (b)	Draw the flow chart for addition and subtraction of two floating point numbers. Also explain about its hardware implementation.	6M	20CS305.4	L2
14 (a)	What is locality of reference? Discuss about types of mapping techniques used in cache memory.	7M	20CS305.5	L2
14 (b)	What is the difference between isolated I/O and memory mapped I/O?	5M	20CS305.5	L2
OR				
15 (a)	Explain about handshaking with a neat block diagram.	6M	20CS305.5	L2
15 (b)	Explain about Direct Memory Access with neat block diagram.	6M	20CS305.5	L2

CSG-14



N S RAJU INSTITUTE OF TECHNOLOGY

(AUTONOMOUS)

SONTYAM , ANANDAPURAM, VISAKHAPATNAM – 531 173

ANSWER KEY AND SCHEME OF EVALUATION

Computer Organization (20CS305)
(Common to CSE, CSE(AI & ML), CSE(DS))

Part A (Short Answer Question 5X2 =10 Marks)

1. Find $(1001101 - 10101001)$ using 2's complement?

2's complement of $10101001 = 1$'s complement of $10101001 + 1$
 $= 01010110 + 00000001$

2's complement of $10101001 = 01010111$

$$\begin{array}{r} 01001101 \\ + 01010111 \\ \hline 10100100 \end{array}$$

Find two's complement of 10100100 as left most bit is 1 and we get -01011100
($77 - 169 = -92$)

So final result is -01011100

Solving problem by finding 2's complement of $10101001 - 1$ M + finding 2's complement of final result $- 1$ M

2. List any three memory-reference instructions

Mention any three memory-reference instructions among AND, ADD, LDA, STA, BUN, BSA and ISZ

For example: AND-Performs AND logic operation on bits in AC and memory word given by the effective address

LDA-Load to AC-Transfers the memory word specified by the effective address to AC.

AND requires operation decoder D_0 and is symbolically represented as $AC \leftarrow AC \wedge M[AR]$

ADD requires operation decoder D_1 and is symbolically represented as $AC \leftarrow AC + M[AR], E \leftarrow C_{out}$

Any three memory reference instructions if mentioned $- 2$ M

3. What are the functions of control memory?

Control memory holds a fixed microprogram that cannot be altered by occasional user. In control memory all the control information is permanently stored. The control memory address register specifies the address of the microinstruction.

Functions of control memory $- 2$ M

4. What is divide overflow?

If a division operand produces a quotient that is too large to fit into the destination operand, divide overflow results. A divide- overflow condition occurs if the high-order half bits of the dividend constitute a number greater than or equal to the divisor. The divide overflow condition takes care of division by zero which must be avoided.

Divide overflow $- 1$ M + Condition for divide overflow $- 1$ M

5. List any two advantages of virtual memory

Advantages-Large programs can be written, as virtual space available is huge compared to physical memory.

More physical memory available, as programs are stored on virtual memory, so they occupy very less space on actual physical memory and allocating memory is easy and cheap.

Mention any two advantages — 2 M

Part B (Long Answer Questions 5X12 = 60 Marks)

6 (a) Explain operations on unsigned binary numbers and perform the subtraction with the following unsigned binary numbers using 2's complement i) $11010 - 10000$ ii) $11010 - 1101$ iii) $100 - 110000$

(i) 2's complement of 10000 = 1's complement of 10000 + 1
 $= 01111 + 00001$

2's complement of 10000 = 10000

$$\begin{array}{r} 11010 \\ + 10000 \\ \hline 1 \overline{)01010} \\ (26-16 = 10) \end{array}$$

(ii) $11010 - 1101$

$$\begin{array}{r} 11010 \\ + 10011 \text{ (2's complement of 01101)} \\ \hline 1 \overline{)01101} \\ (26-13 = 13) \end{array}$$

(iii) $100 - 110000$

$$\begin{array}{r} 000100 \\ + 010000 \text{ (2's complement of 110000)} \\ \hline 0010100 \\ -101100 \text{ (2's complement of 0010100)} \\ \hline (4-48 = -44) \end{array}$$

Each problem solved carries 2 marks - $2 \times 3 = 6$ M

6 (b) Draw the circuit diagram of a 4-bit binary counter with parallel load and explain its working with its function table.

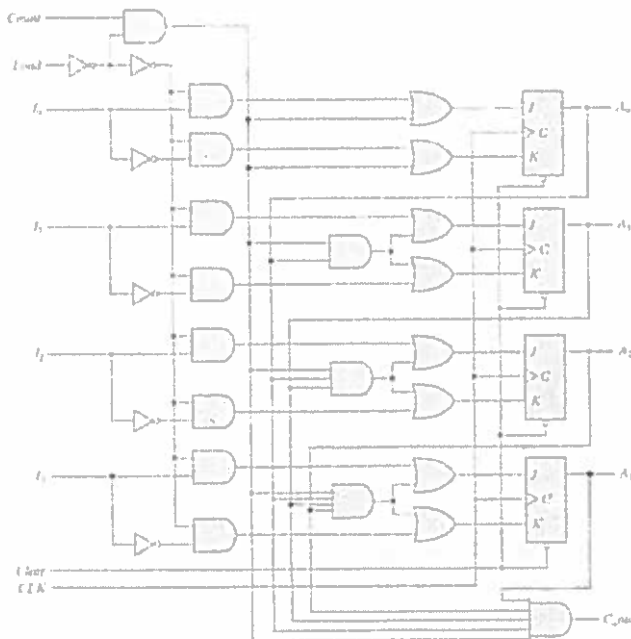


Fig. 4-bit binary counter with parallel load and synchronous clear

Function table

Clock	Clear	Load	Increment	Function
↑	0	0	0	No change
↑	0	0	1	Increment count by 1
↑	1	1	x	Load inputs I_0 through I_3
↑	1	x	x	Clear outputs to zero

4-bit binary counter with parallel load circuit can be designed using D flip-flop also and also through JK flip-flop as shown above

Circuit — 4 M + Function table with its working explanation— 2 M

7 (a) Represent the decimal number 7453 in (i) BCD (ii) Excess-3 (iii) 2421 (iv) Binary number 7453

(i) BCD	0111	0100	0101	0011
(ii) Excess-3 or XS-3	1010	0111	1001	0111
(iii) 2421	1101	0100	1011	0001
(iv) Binary	0001 1101 0001 1101 (4096+2048+1024+256+29=7453)			

BCD,XS-3,2421,Binary – for each number conversion 1.5 mark → 1.5 X 4 = 6 M

7 (b) Explain the fixed point representation with an example.

Fixed-point numbers having decimal points at the right end of the number are treated as integers because the fixed-point numbers having decimal points at the left end of the number are treated as fractions.

Sign and Magnitude Representation : The most significant bit in the number is used for denoting the sign; 0 denotes a positive integer, and 1 denotes a negative integer. The remaining bits in the number provide the magnitude of the number.

Example : $(-24)_{10}$ is defined as $\rightarrow 10011000$

In this example, the most significant bit or leftmost bit 1 means negative, and the magnitude is 24.

The magnitude for both positive and negative values is equal, but they alter only with their signs.

Signed 1's Complement Representation: In this representation, a negative value is received by taking the 1's complement of the equivalent positive number. It can add a signed 1's complement method creates end carry during arithmetic operation that cannot be rejected.

Example: Consider 5-bit numbers for 1's complement.

$(12)_{10} = (01100)_2 = (01100)_2$ is one's complement for positive value, $-(13)_{10} = -(01101)_2 = 10010$ is one's complement for negative value

Signed 2's Complement Representation: In signed 2's complement representation, the 2's complement of a number is discovered by first creating the 1's complement of that number, then incrementing the result by 1.

Example : $(-6)_{10} = -(000110)_2 = (111001)_2 + (000001)_2 = (111010)_2$. So 2's complement of -6 is 111010 in binary

Fixed point representation – 3 M, Example for sign magnitude, signed 1's complement and signed 2's complement – 3 M

8 (a) Explain about arithmetic microoperations in detail.

Arithmetic microoperations- Perform arithmetic operations on numeric data in registers

The basic arithmetic microoperations are: addition, subtraction, increment and decrement.

$R3 \leftarrow R1 + R2$. It states that contents of registers R1 and R2 are added and sum is transferred to register R3

Subtraction Microoperation: $R3 \leftarrow R1 - R2$ or : $R3 \leftarrow R1 + \overline{R2} + 1$

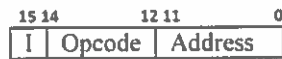
Symbolic designation	Description
$R3 \leftarrow R1 + R2$	Contents of R1 plus R2 transferred to R3
$R3 \leftarrow R1 - R2$	Contents of R1 minus R2 transferred to R3
$R2 \leftarrow R2'$	Complement the contents of R2 (1's complement)
$R2 \leftarrow R2' + 1$	2's complement the contents of R2 (negate)
$R3 \leftarrow R1 + R2' + 1$	R1 plus 2's complement of R2(subtraction)
$R1 \leftarrow R1 + 1$	Increment the content of R1 by one
$R1 \leftarrow R1 - 1$	Decrement the content of R1 by one

Table - Arithmetic microoperations

Arithmetic microoperations – Definition and example – 2 M and microoperations with table and explanation – 4 M

8 (b) Explain memory-reference instructions. Draw the flow chart for memory-reference instructions.

Memory-reference instructions: These instructions refer to memory address as an operand. The other operand is always accumulator.



There are 7 memory-reference instructions as listed below
AND, ADD, LDA, STA, BUN, BSA and ISZ.

Symbol	Operation decoder	Symbolic description
AND	D_0	$AC \leftarrow AC \wedge M[AR]$
ADD	D_1	$AC \leftarrow AC + M[AR], E \leftarrow C_{out}$
LDA	D_2	$AC \leftarrow M[AR]$
STA	D_3	$M[AR] \leftarrow AC$
BUN	D_4	$PC \leftarrow AR$
BSA	D_5	$M[AR] \leftarrow PC, PC \leftarrow AR + 1$
ISZ	D_6	$M[AR] \leftarrow M[AR] + 1$ If $M[AR] + 1 = 0$ then $PC \leftarrow PC + 1$

TABLE Memory-Reference Instructions

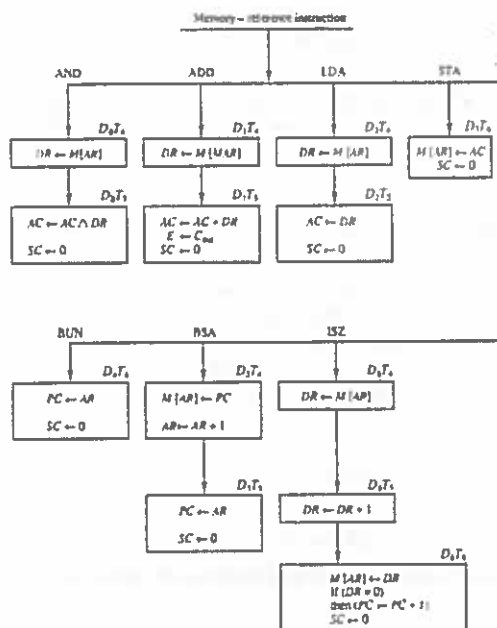


Figure Flowchart for memory-reference instructions.

Memory reference instructions names with explanation – 4 M + Flowchart – 2 M

9 (a) Explain about the 4-bit binary adder-subtractor with neat sketches

The digital circuit that generates the arithmetic addition as well as subtraction of two binary numbers of any length is called a binary adder-subtractor. It can be done by including exclusive-OR gate with each full-adder. The circuit consists of a mode bit M. If M=0 it acts as an adder and if it is 1 it acts as a subtractor. Each exclusive-OR gate receives input M and input B.

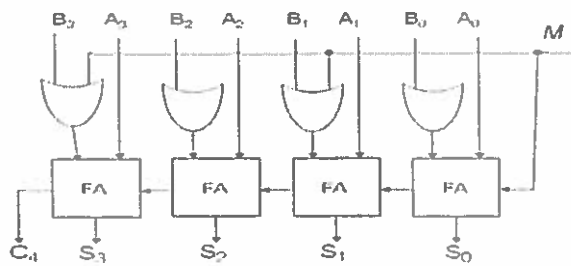


Fig. 4-bit binary adder-subtractor

Definition of binary adder – 1 M + Circuit – 2 M + Explanation – 3 M

9 (b) Draw the diagram of 4-bit arithmetic circuit. Explain the function of circuit with the help of function table.

Select	Input	Output	
$S_1 S_0 C_{in}$	Y	$D = A + Y + C_{in}$	Microoperation
0 0 0	B	$D = A + B$	Add
0 0 1	B	$D = A + B + 1$	Add with carry
0 1 0	B'	$D = A + B'$	Subtract with borrow
0 1 1	B'	$D = A + B' + 1$	Subtract
1 0 0	0	$D = A$	Transfer A
1 0 1	0	$D = A + 1$	Increment A
1 1 0	1	$D = A - 1$	Decrement A
1 1 1	1	$D = A$	Transfer A

Table - Function table for Arithmetic Circuit

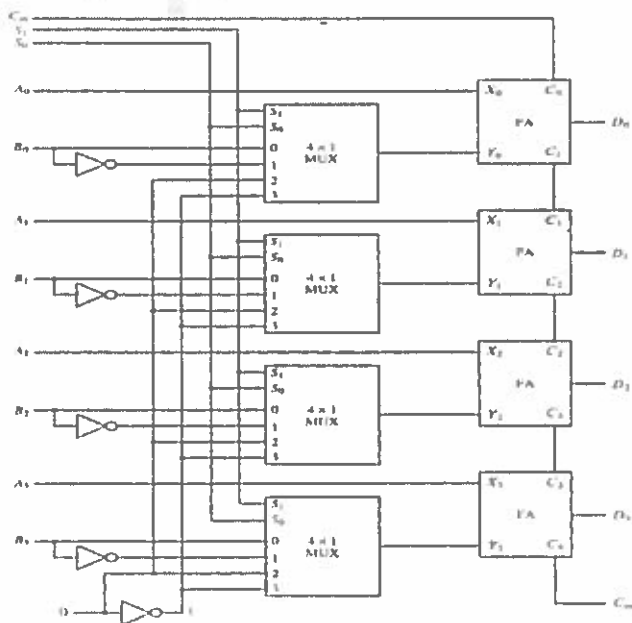


Fig. 4-bit arithmetic circuit

The diagram of a 4-bit arithmetic circuit has four full-adder circuits that constitute the 4-bit adder and four multiplexers for choosing different operations. There are two 4-bit inputs A and B and a 4-bit output D. The four inputs from A go

directly to the X inputs of the binary adder. Each of the four inputs from B are connected to the data inputs of the multiplexers. The multiplexers data inputs also receive the complement of B.

The other two data inputs are connected to logic-0 and logic-1. Logic-0 is a fixed voltage value and the logic-1 signal can be generated through an inverter whose input is 0. The four multiplexers are controlled by two selection inputs, S_1 and S_0 .

The input carry C_{in} goes to the carry input of the FA in the least significant position. The other carries are connected from one stage to the next. The output of the binary adder is calculated from the following arithmetic sum:

$$D = A + Y + C_{in}$$

4-bit arithmetic circuit – 3 M + explanation of circuit with function table – 3 M

10 (a) What are different types of instruction formats? Explain with examples.

Instruction includes a set of operation codes and operands that manage with the operation codes. Instruction format contains fields including opcode, operands, and addressing mode.

Based on number of address, instructions are classified as zero-address, one-address, two-address, three-address and RISC.

Example : $X = (A+B) * (C+D)$

Zero Address Instruction	One Address Instruction	Two Address
Instruction		
PUSH A $TOS \leftarrow A$	LOAD A $AC \leftarrow M[A]$	MOV R1,A
$R1 \leftarrow M[A]$		
PUSH B $TOS \leftarrow B$	ADD B $AC \leftarrow AC + M[B]$	ADD R1,B
$R1 \leftarrow R1 + M[B]$		
ADD $TOS \leftarrow (A + B)$	STORE T $M[T] \leftarrow AC$	MOV R2,C
$R1 \leftarrow M[C]$		
PUSH C $TOS \leftarrow C$	LOAD C $AC \leftarrow M[C]$	ADD R2,D
$R2 \leftarrow R2 + M[D]$		
PUSH D $TOS \leftarrow D$	ADD D $AC \leftarrow AC + M[D]$	MUL R1,R2
$R1 \leftarrow R1 * R2$		
ADD $TOS \leftarrow (C + D)$	MUL T $AC \leftarrow AC * M[T]$	MOV X,R1
$M[X] \leftarrow R1$		
MUL $TOS \leftarrow (C + D) * (A + B)$	STORE X $M[X] \leftarrow AC$	
POP $M[X] \leftarrow TOS$		
Three Address Instruction	RISC Instruction	
ADD R1, A, B $R1 \leftarrow M[A] + M[B]$	LOAD R1, A $R1 \leftarrow M[A]$	
ADD R2, C, D $R2 \leftarrow M[C] + M[D]$	LOAD R2, B $R2 \leftarrow M[B]$	
MUL X,R1,R2 $M[X] \leftarrow R1 * R2$	LOAD R3, C $R3 \leftarrow M[C]$	
	LOAD R4, D $R4 \leftarrow M[D]$	
	ADD R1,R1,R2 $R1 \leftarrow R1 + R2$	
	ADD R3,R3,R2 $R3 \leftarrow R3 + R2$	
	MUL R1,R1,R3 $R1 \leftarrow R1 * R3$	
	STORE X,R1 $M[X] \leftarrow R1$	

Detailed explanation of Instruction format with examples – 8 M

10 (b) Explain about the format of microinstruction

A microinstruction format includes 20 bits in total. They are divided into four elements as displayed in the figure.

3	3	3	2	2	7
F1	F2	F3	CD	BR	AD

F1, F2 and F3 are micro operation fields. They determine the microoperations for the computer. CD is the condition for branching. They choose the status bit conditions. BR is the branch field which determines the type of branch. AD is the address field whose length is of 7 bits.

F1, F2 and F3 if they contain 000 it denotes NOP, F1- 001 it denotes Add, F2-001-Subtract,F3-001 means XOR and so on

A condition field includes 2 bits. They are encoded to define four status bit conditions as given below:

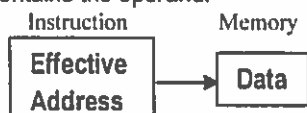
Condition	Symbols	Comments
00 Always(=1)	U	Unconditional branch
01 DR(15)	I	Indirect address bit
10 AC(15)	S	Sign bit of AC
11 AC = 0	Z	Zero value in AC
BR Symbol	Comments	
00 JMP	CAR $\leftarrow$ AD if condition = 1, CAR $\leftarrow$ CAR + 1 if condition = 0	
01 CALL	CAR $\leftarrow$ AD, SBR $\leftarrow$ CAR + 1, if condition = 1, CAR $\leftarrow$ CAR + 1 if condition = 0	
10 RET	CAR $\leftarrow$ SBR (return from subroutine)	
11 MAP	CAR(2-5) $\leftarrow$ DR(11-14), CAR(0,1,6) $\leftarrow$ 0	

Microinstruction format for 20 bits – 2 M + Explanation of fields with symbols used in CD and BR – 2 M

11 (a) Explain five addressing modes with examples in detail.

Addressing modes: Addressing mode refers to the different methods of addressing the operand. The addressing mode specifies a rule for interpreting or modifying the address field of the instruction before the operand is actually executed.

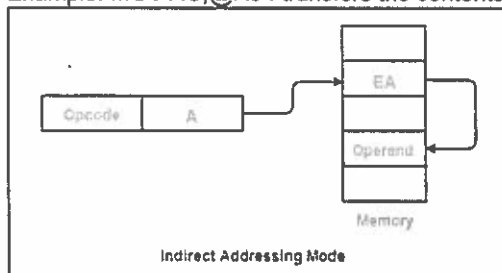
Direct addressing mode (represented by []) : The effective address of the data is part of the instruction itself. Here only one memory reference is used to access the data as the address specifies which register or memory word contains the operand.



Example: ADD AL,[0456]. This instruction adds the contents of location 456 to AL and stores the result in AL

Indirect addressing mode (represented by @ or ()) : In this mode address field of the instruction contains the address of the effective address.

Example: MOV AC,@R3 . transfers the contents of memory location whose address is stored in R3 into accumulator



Immediate addressing mode(represented by #): In this mode the source operand is always the data. Instead of address field an operand field is present that contains the data. The operand or data is specified in the instruction explicitly.

Ex: MOV R2,#38 initializes R2 to 38

Register addressing mode: It indicates the operand data is stored in the register itself, so the instruction contains the address of the register. The data would be retrieved from the register. Retrieving data from the register is fast and the instructions are shorter because no memory is involved.

Ex: ADD R will increment the value stored in the accumulator by the content of register R. $AC \leftarrow AC + R$

Register indirect addressing mode: In this addressing the operand's offset is placed in any of the registers as specified in the instruction. The effective address of the data is in the base register or an index register that is specified by the instruction.

Ex: MOV AX,[BX] (Move the contents of memory locations addressed by the register BX to the register AX)

Different addressing modes explanation and with examples on any five addressing modes – 5 M

11 (b) Explain about various components in the organization of a micro programmed control unit.

Micro programmed control unit: A control unit whose binary control variables are stored in memory is called a micro programmed control unit.

Control Memory: It is the storage in the microprogrammed control unit to store the microprogram.

Microprogram: A sequence of microinstructions constitutes a microprogram.

As alterations of the microprogram are not needed once control unit is in operation, the control memory can be a read-only memory (ROM).

Organization of micro programmed control unit: General configuration of a micro-programmed control unit is shown in the diagram below:

The control memory is assumed to be a ROM, within which all control information is permanently stored.

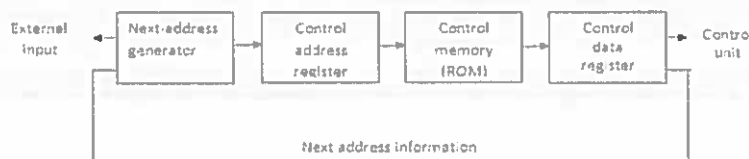


Fig. Micro-programmed control organization

The control memory address register specifies the address of the microinstruction, and the control data register holds the microinstruction read from memory. The microinstruction contains a control word that specifies one or more microoperations for the data processor. Once these operations are executed, the control must determine the next address. The location of the next microinstruction may be the one next in sequence, or it may be located somewhere else in the control memory.

While the microoperations are being executed, the next address is computed in the next address generator circuit and then transferred into the control address register to read the next microinstruction. Thus a microinstruction contains bits for initiating microoperations in the data processor part and bits that determine the address sequence for the control memory.

The next address generator is sometimes called a micro-program sequencer, as it determines the address sequence that is read from control memory. Typical functions of a micro-program sequencer are incrementing the control address register by one, loading into the control address register an address from control memory, transferring an external address, or loading an initial address to start the control operations.

The control data register holds the present microinstruction while the next address is computed and read from memory. It allows the execution of the microoperations specified by the control word simultaneously with the generation of the next microinstruction.

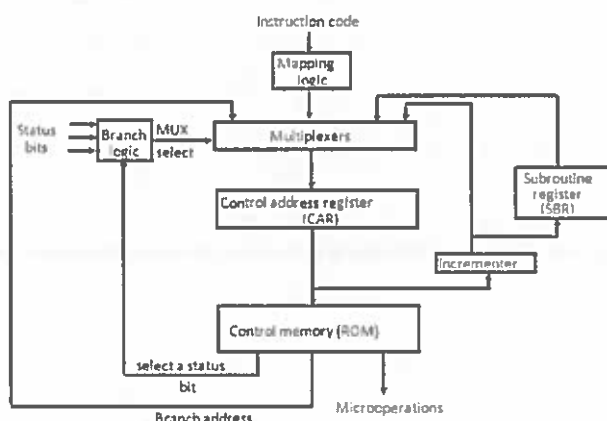


Fig. Selection of address for control memory

Description of different components of microprogrammed control unit with diagram – 5 M + control unit of microprogrammed control unit – 2 M

12 (a) What are the steps involved in the addition of 2's complement notation?

The rule for adding numbers in the signed-2's complement system does not require a comparison or subtraction, only addition and complementation. The procedure is to add the two numbers, including their sign bits, and discard any carry out of the sign bit position. Negative numbers must initially be in 2's complement and if the sum thus obtained after the addition is negative, it is in 2's complement form. Add the two numbers, including their sign bits, and discard any carry out of the sign bit position.

Ex: Add -8 to +3

If 2 two's complement numbers are added, and they both have the same sign (both positive or both negative), then overflow occurs if and only if the result has the opposite sign. Overflow never occurs when adding operands with different signs.

(+3) = 0000 0011 (-8) = 0000 1000
1011

+3 0000 0011 In 2's complement (-8) = 1111 1000
- 8 + 1111 1000

- 5 1111 1011
1111 1011 (-5)

1111 1011 is negative as leftmost bit is 1.

(-5) = 0000 0101 , (-5) in 2's complement = 1111

(-2) = 0000 0010 , (-2) in 2's complement → 1111 1110
- 5 1111 1110
+ - 2 + 1111 1011

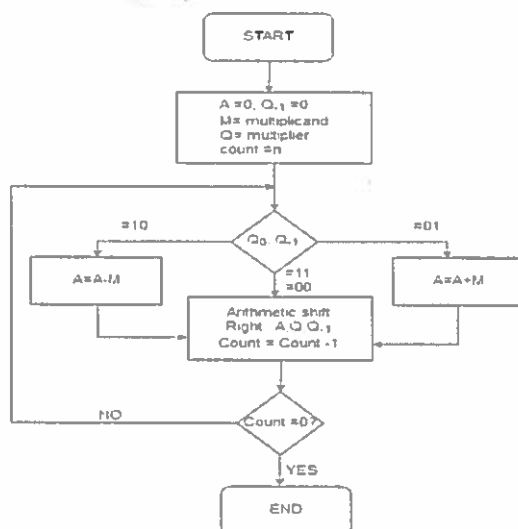
- 7 1)1111 1001 → Discard carry-out
1111 1001 is negative as leftmost bit is 1

Its two's complement is 0000 0101 which is +5

Its two's complement is 0000 0111 which is +7

Rules for signed 2's complement addition – 3 M Example – 3 M

12 (b) Explain Booth multiplication algorithm with an example



Here arithmetic shift right (ashr) is performed on A and multiplier Q including the appended Q_{-1} .

Example : Perform multiplication of 7 and 3

A	Q	Q ₋₁	M	SC	Remarks
0000	0011	0	0111	4	Initial values
1001	0011	0	0111	3	Subtract M { $A \leftarrow A - M$ }
1100	1001	1	0111		Perform Arithmetic shift right(ashr)
1110	0100	1	0111	2	Perform Arithmetic shift right(ashr)
0101	0100	1	0111	1	Addition { $A \leftarrow A + M$ }
0010	1010	0	0111		Perform Arithmetic shift right(ashr)
0001	0101	0	0111	0	Final product is available in A and Q

Flow chart for Booths algorithm – 4 M + Example to show Booths algorithm operation – 2 M

13 (a) What is divide overflow. Explain about division of two signed-magnitude numbers with a neat flow chart.

If a division operand produces a quotient that is too large to fit into the destination operand, divide overflow results. A divide overflow condition occurs if the high-order half bits of the dividend constitute a number greater than or equal to the divisor.

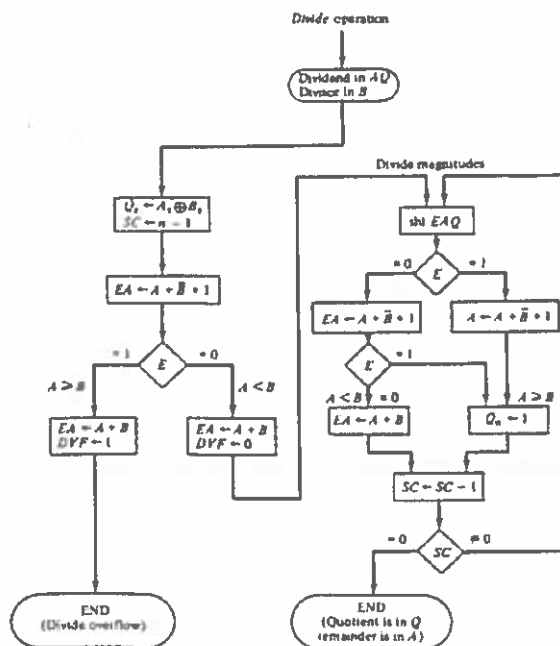


Fig. Flowchart for divide operation

The dividend is in A and Q and the divisor in B. Sign of the result is transferred into Q_n to be part of the quotient. SC is set to specify the number of bits in the quotient. The division of the magnitudes starts by shifting the dividend in AQ to the left with the high-order bit shifted into E.

If $E = 1$ then $EA > B$ and B is subtracted from EA and 1 inserted into Q_n for the quotient bit. If the shift left operation inserts a 0 into E , the divisor is subtracted by adding its 2's complement value and the carry is transferred into E . If $E = 1$ then Q_n is set to 1 as $A \geq B$ and if 0 it indicates that $A < B$ and original number is restored by adding B to A . In this latter case we leave a 0 in Q_n . This process is repeated again with register A which holds the partial remainder. After $n-1$ times, the quotient magnitude is formed in register Q and remainder is found in register A .

Divide Overflow – 2 M + Flowchart and its implementation – 4 M

13 (b) Draw the flow chart for addition and subtraction of two floating point numbers. Also explain about its hardware implementation.

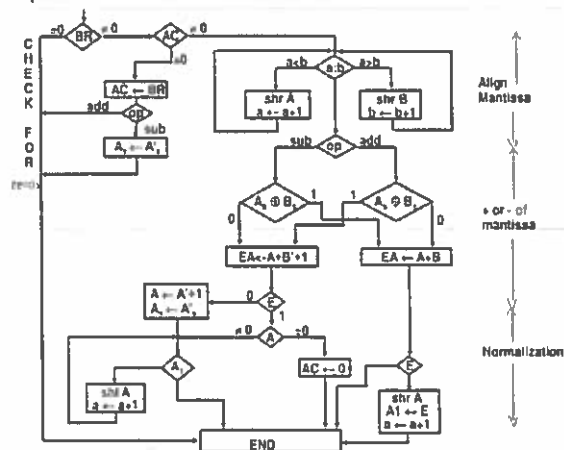


Fig. Addition and subtraction of floating-point numbers

During addition or subtraction, the 2 floating-point operands are in AC and BR . The sum or difference is formed in the AC . The algorithm can be divided into consecutive parts :

1. Check for zeros.
2. Align the mantissas.
3. Add or subtract the mantissas.
4. Normalize the result.

If $BR = 0$, the operation is terminated, with the value in the AC being the result. If $AC = 0$, we transfer the content of BR into AC and also complements its sign if the numbers are to be subtracted. If neither number is equal to zero, align the mantissa. The magnitude comparator attached to exponents a and b provide three outputs that indicate their relative magnitude. If the two

exponents are equal, we go to perform the arithmetic operation. If the exponents are not equal, the mantissa having the smaller exponent is shifted to the right and its exponent incremented. This process is repeated until the 2 exponents are equal. The magnitude part is added or subtracted depending on the signs of the two mantissa. When magnitude is added, then an overflow occurs and it is transferred into the flip-flop E .

Flowchart for addition and subtraction of two floating point numbers with explanation — 6 M

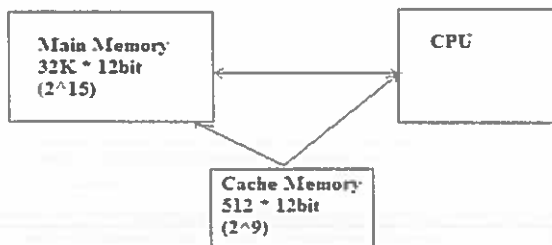
14 (a) What is locality of reference? Discuss about types of mapping techniques used in cache memory.

Locality of reference: It refers to the tendency of the computer program to access the same set of memory locations for a particular time period. The locality of reference property is mainly shown by loops and subroutine calls in a program. When CPU needs to access memory, the cache is examined. If the word is found in the cache, it is read from the fast memory. If the word addressed by the CPU is not found in the cache, the main memory is accessed to read the word.

The transformation of data from main memory to cache memory is referred to as a mapping process. There are three types of mapping:

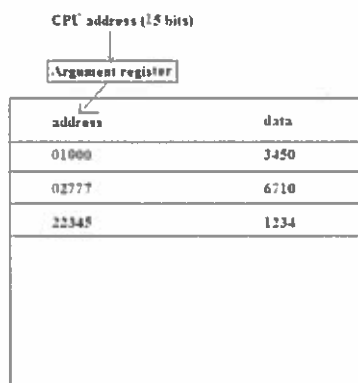
- i) Associative mapping
- ii) Direct mapping
- iii) Set-associative mapping

To help understand the mapping procedure, we have the following example:



Associative mapping: The fastest and most flexible cache organization uses an associative memory. The associative memory stores both the address and data of the memory word. This permits any location in cache to store any word from main memory.

The address value of 15 bits is shown as a five-digit octal number and its corresponding 12-bit word is shown as a four-digit octal number.



A CPU address of 15 bits is placed in the argument register & the associative memory is searched for a matching address. If the address is found, the corresponding 12-bit data is read and sent to the CPU. If not, the main memory is accessed for the word. If the cache is full, an address-data pair must be displaced to make room for a pair that is needed and not presently in the cache.

Direct Mapping

Associative memory is expensive compared to RAM. In general case, there are 2^k words in cache memory and 2^n words in main memory (in our case, $k=9$, $n=15$). The n bit memory address is divided into two fields: k -bits for the index and $n-k$ bits for the tag field.

Each word in cache consists of the data word and its associated tag. When a new word is first brought into the cache, the tags are stored alongside the data bits. When the CPU generates a memory request, the index field is used for the address to access the cache.

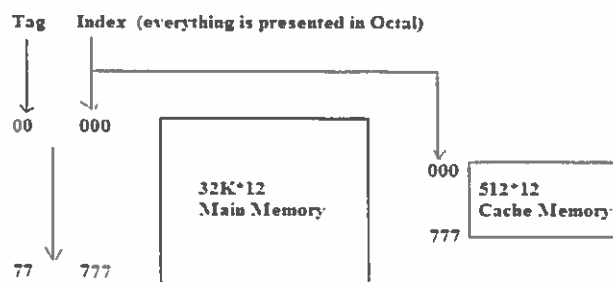


Fig. Addressing relationships between main and cache memories

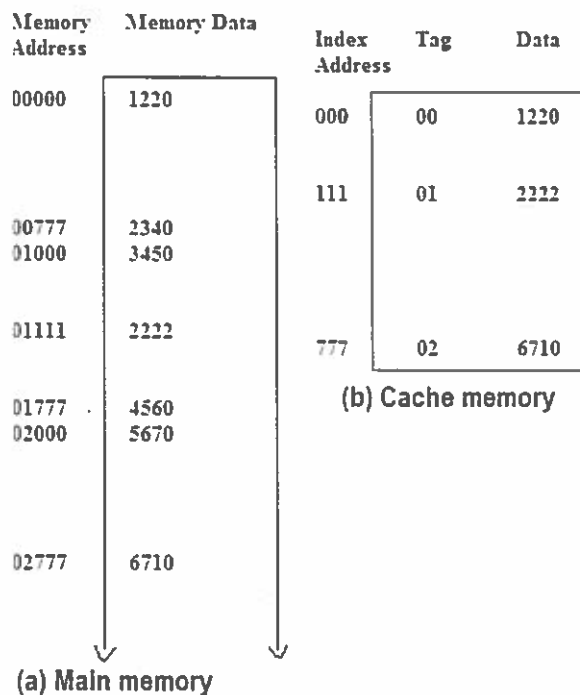


Fig. Direct mapping cache organization

The tag field of the CPU address is compared with the tag in the word read from the cache. If the two tags match, then it's a hit and the desired data word is in cache. Otherwise it's a miss and required word is read from main memory. It is then stored in the cache together with the new tag, replacing the previous value.

Consider the numerical example shown above. The word at address zero is presently stored in the cache(index=000, tag=00, data=1220). Let us assume that CPU wants to access the word at address 01000. The index address is 000, so it is used to access the cache. The two tags are then compared. The cache tag is 00 but the address tag is 01, which does not produce a match. Hence, the main memory is accessed and the data word 3450 is transferred to the CPU. The cache word at index address 000 is then replaced with a tag of 01 and data of 3450.

Set-Associative Mapping

The disadvantage of direct mapping is that two words with the same index in their address but with different tag values cannot reside in cache memory at the same time. Set-Associative Mapping is an improvement over the direct-mapping in that each word of cache can store two or more word of memory under the same index address

Memory Address	Memory Data	Index Address	Tag	Data	Tag	Data
00000	1220	000	01	3450	02	5670
00777	2340	111	01	2222		
01000	3450					
01111	2222	777	02	6710	00	2340
01777	4560					
02000	5670					
02777	6710					

Fig. Two-way set-associative mapping cache.

Each index address refers to two data words and their associated tags

Each tag requires six bits and each data word has 12 bits, so the word length is $2(6+12) = 36$ bits

In general, a set-associative cache of set size k will accommodate k words of main memory in each word of cache.

When the CPU generates a memory request, the index value of the address is used to access the cache. The tag field of the CPU address is then compared with both tags in the cache to determine if a match occurs.

The comparison logic is done by an associative search of the tags in the set similar to an associative memory search: thus the name "set-associative." The hit ratio will improve as the set size increases because more words with the same index but different tags can reside in cache.

When a miss occurs in a set-associative cache and the set is full, it is necessary to replace one of the tag-data items with a new value. We can use replacement policies like FIFO, LRU algorithms for replacing the item.

Locality of reference — 1 M + Associative, Direct and Set-Associative mapping with examples — 6 M

14 (b) What is the difference between isolated I/O and memory mapped I/O?

To communicate with I/O, the processor must communicate with the memory unit. Like the I/O bus, the memory bus contains data, address and read/write control lines. There are 3 ways that computer buses can be used to communicate with memory and I/O:

- Use two Separate buses , one for memory and other for I/O.
- Use one common bus for both memory and I/O but separate control lines for each.
- Use one common bus for memory and I/O with common control lines I/O Processor

Isolated I/O	Memory Mapped I/O
1. Memory and I/O have separate address space	1. Both isolated and memory mapped I/O have same address space.
2. Separate instruction control read and write operation in I/O and memory	2. Same instructions can control both I/O and memory.
3. All address can be used by memory	3. Due to addition of I/O addressable memory locations becomes less
4. More efficient due to separate buses	4. Lesser efficient
5. Larger in size due to more buses	5. Smaller in size

Differences between isolated I/O and memory mapped I/O – 5 M

15 (a) Explain about handshaking with a neat block diagram

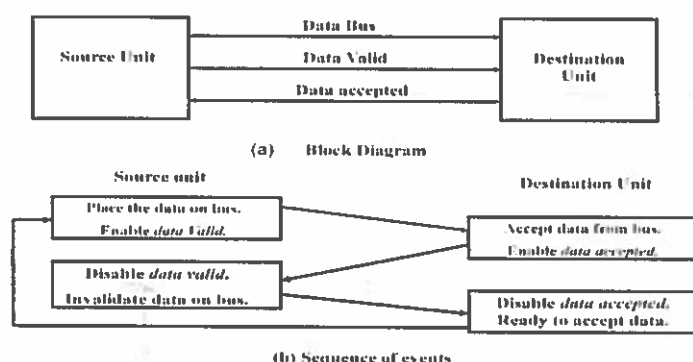
The handshaking method solves the problem of strobe method by introducing a second control signal that provides a reply to the unit that initiates the transfer.

Principle of Handshaking:

The basic principle of the two-wire handshaking method of data transfer is as follows:

One control line is in the same direction as the data flows in the bus from the source to destination. It is used by source unit to inform the destination unit whether there a valid data in the bus. The other control line is in the other direction from the destination to the source. It is used by the destination unit to inform the source whether it can accept the data. **Source Initiated Transfer Source-initiated transfer using handshaking:**

The sequence of events shows four possible states that the system can be at any given time. The source unit initiates the transfer by placing the data on the bus and enabling its data valid signal. The data accepted signal is activated by the destination unit after it accepts the data from the bus. The source unit then disables its data accepted signal and the system goes into its initial state.



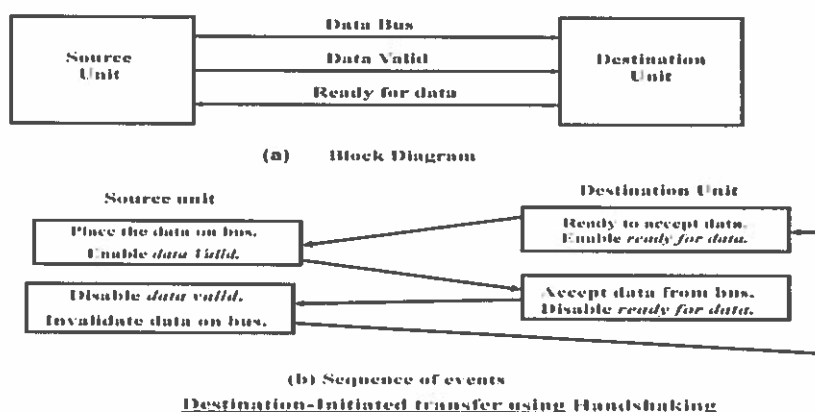
Destination Initiated Transfer Using Handshaking:

The source unit in this case does not place data on the bus until after it receives the ready for data signal from the destination unit. The handshaking procedure follows the same pattern as in the source initiated case.

Advantage of the Handshaking method:

The handshaking scheme provides degree of flexibility and reliability because the successful completion of data transfer relies on active participation by both units.

If any of one unit is faulty, the data transfer will not be completed. Such an error can be detected by means of a *Timeout mechanism* which provides an alarm if the data is not completed within time.



Handshaking – 2 M + Source initiated and destination handshaking with advantages – 4 M

15 (b) Explain with a block diagram the DMA transfer in a computer system

Direct memory access (DMA) is a feature of computer systems that allows certain hardware subsystems to access main system memory (random-access memory) independently of the central processing unit (CPU). In the Direct Memory Access (DMA) the interface transfer the data into and out of the memory unit through the memory bus. The transfer of data between a fast storage device such as magnetic disk and memory is often limited by the speed of the CPU. Removing the CPU from the path and letting the peripheral device manage the memory buses directly would improve the speed of transfer. This transfer technique is called Direct memory access (DMA). During the DMA transfer, the CPU is idle and has no control of the memory buses. A DMA controller takes over the buses to manage the transfer directly between the I/O device and memory. The two control signals Bus Request (BR) and Bus Grant (BG) in the CPU facilitates the DMA transfer.

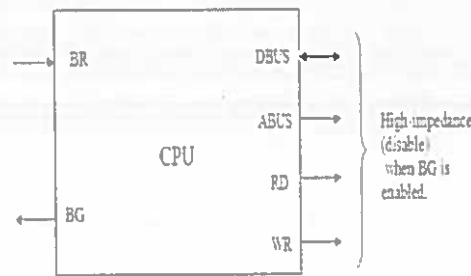


Fig CPU bus signals for DMA transfer

The Bus Request (BR) input is used by the DMA controller to request the CPU. When this input is active, the CPU terminates the execution of the current instruction and places the address bus, data bus and read write lines into a high impedance state. High Impedance state means that the output is disconnected.

When the BG (Bus Grant) input is 0, the CPU can communicate with the DMA registers through the data bus to read from or write to the DMA registers. When BG =1, the DMA can communicate directly with the memory by specifying an address in the address bus and activating the RD or WR control.

DMA Transfer:

The CPU communicates with the DMA through the address and data buses as with any interface unit. The DMA has its own address, which activates the DS and RS lines. The CPU initializes the DMA through the data bus. Once the DMA receives the start control command, it can transfer between the peripheral and the memory.

When BG = 0 the RD and WR are input lines allowing the CPU to communicate with the internal DMA registers. When BG=1, the RD and WR are output lines from the DMA controller to the random access memory to specify the read or write operation of data.

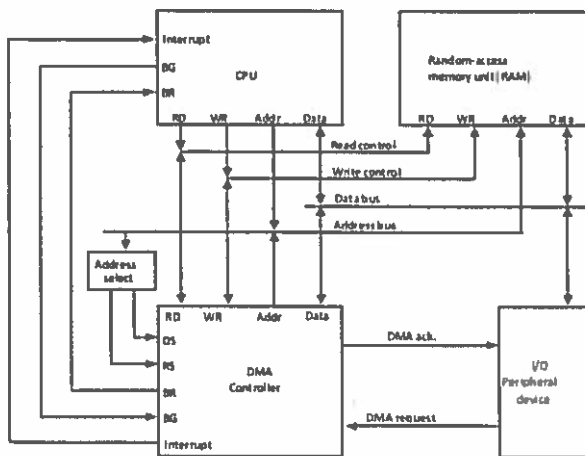


Fig. DMA transfer in a computer system

DMA – 2 M + DMA transfer block diagram with detailed explanation of transfer – 4 M

A. Luis Tuman



Semester End Examination, January/February, 2022

Degree	B. Tech. (U. G.)	Program	CE, EEE & MECH	Academic Year	2021 - 2022
Course Code	20BSX13	Test Duration	3 Hrs. Max. Marks 70	Semester	III
Course	Numerical Methods & Transforms				

Part A (Short Answer Questions 5 x 2 = 10 Marks)

No.	Questions (1 through 5)	Learning Outcome (s)	DoK
1	Write the iteration formulas of Regula Falsi method and Secant method	20BSX13.1	L1
2	Prove that $\Delta = \nabla E$	20BSX13.2	L2
3	Write Simpson's three-eighth rule	20BSX13.3	L1
4	Evaluate $L\{(t-1)^3 u(t-1)\}$	20BSX13.4	L2
5	Write the expressions for Fourier integral, Fourier Sine and Fourier Cosine integrals.	20BSX13.5	L1

Part B (Long Answer Questions 5 x 12 = 60 Marks)

No.	Questions (6 through 15)	Marks	Learning Outcome (s)	DoK
6 (a)	Using bisection method, compute a real root of the equation $xe^x - 2 = 0$, correct to three decimal places.	6M	20BSX13.1	L2
6 (b)	Find a real root of the equation $x \tan x + 1 = 0$ by Newton - Raphson method, correct to three decimal places, near $x = \pi$.	6M	20BSX13.1	L3

OR

7 (a)	Find a real root of the equation $x^3 + x - 1 = 0$ that is near to $x = 1$ by successive approximation method, correct to four decimal places.	6M	20BSX13.1	L2
7 (b)	Solve the following equations by Jacobi method: $5x - y + z = 10$; $2x + 4y = 12$; $x + y + 5z = -1$. Start with the solution (2, 3, 0).	6M	20BSX13.1	L3

Evaluate

8 (a)	$\Delta^2 \left[\frac{1}{x^2 + 5x + 6} \right] + \Delta [e^{3x} \log 2x]$	6M	20BSX13.2	L3

The area A of a circle of diameter d is given for the following values.
Calculate the area of a circle of diameter 7 using Newton's forward interpolation formula.

8 (b)		6M	20BSX13.2	L2

d	5	6	9	11
A	12	13	14	16

OR

Using the method of separation of symbols, prove that

9 (a)	$u_0 + u_1 + u_2 + u_3 + \dots + u_n =$ $(n+1)C_1 u_0 + (n+1)C_2 \Delta u_0 + (n+1)C_3 \Delta^2 u_0$ $+ \dots + (n+1)C_{n+1} \Delta^n u_0$	6M	20BSX13.2	L3

Find the number of students who obtained marks between 40 and 45

9 (b)	Marks	30-40	40-50	50-60	60-70	70-80	6M	20BSX13.2	L2
	No. of students	31	42	51	35	31			

Evaluate

10 (a)	$\int_0^6 \frac{1}{4x+5} dx$	6M	20BSX13.3	L2

by Trapezoidal and Simpson's 3/8th rule, by dividing the interval (0, 6) into 6 parts

10 (b)	Solve $y' = xy + 1, y(0) = 1$ using Picard's method and compute $y(0.2)$.	6M	20BSX13.3	L3
OR				
11 (a)	Evaluate $\int_0^1 \frac{1}{1+x} dx$ by Simpson's $\frac{1}{3}$ rd and Simpson's $\frac{3}{8}$ th rule.	6M	20BSX13.3	L2
11 (b)	Using Euler method, evaluate $y(0.1)$ and $y(0.2)$ given that $y' = x + y, y(0) = 1$.	6M	20BSX13.3	L3
12 (a)	Show that $\int_0^\infty t^2 e^{-4t} \sin 2t dt = \frac{1}{250}$	6M	20BSX13.4	L3
12 (b)	Using convolution theorem, evaluate the inverse Laplace transform of $\frac{s}{(s^2 + 16)^2}$	6M	20BSX13.4	L2
OR				
13 (a)	Evaluate the Laplace transform of $\frac{e^{at} \sin bt}{t}$	6M	20BSX13.4	L2
13 (b)	Solve $y'''(t) + y(t) = 1$, if $y(0) = y'(0) = y''(0) = 0$ using transform method.	6M	20BSX13.4	L3
14 (a)	Show that $f(x) = e^{\frac{-x^2}{2}}$ is self-reciprocal with respect to Fourier transform.	6M	20BSX13.5	L3
14 (b)	Find the Fourier cosine transform of $f(x) = \begin{cases} x, & \text{for } 0 < x < 1 \\ 2 - x, & \text{for } 1 < x < 2 \\ 0, & \text{for } x > 2 \end{cases}$	6M	20BSX13.5	L2
OR				
15 (a)	Find the Fourier Transform of $f(x) = x^2, -a < x < a$. Note that $f(x) = 0$ otherwise.	6M	20BSX13.5	L2
15 (b)	Find the Fourier sine transform of $f(x) = \frac{e^{-ax}}{x}$.	6M	20BSX13.5	L3

Semester End Examination, February, 2022

NSRIT

Degree B.Tech (UG.) Program - CE, EEE & MECH

A.Y. 2021-2022

Course Code 20BSX13

Duration - 3 Hrs.

Marks: 70

Semester: III

Course Numerical methods & Transform.

Part - : A (Short Answer Questions) -

1. The iteration formula of Regula-Falsi method is

$$x_{i+1} = \frac{x_{i-1} f(x_i) - x_i f(x_{i-1})}{f(x_i) - f(x_{i-1})}$$

1 Mark

The iteration formula of Stant method is

$$x_{i+1} = x_{i-1} - \frac{x_i - x_{i-1}}{f(x_i) - f(x_{i-1})} \cdot f(x_{i-1})$$

1 Mark

2. If h is the interval of differencing, then

$$\Delta f(x) = f(x+h) - f(x), \quad \nabla f(x) = f(x) - f(x-h), \quad E f(x) = f(x+h).$$

$$\text{now } \nabla E f(x) = \nabla \{ f(x+h) \} = f(x+h) - f(x) = \Delta f(x)$$

$$\therefore \Delta = \nabla E.$$

2 Mark

3. Given the data

x	x_0	x_1	x_2	x_3	...	x_n
y	y_0	y_1	y_2	y_3	...	y_n

where n is a multiple of 3,

If h is the interval of differencing, then $h = \frac{b-a}{n}$ &

by Simpson's $3/8^{\text{th}}$ rule,

$$\int_{a=x_0}^{b=x_n} y \, dx = \frac{3h}{8} \left[(y_0 + y_n) + 3(y_1 + y_2 + y_4 + y_5 + \dots) + 2(y_3 + y_6 + y_9 + \dots) \right]$$

2 Mark

4. We have $L\{f(t-a) \cdot u(t-a)\} = e^{-as} F(s) = e^{-as} L\{f(t)\}$

$$\therefore L\{(t-1)^3 u(t-1)\} = e^{-s} L\{t^3\} = e^{-s} \left(\frac{3!}{s^4} \right) = \frac{6e^{-s}}{s^4}$$

2 Mark

5. Fourier integral representation of $f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(t) \cos \lambda(t-x) \, dt \, d\lambda$

Fourier sine integral is $f(x) = \frac{2}{\pi} \int_0^{\infty} \sin \lambda x \, d\lambda \int_0^{\infty} f(t) \sin \lambda t \, dt$

1 Mark

Fourier cosine integral is $f(x) = \frac{2}{\pi} \int_0^{\infty} \cos x \lambda \int_0^{\infty} f(t) \cos \lambda t dt dt$.

— (1 mark)

Part-B:-

6. (a) $f(x) = xe^x - 2$

$f(0) = (0)e^0 - 2 = 0 - 2 = -2 < 0$, $f(1) = (1)e^1 - 2 = e - 2 > 0$

$\therefore$ A real root lies in the interval $(0, 1)$.

— (1 mark)

Let $x_0 = 0.5$, $f(x_0) = (0.5)e^{0.5} - 2 = (0.5)(1.64872) - 2$
 $= -1.1756 < 0$.

$\therefore$ root lies in $(0.5, 1)$

Let $x_1 = 0.75$, $f(x_1) = (0.75)e^{0.75} - 2 = -0.4122 < 0$.

$\therefore$ root lies in $(0.75, 1)$

— (1M)

Let $x_2 = 0.875$, $f(x_2) = (0.875)e^{0.875} - 2 = 0.099 > 0$

$\therefore$ root lies in $(0.75, 0.875)$

— (1M)

Let $x_3 = 0.8125$, $f(x_3) = (0.8125)e^{0.8125} - 2 = -0.169 < 0$

$\therefore$ root lies in $(0.8125, 0.875)$

— (1M)

Let $x_4 = 0.8451$, $f(x_4) = (0.8451)e^{0.8451} - 2$

$= -0.0324 < 0$

— (1M)

$\therefore$ root lies in $(0.8451, 0.875)$

Let $x_5 = 0.86005$, $f(x_5) = (0.86005)e^{0.86005} - 2$

$= 0.0325 > 0$

— (1M)

$\therefore$ root lies in $(0.8451, 0.86005)$

Let $x_6 = 0.852575$, $f(x_6) = (0.852575)e^{0.852575} - 2$

$= -0.0001 < 0$

$\therefore$ root lies in $(0.852575, 0.86005)$

Let $x_7 = 0.8563125$, $f(x_7) = (0.8563125)e^{0.8563125} - 2$

$= 0.0161 > 0$

$\therefore$ root lies in $(0.852575, 0.8563125)$

Let $x_8 = 0.8544$, $f(x_8) > 0$

root is 0.85

6. (b) Given $f(x) = x \tan x + 1 = 0 \quad \therefore f'(x) = x \sec^2 x + \tan x$

Now $f(2) = 2 \tan 2 + 1 = -3.370079 < 0$ and

$f(3) = 3 \tan 3 + 1 = 0.57237 > 0.$

$\therefore$ The root lies between 2 and 3. We take the average of 2 & 3.

Let $x_0 = \frac{2+3}{2} = 2.5.$

Using Newton-Raphson method, $x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$, we obtain

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 2.5 - \frac{-0.86755}{3.14808} = 2.77558$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 2.77558 - \left(\frac{-0.06383}{2.80004} \right) = 2.798$

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 2.798 - \left(\frac{-0.0010003052}{2.7983} \right) = 2.798$

Since $x_2 = x_3$, therefore, the real root is 2.798.

7. (c) $f(x) = x^3 + x - 1$ $f(0) = -1$, $f(0.5) = -0.375$, $f(1) = 1$ $\therefore$

root lies in $(0.5, 1)$.

Rewriting the equation as $x^3 = 1 - x$ i.e., $x = (1-x)^{1/3} = \phi(x)$,

we have $\phi'(x) = \frac{1}{3} (1-x)^{-2/3} (-1) = -\frac{1}{3(1-x)^{2/3}}$

$4|\phi'(x)| = \frac{1}{3} \left| \frac{1}{(1-x)^{2/3}} \right| < 1$ in $(0, 1)$.

Hence, the iteration method can be applied & we start with $x_0 = 0.5$.

$x_1 = \phi(x_0) = 0.7937$

$x_2 = \phi(x_1) = 0.5908$

$x_3 = \phi(x_2) = 0.7424$

$x_4 = \phi(x_3) = 0.6363$

$x_5 = \phi(x_4) = 0.7138$

$x_6 = \phi(x_5) = 0.659$

$x_7 = \phi(x_6) = 0.6906$

$x_8 = \phi(x_7) = 0.6705$

$x_9 = \phi(x_8) = 0.6907$

$x_{10} = 0.6762$

$x_{11} = 0.6866$

$x_{12} = 0.6793$

$x_{13} = 0.6844$

$x_{14} = 0.6808$

Since $x_{13} = x_{14}$,

the desired

root is 0.68.

$$\begin{aligned} & \frac{16}{3} + \frac{208}{15} + \frac{14}{123} - \frac{16}{15} \\ &= \frac{-2}{3} + \frac{192}{15} = \frac{-10 + 192}{15} = \frac{182}{15} = 12.133 \end{aligned}$$

7.16) The given system of equations is

$$5x - y + z = 10, \quad 2x + 4y = 12, \quad x + y + 5z = -1.$$

In eq. (i), $|5| > |-1| + |1|$, In eq. (ii), $|4| > 2$

In eq. (iii), $|5| > |1| + |1|$.

Hence the given equation is diagonally dominant.

The equations can be expressed as

$$x = \frac{1}{5}(10 + y - z), \quad y = \frac{1}{4}(12 - 2x), \quad z = \frac{1}{5}(-1 - x - y)$$

Given that initial approximation is

$$x_0 = 2, \quad y_0 = 3, \quad z_0 = 0.$$

First approx:- $x_1 = \frac{1}{5}(10 + y_0 - z_0) = \frac{1}{5}(10 + 3 - 0) = \frac{13}{5} = 2.6$

$$y_1 = \frac{1}{4}(12 - 2x_1) = \frac{8}{4} = 2$$

$$z_1 = \frac{1}{5}(-1 - 2 - 3) = \frac{-6}{5} = -1.2$$

Second approx:- $x_2 = \frac{1}{5}(10 + y_1 - z_1) = \frac{1}{5}(10 + 2 + 1.2) = \frac{13.2}{5} = 2.64$

$$y_2 = \frac{1}{4}(12 - 2x_2) = \frac{1}{4}(12 - 2(2.64)) = \frac{12 - 5.28}{4} = \frac{6.72}{4} = 1.7$$

$$z_2 = \frac{1}{5}(-1 - 2.64 - 1.7) = \frac{-5.34}{5} = -1.068$$

Third approx:- $x_3 = \frac{1}{5}(10 + y_2 - z_3) = \frac{1}{5}(10 + 1.7 + 1.068) = \frac{12.768}{5} = 2.5536$

$$y_3 = \frac{1}{4}(12 - 2x_3) = \frac{1}{4}(12 - 2(2.5536)) = \frac{12 - 5.1072}{4} = \frac{6.8928}{4} = 1.7232$$

$$z_3 = \frac{1}{5}(-1 - x_3 - y_3) = \frac{1}{5}(-1 - 2.5536 - 1.7232) = \frac{-5.2768}{5} = -1.0554$$

Fourth approx:-

$$x = 2.556, \quad y = 1.722, \quad z = -1.055$$

8. (a) $\Delta f(x) = f(x+1) - f(x)$

$$\Rightarrow \Delta \left(\frac{1}{x^2+5x+6} \right) = \frac{1}{(x+1)^2+5(x+1)+6} - \frac{1}{x^2+5x+6} = \frac{1}{x^2+7x+12} - \frac{1}{x^2+5x+6}$$

$$= \frac{-2x-6}{(x^2+7x+12)(x^2+5x+6)} = \frac{-2(x+3)}{(x+3)(x+4)(x+2)(x+3)} = \frac{-2}{(x+2)(x+3)(x+4)}$$

$$\Delta^2 \left(\frac{1}{x^2+5x+6} \right) = \Delta \left(\frac{-2}{(x+2)(x+3)(x+4)} \right) = \frac{-2}{(x+3)(x+4)(x+5)} + \frac{2}{(x+2)(x+3)(x+4)}$$

$$= \frac{-2(x+2) + 2(x+5)}{(x+2)(x+3)(x+4)(x+5)} = \frac{6}{(x+2)(x+3)(x+4)(x+5)}$$

$$\Delta(e^{3x} \log 2x) = e^{3(x+1)} \log 2(x+1) - e^{3x} \log 2x. \quad \text{--- 3 marks}$$

$$= e^{3x} e^3 \cdot \log(2x+2) - e^{3x} \log 2x. \quad \text{--- 2 marks}$$

$$= e^{3x} \cdot e^3 \log(2x+2) - e^{3x} e^3 \log 2x + e^{3x} \cdot e^3 \log 2x - e^{3x} \log 2x$$

$$= e^{3x} \cdot e^3 \log \left(\frac{x+1}{x} \right) + e^{3x} (e^3 - 1) \log 2x.$$

$$= e^{3x} \left(e^3 \log \left(1 + \frac{1}{x} \right) + (e^3 - 1) \log 2x \right).$$

$$\therefore \Delta^2 \left(\frac{1}{x^2+5x+6} \right) + \Delta(e^{3x} \log 2x)$$

$$= \frac{6}{(x+2)(x+3)(x+4)(x+5)} + e^{3x} \left(e^3 \log \left(1 + \frac{1}{x} \right) + (e^3 - 1) \log 2x \right) \quad \text{--- 1 mark}$$

8. (b)

diameter (d)	5	6	9	11
Area (A)	12	13	14	16

We know that $\text{Area} = \pi r^2 = \pi \left(\frac{d}{2} \right)^2 = \frac{\pi d^2}{4}$

$x_0 = 5, x_1 = 6, x_2 = 9, x_3 = 11$

$y_0 = 12, y_1 = 13, y_2 = 14, y_3 = 16$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
5	12	1		
6	13	1	0	
9	14	2	1	1
11	16			

$$\Rightarrow y_0 = 12, \Delta y_0 = 1, \Delta^2 y_0 = 0, \Delta^3 y_0 = 1$$

By Newton's forward interpolation formula,

$$y = y_0 + p \Delta y_0 + \frac{p(p-1)}{2!} \Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!} \Delta^3 y_0 + \dots \quad |f'(x)| = \left| \frac{-3(1)^2}{-1/3} \right|$$

~~Since x -values are not equally distributed, this method is not applicable.~~

$$x^{3/2} = 1 - x \Rightarrow x = \sqrt{1-x}$$

$$\phi(x) = (1-x)^{1/3} \Rightarrow \phi'(x) = \frac{1}{3}(1-x)^{-2/3}(-1) = -\frac{1}{3}(1-x)^{-2/3}$$

$$x = 1/2, \phi(1/2) = 1/2, \phi'(1/2) = -2/3$$

$$\begin{array}{r} 1 \\ 0.02 \\ 0.0027 \\ 0.00027 \\ \hline 1.02297 \end{array}$$

Since x values are not equally distributed, Newton's forward interpolation formula is not applicable.

So, we can use either Lagrange's interpolation formula or Newton's divided difference formula.

By Lagrange's interpolation formula,

$$\begin{aligned}
 y &= \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} y_1 \\
 &\quad + \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} y_3 \quad \text{--- (2 marks)} \\
 &= \frac{(x-6)(x-9)(x-11)}{(5-6)(5-9)(5-11)} \cdot 12 + \frac{(x-5)(x-9)(x-11)}{(6-5)(6-9)(6-11)} \cdot 13 \\
 &\quad + \frac{(x-5)(x-6)(x-11)}{(9-5)(9-6)(9-11)} \cdot 14 + \frac{(x-5)(x-6)(x-9)}{(11-5)(11-6)(11-9)} \cdot 16 \\
 &= \frac{(x-6)(x-9)(x-11)}{(-1)(-4)(-6)} \cdot 12 + \frac{(x-5)(x-9)(x-11)}{(1)(-3)(-5)} \cdot 13 \\
 &\quad + \frac{(x-5)(x-6)(x-11)}{(4)(3)(-2)} \cdot 14 + \frac{(x-5)(x-6)(x-9)}{(6)(5)(2)} \cdot 16 \quad \text{--- (2 marks)}
 \end{aligned}$$

$$\begin{aligned}
 y &= -\frac{2}{3} (x-6)(x-9)(x-11) + \frac{13}{15} (x-5)(x-9)(x-11) \\
 &\quad - \frac{7}{12} (x-5)(x-6)(x-11) + \frac{4}{15} (x-5)(x-6)(x-9)
 \end{aligned}$$

when diameter $x = 7$,

$$\begin{aligned}
 \text{Area, } y &= -\frac{2}{3} (1)(-2)(-4) + \frac{13}{15} (2)(-2)(-4) \\
 &\quad - \frac{7}{12} (2)(1)(-4) + \frac{4}{15} (2)(1)(-2) \\
 &= -\frac{16}{3} + \frac{104}{15} - \frac{14}{3} - \frac{16}{15} = \frac{-16 + 104 - 72 - 16}{15} = \frac{2}{15} = 0.1333 \quad \text{--- (2 marks)}
 \end{aligned}$$

(or) Newton's Divided difference table:-

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
5	12	$\frac{13-12}{6-5} = 1$	$\frac{0.3333-1}{9-5} = -0.6667$	
6	13	$\frac{14-13}{9-6} = \frac{1}{3} = 0.3333$	$= -0.16667$	$\frac{13334}{0.22222+0.16667}$
9	14	$\frac{16-14}{11-9} = 1$	$\frac{1-0.3333}{11-6} = \frac{0.6667}{5}$	$11-5.$ $= 0.0511125$
11	16		$= 0.22222$ $13334.$	

3 marks

$$y = y_0 + (x-x_0) \Delta(x_0, x_1) + (x-x_0)(x-x_1) \Delta^2(x_0, x_1, x_2) + (x-x_0)(x-x_1)(x-x_2) \Delta^3(x_0, x_1, x_2, x_3)$$

1 mark

$$= 12 + (x-5)(1) + (x-5)(x-6)(-0.16667) + (x-5)(x-6)(x-9)(0.0511125)$$

Now, y at $x=7$ is

$$y = 12 + (2)(1) + (7-5)(7-6)(-0.16667) + (7-5)(7-6)(7-9)(0.0511125)$$

$$= 12 + 2 + (2)(-1)(-0.16667) + (2)(1)(-2)(0.0511125)$$

$$= 12 + 2 + 0.33334 - 0.20445 = 14.28905$$

2 marks

9. (a) $u_0 + u_1 + u_2 + \dots + u_n$

$$= u_0 + E u_0 + E^2 u_0 + \dots + E^n u_0$$

$$= (1 + E + E^2 + \dots + E^n) u_0 = \left(\frac{1-E^{n+1}}{1-E} \right) u_0 = \left(\frac{E^{n+1}-1}{\Delta} \right) u_0$$

$$= \left(\frac{(1+\Delta)^{n+1}-1}{\Delta} \right) u_0$$

1 mark

2 marks

1 mark

$$\begin{aligned}
 &= \left\{ (n+1)c_0 + (n+1)c_1\Delta + (n+1)c_2\Delta^2 + \dots + (n+1)c_{n+1}\Delta^{n+1} \right\} u_0 \\
 &= \left\{ (n+1)c_1 + (n+1)c_2\Delta + (n+1)c_3\Delta^2 + \dots + (n+1)c_{n+1}\Delta^n \right\} u_0 \\
 &= (n+1)c_1 u_0 + (n+1)c_2 \Delta u_0 + (n+1)c_3 \Delta^2 u_0 + \dots + (n+1)c_{n+1} \Delta^n u_0.
 \end{aligned}$$

2 marks

9. (b)

marks	30-40	40-50	50-60	60-70	70-80
no. of students	31	42	51	35	31

Let x be "marks less than"

$y = \text{no. of students}$

x	40	50	60	70	80
y	31	73	124	159	190

1 mark

Forward Difference Table.

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
40	31	42	9	-25	37
50	73	51	-16	12	
60	124	35	-4		
70	159	31			
80	190				

$$\begin{aligned}
 p &= \frac{x - x_0}{h} \\
 &= \frac{45 - 40}{10} \\
 &= 0.5
 \end{aligned}$$

2 marks

By Newton's forward interpolation formula,

$$y(x) = y_0 + p \Delta y_0 + \frac{p(p-1)}{2!} \Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!} \Delta^3 y_0 + \frac{p(p-1)(p-2)(p-3)}{4!} \Delta^4 y_0 + \dots$$

1 mark

$$\therefore y(45) = 31 + \left(\frac{1}{2}\right)(44) + \frac{\frac{1}{2}(\frac{1}{2}-1)}{2}(9) + \frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}-2)}{6}(-25) \\ + \frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}-2)(\frac{1}{2}-3)}{24}(37)$$

$$= 31 + 21 + \left(-\frac{9}{8}\right) + \frac{8}{48}(-25) - \frac{5}{120}(37)$$

$$= 51 + 1 - \frac{9}{8} - \frac{25}{16} - \frac{185}{120}$$

$$= 52 - \frac{43}{16} - \frac{185}{120} = 52 - \frac{44.75}{120}$$

$$= 52 - \frac{49}{120} = 52 - 4.132$$

$$= 47.87 \approx 48$$

$\therefore$ No. of students

< 45 is 48

But No. of student < 40 is 31

$\therefore$ NO. of students between 40 & 45 is

17

2 marks

10. (a) $x_0 = 0, x_n = 6$ $\therefore h = \frac{b-a}{n} = \frac{6-0}{6} = 1$.

$x_0 = 0, x_1 = 1, x_2 = 2, x_3 = 3, x_4 = 4, x_5 = 5, x_6 = 6$

x	y	$xy + 5$	$y = \frac{1}{xy+5}$
0	0	5	$\frac{1}{5} = 0.2$
1	4	9	$\frac{1}{9} = 0.1111$
2	8	13	$\frac{1}{13} = 0.0769$
3	12	17	$\frac{1}{17} = 0.0588$
4	16	21	$\frac{1}{21} = 0.0476$
5	20	25	$\frac{1}{25} = 0.04$
6	24	29	$\frac{1}{29} = 0.0345$

2 marks

$$y_2 = y_0 + \int_0^x f(x, y) dx = 1 + \int_0^x f(x, 1 + x + \frac{x^2}{2}) dx$$

$$= 1 + \int_0^x \left[x \left(1 + x + \frac{x^2}{2} \right) + 1 \right] dx = 1 + \int_0^x \left(\frac{x^3}{2} + x^2 + x + 1 \right) dx$$

$$= 1 + \left(\frac{x^4}{8} + \frac{x^3}{3} + \frac{x^2}{2} + x \right)_0^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{8}$$

$$y_3 = y_0 + \int_0^x f(x, y_2) dx = 1 + \int_0^x f(x, 1 + x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{8}) dx$$

$$= 1 + \int_0^x \left[x \left(1 + x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{8} \right) + 1 \right] dx$$

$$= 1 + \int_0^x \left[x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{8} + \frac{x^5}{8} + 1 \right] dx$$

$$= 1 + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{8} + \frac{x^5}{15} + \frac{x^6}{48} + x$$

$$y(0.2) = 1 + \frac{(0.2)^2}{2} + \frac{(0.2)^3}{3} + \frac{(0.2)^4}{8} + \frac{(0.2)^5}{15} + \frac{(0.2)^6}{48} + 0.2$$

$$= 1 + \frac{0.04}{2} + \frac{0.008}{3} + \frac{0.0016}{8}$$

$$= 1 + 0.02 + 0.0027 + 0.0002 + \dots$$

$$\approx \underline{\underline{1.0229}}$$

(a) Trapezoidal Rule:-

$$\begin{aligned} \int_0^6 \frac{1}{4x+5} dx &= \frac{h}{2} (y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1}) \\ &= \frac{h}{2} (y_0 + y_n) + 2(y_1 + y_2 + y_3 + y_4 + y_5) \\ &= \frac{1}{2} [(0.2 + 0.0345) + 2(0.1111 + 0.0769 + 0.0588 + 0.0476 + 0.04)] \\ &= \frac{1}{2} [0.2345 + 2(0.3344)] = \frac{1}{2} [0.2345 + 0.6688] \\ &= \frac{1}{2} [0.9033] = 0.45165. \end{aligned}$$

(1 mark)

(1 mark)

(b) Simpson's $\frac{3}{8}$ rule:-

$$\begin{aligned} \int_0^6 \frac{1}{4x+5} dx &= \frac{3h}{8} (y_0 + y_6) + 3(y_1 + y_2 + y_4 + y_5) + 2(y_3) \\ &= \frac{3}{8} [(0.2 + 0.0345) + 3(0.1111 + 0.0769 + 0.0476 + 0.04) + 2(0.0588)] \\ &= \frac{3}{8} [0.2345 + 3(0.2756) + 0.1176] = \frac{1.1789}{8} \\ &= 0.14736 \end{aligned}$$

(1 mark)

(1 mark)

10. (b) $\frac{dy}{dx} = xy + 1$, $y(0) = 1$

$\therefore f(x, y) = xy + 1$, $x_0 = 0$, $y_0 = 1$

By Picard's method, $y_n = y_0 + \int_{x_0}^{x_n} f(x, y_{n-1}) dx$

(1 mark)

(i) $y_1 = y_0 + \int_{x_0}^{x_1} f(x, y_0) dx = 1 + \int_0^1 f(x, 1) dx$

$$\begin{aligned} &= 1 + \int_0^1 (x(1) + 1) dx = 1 + \left(\frac{x^2}{2} + x \right)_0^1 \\ &= 1 + 1 + \frac{1}{2} = 1 + \frac{3}{2} \end{aligned}$$

(1 mark)

11. (a) $x_0 = 0, x_n = 1, y = \frac{1}{1+x}$

we divide the interval $[0, 1]$ into six sub-intervals

The values of x and y are tabulated below:

x	0	1/6	2/6	3/6	4/6	5/6	1
$y = \frac{1}{1+x}$	1	0.8571	0.75	0.6666	0.6	0.5454	0.5
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

(i) By Simpson's $\frac{1}{3}$ rd rule,

$$\int_0^1 \frac{1}{1+x} dx = \frac{h}{3} [(y_0 + y_6) + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4)]$$

$$= \frac{1}{18} [(1 + 0.5) + 4(0.8571 + 0.6666 + 0.5454) + 2(0.75 + 0.6)]$$

$$= 0.6931$$

(ii) By Simpson's $\frac{3}{8}$ th rule,

$$\int_0^1 \frac{1}{1+x} dx = \frac{3h}{8} [(y_0 + y_6) + 3(y_1 + y_2 + y_4 + y_5) + 2(y_3)]$$

$$= \frac{3}{(6)(8)} [(1 + 0.5) + 3(0.8571 + 0.75 + 0.6 + 0.5454) + 2(0.6666)]$$

$$= \frac{1}{16} [1.5 + 8.2525 + 1.3332] = \frac{11.0907}{16}$$

$$= 0.6932$$

11. (b) Here $f(x, y) = x + y, x_0 = 0, y_0 = 1$.

we take step size $h = 0.1$

Euler's algorithm (iteration formula) is

$$y_{n+1} = y_n + h f(x_n, y_n)$$

Taking $n=0$, $y_1 = 2 + h f(x_0, y_0)$

i.e., $y(0.1) = 1 + 0.1 f(0, 1)$

$\Rightarrow y(0.1) = 1 + 0.1 (0 + 1) = 1.1$

Next, we have $x_1 = x_0 + h = 0 + 0.1 = 0.1$

$\therefore y_1 = y(x_1) = y(0.1) = 1.1$

$y_2 = y_1 + h f(x_1, y_1)$ (by taking $n=1$ in (i))

$= 1.1 + (0.1) f(0.1, 1.1)$

$= 1.1 + (0.1) (0.1 + 1.1) = 1.1 + 0.12 = 1.22$

$\therefore y(x_2) = y(x_2) = y_2 = 1.22$

So, $y(0.1) = 1.1$ & $y(0.2) = 1.22$.

12. (b) Show that $\int_0^{\infty} t^2 e^{-4t} \sin 2t \, dt = \frac{1}{2\sqrt{5}}$

Proof:- $L(\sin 2t) = \frac{2}{s^2 + 4}$

$L(t \sin 2t) = -\frac{d}{ds} \left(\frac{2}{s^2 + 4} \right) = -2 \cdot \frac{-1}{(s^2 + 4)^2} \cdot 2s$
 $= \frac{4s}{(s^2 + 4)^2}$

$L(t^2 \sin 2t) = -\frac{d}{ds} \left(\frac{4s}{(s^2 + 4)^2} \right)$
 $= - \left\{ \frac{(s^2 + 4)^2 \cdot (4) - (4s) \cdot 2(s^2 + 4)(2s)}{(s^2 + 4)^4} \right\}$
 $= - \left\{ \frac{4(s^2 + 4) - 16s^2}{(s^2 + 4)^3} \right\} = \left\{ \frac{12s^2 - 16}{(s^2 + 4)^3} \right\}$

— (3 Marks)

$$\text{Now } \int_0^{\infty} t^2 e^{-4t} \sin 2t \, dt = \int_0^{\infty} e^{-st} (t^2 \sin 2t) \, dt \quad \text{at } s=4$$

$$= \left\{ L(t^2 \sin 2t) \right\}_{\text{at } s=4}$$

$$= \left\{ \frac{12s^2 - 16}{(s^2 + 4)^3} \right\}_{\text{at } s=4} = \frac{12(16) - 16}{(16 + 4)^3}$$

$$= \frac{176}{(20)^3} = \frac{176}{8000} = \frac{11}{500}$$

3 marks

12.(b) let us take $F(s) = \frac{1}{s^2 + 16}$ & $G(s) = \frac{s}{s^2 + 16}$

& that $\frac{s}{(s^2 + 16)^2} = F(s) \cdot G(s).$

$$f(t) = L^{-1}\{F(s)\} = L^{-1}\left\{\frac{1}{s^2 + 16}\right\} = \frac{1}{4} \sin 4t$$

$$g(t) = L^{-1}\{G(s)\} = L^{-1}\left\{\frac{s}{s^2 + 16}\right\} = \cos 4t$$

2 M

By convolution theorem,

$$L^{-1}\left\{\frac{s}{(s^2 + 16)^2}\right\} = L^{-1}\{F(s) \cdot G(s)\} = f(t) * g(t)$$

$$= \int_0^t f(u) g(t-u) \, du \quad \text{--- 1 M}$$

$$= \int_0^t \frac{1}{4} \sin(4u) \cdot \cos 4(t-u) \, du$$

$$= \frac{1}{4} \int_0^t \sin(4u) \cos(4t-4u) \, du$$

$$= \frac{1}{8} \int_0^t 2 \sin(4u) \cos(4t-4u) \, du$$

$$= \frac{1}{8} \int_0^t [\sin(4t) + \sin(8u-4t)] \, du$$

$$= \frac{1}{8} \left[u \cdot \sin 4t - \frac{\cos(4u - 4t)}{8} \right]_{u=0}^t$$

$$= \frac{1}{8} \left[\left\{ t \sin 4t - \frac{\cos 4t}{8} \right\} - \left\{ 0 - \frac{\cos 4t}{8} \right\} \right]$$

$$= \frac{t}{8} \sin 4t$$

3 marks

13. (a) $\mathcal{L}(\sin bt) = \frac{b}{s^2 + b^2}$

1 mark

$$\mathcal{L}\left(\frac{\sin bt}{t}\right) = \int_s^\infty F(s) ds = \int_s^\infty \left(\frac{b}{s^2 + b^2}\right) ds$$

$$= \frac{b}{b} \left[\tan^{-1}\left(\frac{s}{b}\right) \right]_s^\infty = \frac{\pi}{2} - \tan^{-1}\left(\frac{s}{b}\right)$$

$$= \cot^{-1}\left(\frac{s}{b}\right)$$

2 marks

$$\mathcal{L}\left(e^{at} \frac{\sin bt}{t}\right) = \left\{ \mathcal{L}\left(\frac{\sin bt}{t}\right) \right\}_{s \rightarrow s-a}$$

$$= \left\{ \cot^{-1}\left(\frac{s}{b}\right) \right\}_{s \rightarrow s-a}$$

$$= \cot^{-1}\left(\frac{s-a}{b}\right)$$

2 marks

13. (b) Given DE is $y'''(t) + y(t) = 1$

by applying LT, we get

$$\mathcal{L}\{y'''(t)\} + \mathcal{L}\{y(t)\} = \mathcal{L}(1)$$

$$\Rightarrow s^3 Y(s) - s^2 y(0) - s y'(0) - y''(0) + Y(s) = \frac{1}{s}$$

$$\Rightarrow (s^3 + 1) Y(s) - s^2 y(0) - s y'(0) - y''(0) = \frac{1}{s}$$

Given that $y(0) = 0$, $y'(0) = 0$, $y''(0) = 0$

$$\Rightarrow (s^3 + 1) Y(s) = \frac{1}{s} \Rightarrow Y(s) = \frac{1}{s(s^3 + 1)}$$

2 marks

by applying inverse LT, we get

$$\mathcal{L}^{-1}\{Y(s)\} = \mathcal{L}^{-1}\left\{\frac{1}{s(s^3+1)}\right\} \quad \text{--- (1)}$$

$$\text{Let } \frac{1}{s(s^3+1)} = \frac{1}{s(s+1)(s^2-s+1)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C(s+D)}{s^2-s+1}$$

$$\Rightarrow \frac{1}{s(s^3+1)} = \frac{A(s+1)(s^2-s+1) + Bs(s^2-s+1) + (C(s+D))s(s+1)}{s(s+1)(s^2-s+1)}$$

$$\Rightarrow 1 = A(s+1)(s^2-s+1) + B(s^3-s^2+s) + C(s^2(s+1) + Ds(s+1))$$

for $s=0$,

$$1 = A(1)(1) \Rightarrow A=1$$

for $s=-1$,

$$1 = B(-1-1-1) \Rightarrow B = -\frac{1}{3}$$

By comparing s^3 coefficients, we have

$$0 = A+B+C \Rightarrow C = -A-B = -1 + \frac{1}{3} = -\frac{2}{3}$$

$$\Rightarrow C = -\frac{2}{3}$$

By comparing s^2 coefficients, we have

$$0 = -B+C+D \Rightarrow D = B-C = -\frac{1}{3} + \frac{2}{3} = \frac{1}{3}$$

$$\therefore D = \frac{1}{3}$$

$$\therefore \frac{1}{s(s^3+1)} = \frac{1}{s} + \frac{(-\frac{1}{3})}{s+1} + \frac{(-\frac{2}{3})s + (\frac{1}{3})}{s^2-s+1} \quad \text{--- (2) Ans}$$

$$\begin{aligned} \therefore \text{By (1), } \mathcal{L}^{-1}\{Y(s)\} &= \mathcal{L}^{-1}\left\{\frac{1}{s} - \frac{1}{3}\left(\frac{1}{s+1}\right) + \frac{-\frac{2s+1}{3}}{s^2-s+1}\right\} \\ &= \mathcal{L}^{-1}\left(\frac{1}{s}\right) - \frac{1}{3}\mathcal{L}^{-1}\left(\frac{1}{s+1}\right) - \frac{2}{3}\mathcal{L}^{-1}\left[\frac{s-\frac{1}{2}}{s^2-s+1}\right] \\ &= \cancel{1} - \frac{e^{-t}}{3} - \frac{2}{3}\mathcal{L}^{-1}\left[\frac{s-\frac{1}{2}}{(s-\frac{1}{2})^2 + \frac{3}{4}}\right] \end{aligned}$$

$$= 1 - \frac{e^t}{3} - \frac{2}{3} + e^{t/2} L^{-1} \left\{ \frac{1}{s^2 + (\frac{\sqrt{3}}{2})^2} \right\}$$

$$= 1 - \frac{e^t}{3} - \frac{2}{3} + e^{t/2} \cos\left(\frac{\sqrt{3}}{2} \cdot t\right).$$

2 Marks

14.6) $f(x) = e^{-x^2/2}$

$$F\{e^{-x^2/2}\} = \int_{-\infty}^{\infty} e^{-x^2/2} \cdot e^{isx} dx = \int_{-\infty}^{\infty} e^{-\frac{x^2}{2} + isx} dx$$

$$= \int_{-\infty}^{\infty} e^{-\frac{1}{2}(x^2 - 2isx)} dx = \int_{-\infty}^{\infty} e^{-\frac{1}{2}(x - is)^2 - i^2 s^2} dx$$

$$= \int_{-\infty}^{\infty} e^{-\frac{1}{2}(x - is)^2} \cdot e^{-s^2/2} dx$$

3 Marks

Put $x - is = t \Rightarrow dx = dt$

1 Mark

$$= e^{-s^2/2} \int_{-\infty}^{\infty} e^{-t^2/2} dt$$

$$= e^{-s^2/2} \sqrt{\pi}$$

$$\left(\because \int_{-\infty}^{\infty} e^{-x^2/2} dx = \sqrt{\pi} \right)$$

$$\therefore F\{e^{-x^2/2}\} = \sqrt{\pi} \cdot e^{-s^2/2}$$

2 Marks

$\Rightarrow$ Fourier Transform of $e^{-x^2/2}$ is a constant times $e^{-s^2/2}$.

Also the function $e^{-x^2/2}$ and $e^{-s^2/2}$ are the same. Hence, it follows that $e^{-x^2/2}$ is self-reciprocal, under the Fourier Transform.

15. (b) $f(x) = \begin{cases} x & ; 0 < x < 1 \\ 2-x & ; 1 < x < 2 \\ 0 & ; 2 < x < \infty \end{cases}$

The Fourier cosine transform of $f(x)$ is given by.

$$F_C\{f\} = \int_0^{\infty} f(x) \cos sx \, dx$$

— 1 Mark

$$= \int_0^1 x \cos sx \, dx + \int_1^2 (2-x) \cos sx \, dx + \int_2^{\infty} (0) \cos sx \, dx$$

— 2 Marks

$$= \left[x \frac{\sin sx}{s} - \left(-\frac{\cos sx}{s^2} \right) \right]_0^1 + \left[(2-x) \left(\frac{\sin sx}{s} \right) - (-1) \left(-\frac{\cos sx}{s^2} \right) \right]_1^2 + 0$$

$$= \left(\frac{\sin s}{s} + \frac{\cos s}{s^2} - \frac{1}{s^2} \right) + \left(-\frac{\cos 2s}{s^2} - \frac{\sin s}{s} + \frac{\cos s}{s^2} \right)$$

$$= \frac{2 \cos s - \cos 2s}{s^2}$$

— 3 Marks

15. (a) $f(x) = \begin{cases} x^2 & ; -a < x < a \\ 0 & ; \text{elsewhere} \end{cases}$

The Fourier Transform of $f(x)$ is given by

$$F[f(x)] = \int_{-\infty}^{\infty} f(x) \cdot e^{isx} \, dx = \int_{-a}^a x^2 e^{isx} \, dx$$

$$= \left[(x^2) \left(\frac{e^{isx}}{is} \right) - (2x) \left(\frac{e^{isx}}{i^2 s^2} \right) + (2) \left(\frac{e^{isx}}{i^3 s^3} \right) \right]_{x=-a}^a$$

$$= \left[\frac{x^2 e^{isx}}{is} + \frac{2x e^{isx}}{s^2} - \frac{2 e^{isx}}{i s^3} \right]_{x=-a}^a$$

$$= \left[\frac{a^2 e^{ias}}{is} + \frac{2a e^{ias}}{s^2} - \frac{2 e^{ias}}{i s^3} \right] - \left[\frac{a^2 e^{-ias}}{is} - \frac{2a e^{-ias}}{s^2} - \frac{2 e^{-ias}}{i s^3} \right]$$

$$= \frac{a^2}{is} \left\{ e^{isa} - e^{-isa} \right\} + \frac{2a}{s^2} \left\{ e^{isa} + e^{-isa} \right\} - \frac{2}{is^3} \left\{ e^{isa} - e^{-isa} \right\}$$

$$\begin{aligned} e^{ia} &= \cos a + i \sin a, \quad e^{-ia} = \cos a - i \sin a \\ \Rightarrow e^{ia} + e^{-ia} &= 2 \cos a, \quad e^{ia} - e^{-ia} = 2i \sin a \\ \therefore e^{isa} - e^{-isa} &= 2i \sin(sa), \quad e^{isa} + e^{-isa} = 2 \cos(sa) \end{aligned}$$

$$= \frac{a^2}{is} \left\{ 2i \sin(sa) \right\} + \frac{2a}{s^2} \left\{ 2 \cos(sa) \right\} - \frac{2}{is^3} \left\{ 2i \sin(sa) \right\}$$

$$= \frac{2a^2 \sin(sa)}{s} + \frac{4a \cos(sa)}{s^2} - \frac{4 \sin(sa)}{s^3}$$

6 Marks

15.(b) $f(x) = \frac{e^{-ax}}{x}$.

Its Fourier sine Transform is

$$F_s[f(x)] = \int_0^\infty f(x) \sin sx \, dx = \int_0^\infty \frac{e^{-ax}}{x} \sin sx \, dx = F(s) \quad \text{--- (1 Mark)}$$

Differentiating both sides w.r.t 's', we get.

$$\begin{aligned} \frac{d}{ds} \{F(s)\} &= \int_0^\infty \frac{x e^{-ax} \cos sx}{x} \, dx = \int_0^\infty e^{-ax} \cos sx \, dx \\ &= \frac{a}{s^2 + a^2} \quad \text{--- (2 Marks)} \end{aligned}$$

Integrating w.r.t 's', we get.

$$F(s) = \int_0^\infty \frac{a}{s^2 + a^2} \, ds = \tan^{-1}\left(\frac{s}{a}\right) + C$$

Let $F(s) = 0$ when $s = 0 \quad \therefore C = 0$

$$\therefore F(s) = \tan^{-1}\left(\frac{s}{a}\right)$$

MC
(20 Marks)

Semester End Examination, January/February, 2022

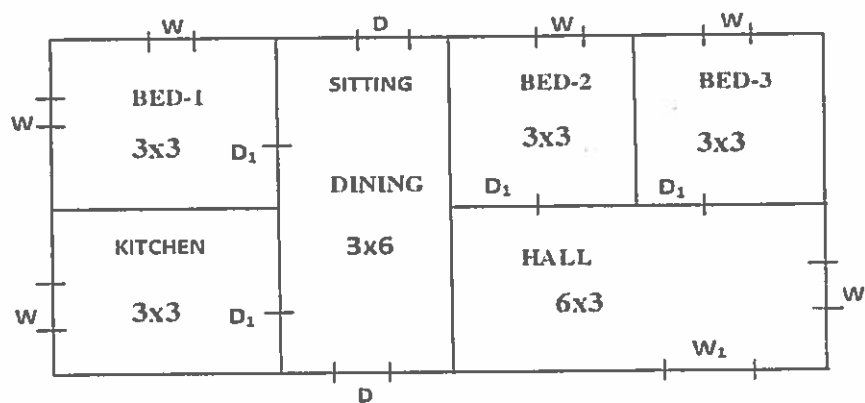
Degree	B. Tech. (U. G.)	Program	Civil Engineering	Academic Year	2021 – 2022
Course Code	20CE302	Test Duration	3 Hrs. Max. Marks 70	Semester	III
Course	Building Planning And Drawing				

Part A (Short Answer Questions 14 x 3 = 42 Marks) - Answer any three among the following

No.	Questions (1 through 5)	Marks	Learning Outcome (s)	DoK
1(a)	Explain the importance of dimensioning & conventional representations in the drawing and Draw the conventional signs for the following :- i) Brick ii) iii) Concrete iii) Glass iv) Timber.	7 M	20CE302.1	L1
1(b)	Draw the isometric view of 2 brick English Bond with 5 as minimum number of layers.	7 M	20CE302.1	L2
2(a)	Write in detail about for guidelines for planning the buildings to suit their functional requirements	7 M	20CE302.2	L2
2(b)	List out the bye- laws and regulations which include all the features of various types of Buildings.	7 M	20CE302.2	L2
3(a)	What is meant by ASPECT in planning of a residential building?	7 M	20CE302.3	L2
3(b)	In any residential building, What are the essentials, which need consideration in grouping of dwelling unit	7 M	20CE302.3	L2
4(a)	Explain the different features of a king post roof truss	7 M	20CE302.4	L2
4(b)	Draw the working drawing of a Glazes door	7 M	20CE302.4	L2
5(a)	Draw the layout of a health centre or Doctor Clinic	7 M	20CE302.5	L2
5(b)	Explain the Principle of Planning a hospital	7 M	20CE302.5	L2

Part B (Long Answer Questions 1 x 28 = 28 Marks) – Answer any one among the following

No.	Questions (6 through 7)	Marks	Learning Outcome (s)	DoK
6(a)	Draw a neat sketch of partly paneled and glaze door of size 1.0 x 2.0 m	14 M	20CE302.4	L2
6(b)	Draw to a suitable scale, the section and elevation of a fully paneled double shuttered door	14 M	20CE302.4	L2
7	Draw the Plan and Elevation of the residential Building with the following dimensions. Assume suitable dimensions as minimum dimensions as BIS	28 M	20CE302.5	L2



REFERENCE:-

D-1000X2000 mm; D₁-900X2000 mm; W-900X1200 mm; W₁-2000X1000 mm;
 V-800X300 mm; All dimensions of rooms are e in m



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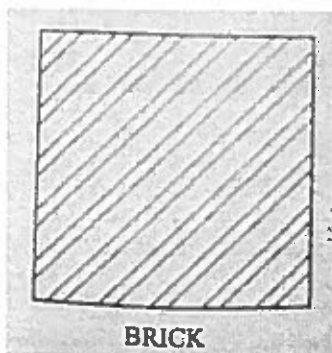
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ANSWER KEY AND SCHEME OF EVALUATION

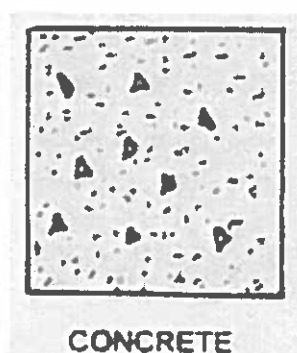
1a) Importance of dimensioning & conventional representations in the drawing:

- Indicating the sizes of the object and other details essential for its construction and function using lines, numerals, symbols etc in drawing is called dimensioning.
- The purpose of dimensioning is to provide a clear and complete description of an object.
- Conventional representations such as conventional signs and conventional symbols are used in building drawings to represent various building materials and to represent the actual object in the building drawings respectively

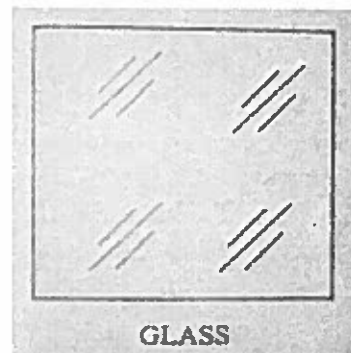
Conventional signs:



BRICK



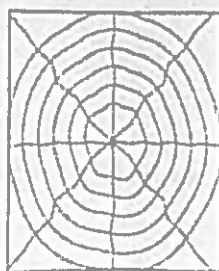
CONCRETE



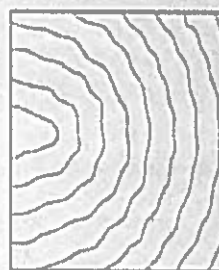
GLASS



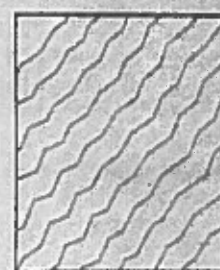
WOOD ACROSS GRAIN



WOOD ACROSS GRAIN



WOOD ALONG GRAIN



WOOD ALONG GRAIN



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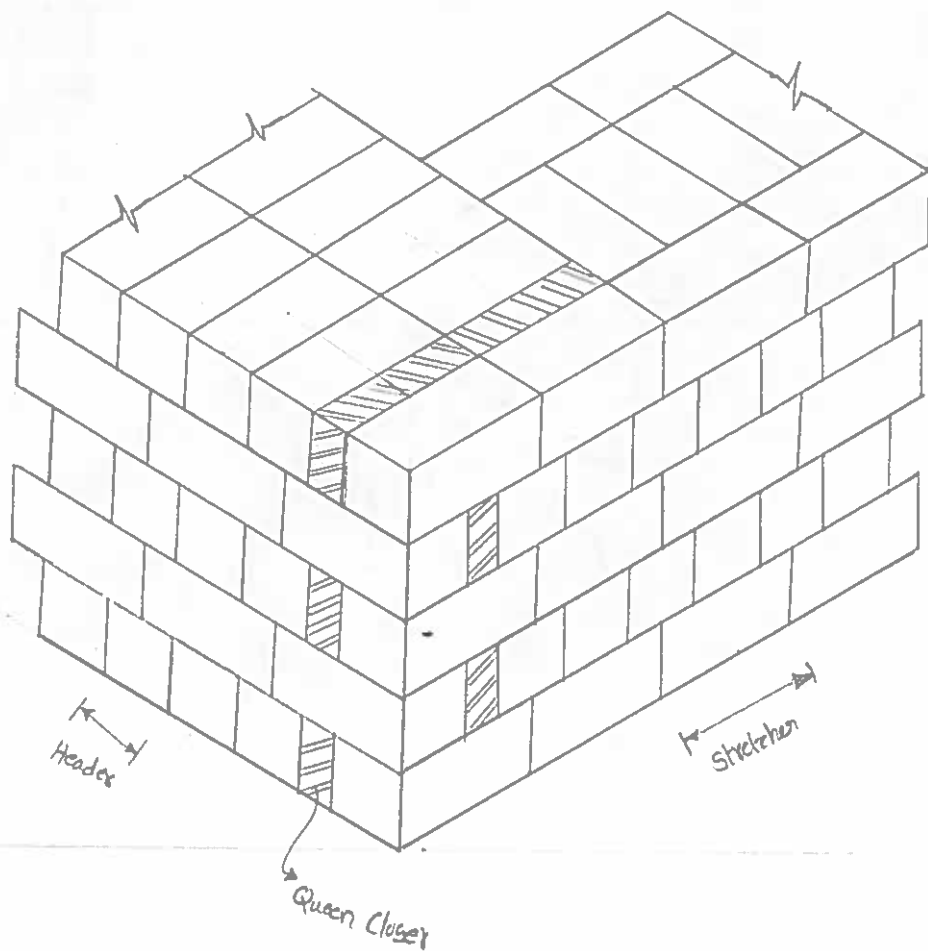
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ANSWER KEY AND SCHEME OF EVALUATION

1b) Isometric view of 2 brick of English bond with 5 minimum layers.

7M



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ANSWER KEY AND SCHEME OF EVALUATION

2a) Planning of building depends on :

- 2M {
- Its functional objects and requirements
 - Its component parts, their sizes and relationship between the different rooms
 - Shape of plot, topography
 - Climatic conditions of the place
 - Its location and neighborhood
 - Whether single storeyed or multi-storeyed, detached or semi-detached or row houses

Minimum standards of various parts of the buildings and their requirements:

- 5M
- The minimum depth of footing is kept 1.0 m below ground level, the width of foundation should be such that the stress in the soil is within its safe bearing capacity
 - Plinth height may be 300 mm to 600 mm but 450 mm more common. For water logging prone and important buildings the minimum plinth height is 600 mm
 - Plinth area should be a minimum of 1.2 times the floor area of the building
 - Width of cement concrete bed of foundation = width of brick footing immediately above the concrete bed + 100 mm + 100 mm
 - Minimum width of footing = $2t + 2j$
Where t = thickness of wall (200 mm) j = offset of concrete (less than or equal to 100 mm)
 - Thickness of the damp proof course 20 mm to 25 mm
 - Thickness of cement concrete floor is a minimum of 100 mm + floor finish over it. Floor finish generally with 10 mm to 20mm thick cement mortar or tiles
 - Minimum thickness of load bearing masonry walls is 200 mm and thickness of parapet wall and partition walls may be generally of 100 mm
 - Ceiling height for main rooms of a residential building should be 3000 mm to 3600 mm. for bath water closet the ceiling height may be 2000 mm to 2750 mm
 - Main entrance door of a residential building should be 1000 mm X 2100 mm, other doors should have minimum width of 900 x 2000 mm
 - Window width = $1/8$ (width of room + height of room)
 - Generally, the minimum size of a window is 750 X 1000 mm
 - Window sill level is 900 to 1200 mm above floor level
 - The minimum size of ventilator is 1000 x 500 mm in order to provide good ventilation for any habitable room, for bath and water closet is 450 X 300 mm
 - Thickness of R.C.C lintel beam is 100 mm to 250 mm and 150 mm is more common
 - Thickness of R.C.C roof slab is 100 mm to 150 mm and that of sunshades 25 mm to 50 mm
 - Sunshades project through 450 mm to 600 mm from facing of external walls



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ANSWER KEY AND SCHEME OF EVALUATION

- The height of a parapet wall should not be less than 0.60 m and not more than 1.20 m above the finished floor level

2b) Bye- laws and regulations various types of Buildings:

- 2M {
- The rules and regulations framed by town planning authorities covering the requirements of building, ensuring the safety of the public through open spaces, minimum size of rooms and height and area limitations, are known as Building Bye Laws.

The rules regulate:

- 5M
- Classify the building with unit as a family & mention the requirements.
 - Classify the rooms according to use & then specify minimum standards of each room with respective to size, height, floor area, ventilation & light
 - Specifying height of compound wall & gate.
 - Controlling height of structures & laying maximum limit of height.
 - Controlling projections in marginal spaces.
 - Insisting on suitable FSI or FAR.
 - Specify suitable arrangements with respect to drainage & water supply.
 - Making compulsory the appointment of an architect or engineer for work of specified nature & magnitude.
 - Specifying materials & workmanship as per standard specifications for the construction.
 - Specify light plane, setbacks & Marginal spaces.
 - Specifying minimum size of plots, their dimensions & frontages.

3a) Aspect in planning of a residential building

- 7M
- Aspect is the positioning of rooms in buildings with respect to 4 directions in such a way that the occupants of buildings would enjoy the natural comforts like sunshine, breeze, scenery, etc. to the maximum possible extent.
 - A room receiving light and air from any particular direction is said to have the aspect of that direction. All the rooms of a dwelling need a particular aspect. Some necessary aspects of commonly constructed rooms are listed below:

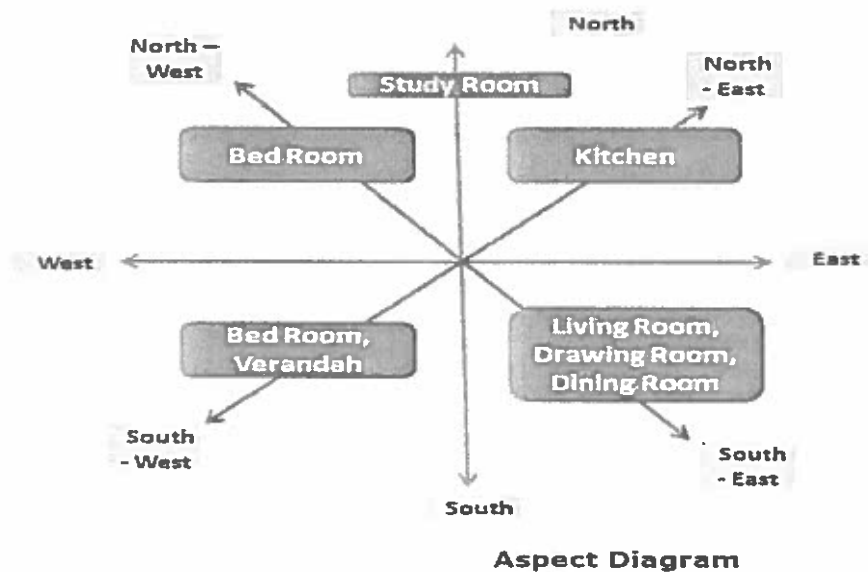


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ANSWER KEY AND SCHEME OF EVALUATION



- Living Room: It should have a southern or south-east aspect. The sun is towards the south during winter and north during summer which will provide sunshine during winter and cooler during summertime.
- Bedroom: It should have a west or south-west aspect, as the breeze required particularly in summer would prevail from this side.
- Kitchen: It should have an eastern aspect so as to admit morning sun refresh and purify the air.
- Gallery or Verandah: It should be north or north-east aspect.
- Classroom, Reading Room: It should be laid with the north aspect as light received from the north will be diffused and evenly distributed.

2M 3b) Dwelling unit means a house or apartment used to provide living accommodations in a building or structure, but it does not include a unit in a hotel, motel, or other establishment in which more than 50% of the units are used on a transient basis.

Essentials which need consideration in grouping of dwelling unit:

For the residential building, planning should be grouped as (a) Living area; (b) Sleeping area; (c) Service area; and (d) Circulation area

5M a) Living area:



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This area is for general use. The living and drawing room should be planned near the entrance taking into view the following considerations.

- It should not provide direct passage/access to the bed room and w.c.or bath.
- It should be adjacent to the dinning room.
- It should be comfortable and spacious in order to accommodate furniture and also allow proper circulation area.
- It should be sufficiently lighted and offer an attractive view of the surroundings landscape, garden, etc.
- It should have a southern and northern aspect.

b) Sleeping area:

- This area provides bed rooms for sleeping and relaxing.
- Bed rooms may be with attached toilets i.e. bath room and WC. Their size depends upon the number of beds.
- They should be located so as to give privacy and should accommodate beds, easy chair, cup boards etc.
- They should have a North-West or South-West aspect.

Service area

- This includes the area of the kitchen, dining rooms, bathrooms, and toilet.
- The dining room should be close to the kitchen. This will make it much easier to carry food.
- The kitchen room should be avoided near the living room, bedroom, and study room to avoided smoke and smells

c) Circulation area

- This is an area for passage, lobby, corridor, etc., It should be minimum but well ventilated and lighted

4a) Features of a king post roof truss:

- A king post truss consists of a horizontal beam called tie beam, two inclined rafters called principal rafters, a vertical member called king post and two inclined struts.
- The truss derives its name from the central vertical upright member called king post.
- It is the oldest form of a roof structure that can be used up to a span of 9 m.
- The roof trusses are spaced at not more than 3 m. centre to centre and are made to rest on stone, wood or R.C.C. bed blocks or bed plates. They are connected by purlins and ridge piece.
- Principal rafters support the frame work of the roof. To prevent the bending of the principal rafters, two inclined braces are provided called struts.

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ANSWER KEY AND SCHEME OF EVALUATION

- Similarly, to prevent the sagging of the tie beam king post, a central vertical member is connected to the centre of tie beam with the ridge.
- Purlins are horizontal timber members which are usually placed over the principle rafters and they support the common rafters.
- Ridge beam is provided at the apex of the roof to provide end support to common rafters
- Common rafters are a number of inclined rafters which are placed at distances of about 1 m or closely depending upon the load, i.e., of the roof covering.
- Cleats are wooden pieces which are fixed on principal rafters to prevent the purlins from tilting or sliding down.



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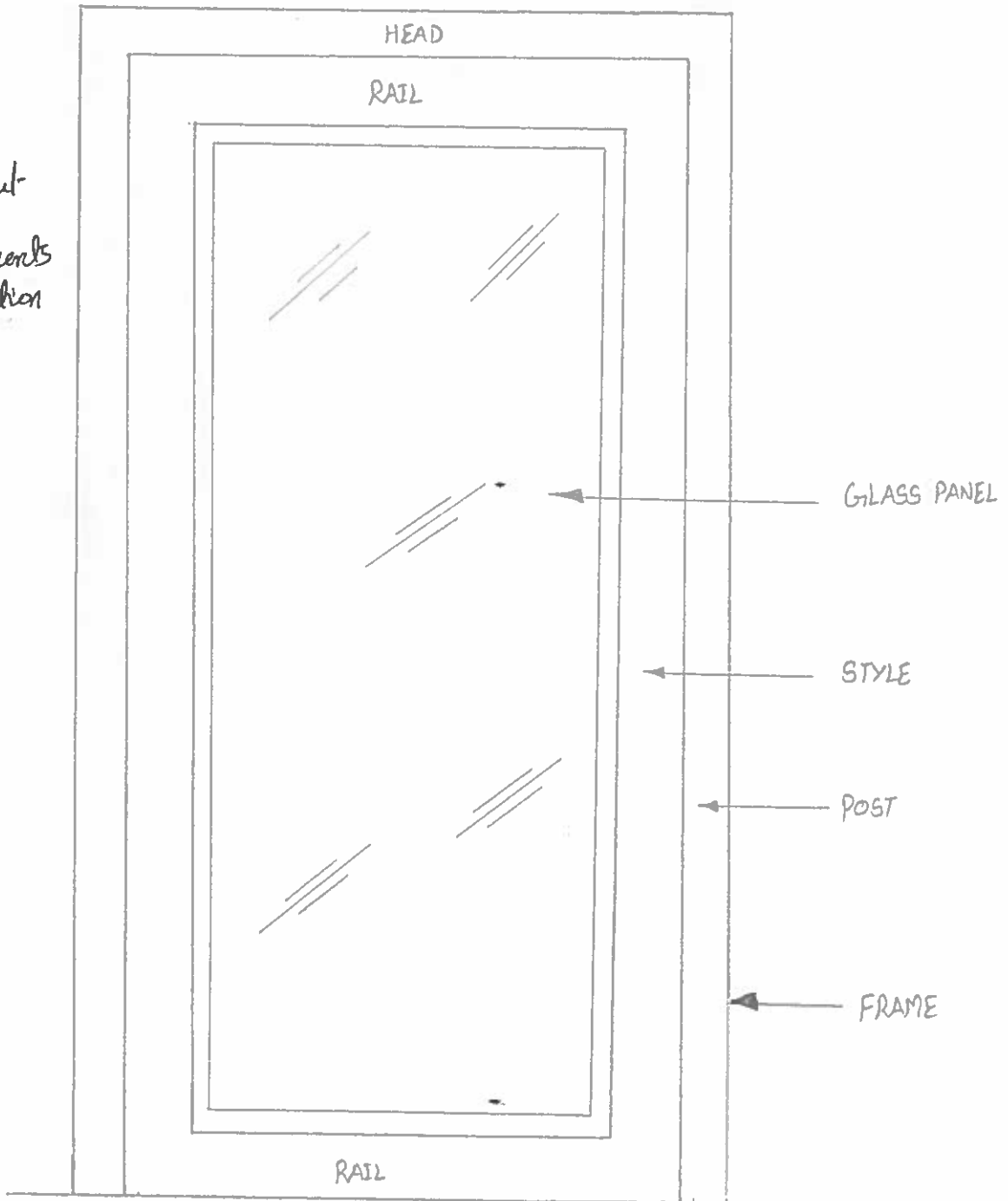
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ANSWER KEY AND SCHEME OF EVALUATION

4b)

4M - Layout
3M - Components
Indication

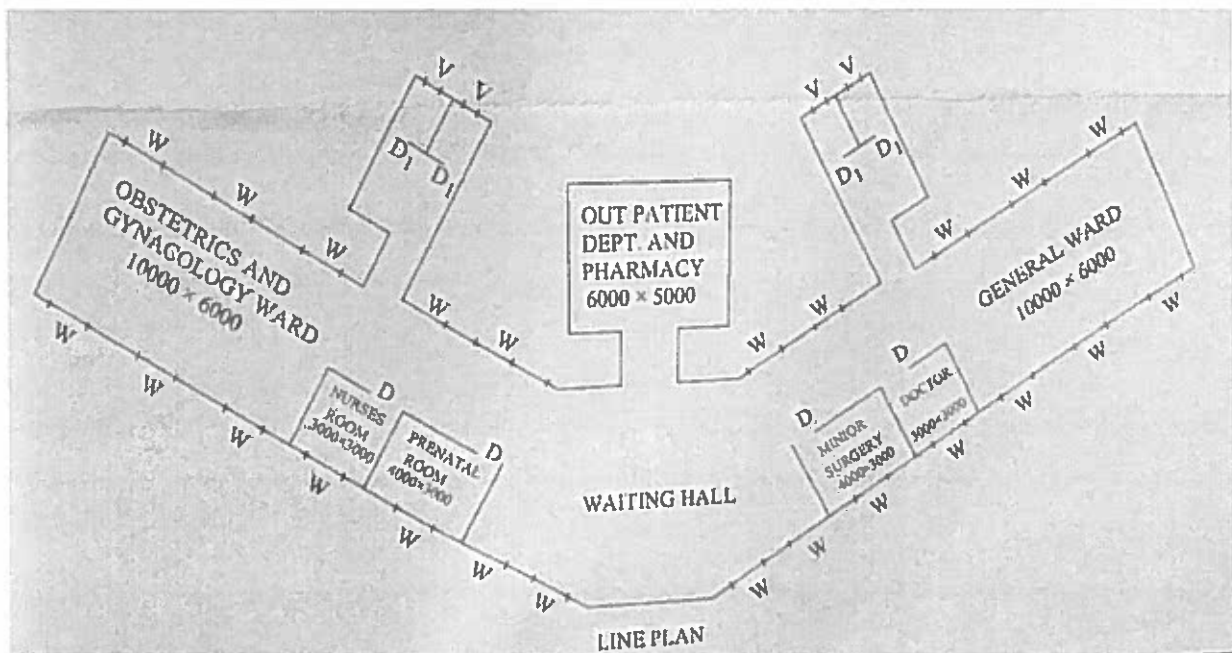


GLAZED DOOR



ANSWER KEY AND SCHEME OF EVALUATION

5a) Layout of health centre



5b) Principles of planning a hospital:

1. Aspect:

- Entrance door should be wide enough as a patient is usually accompanied by escorts or may be brought horizontally on stretcher
- Large windows in sufficient numbers are essentially for a hospital

7M

2. Privacy:

- Privacy is a vital for obstructs and gynaecology and at every level of surgery

3. Roominess:

- Sick men require more room compared to healthy men. Hence wards should be more spacious

4. Grouping:

- Casualty ward should be nearer to the entrance
- Obstetrics and gynaecology are grouped. Paediatrics is also to be planned
- Outpatient department and pharmacy should be close to each other

5. Circulation:

- Separate entrance is to be provided for doctors, nurses and other hospital staff and for patients and their accompanying relatives or friends
- Emergency entrance leading to intensive care unit should be provided nearer to the lift



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ANSWER KEY AND SCHEME OF EVALUATION

- In addition to stairs, ramps and lifts big enough to harbour stretcher and trolley should be provided for easy and efficient vertical circulation

6. Sanitation:

- Noiselessness, free circulation of natural breeze and exposures to the morning sun keeps patient cheerful helping them to recover quickly.
- Serene environment is to be preserved by planning the hospital away from crowded areas and places of excessive noise.
- A patient cannot tolerate loud sound or even less intense noise over a longer period. Cleanliness is a must for a hospital.
- Floors which can be cleaned and walls of bright appearance and bright ceiling free from defect is essential and suitable building material are choose to achieve the same

7. Elegance:

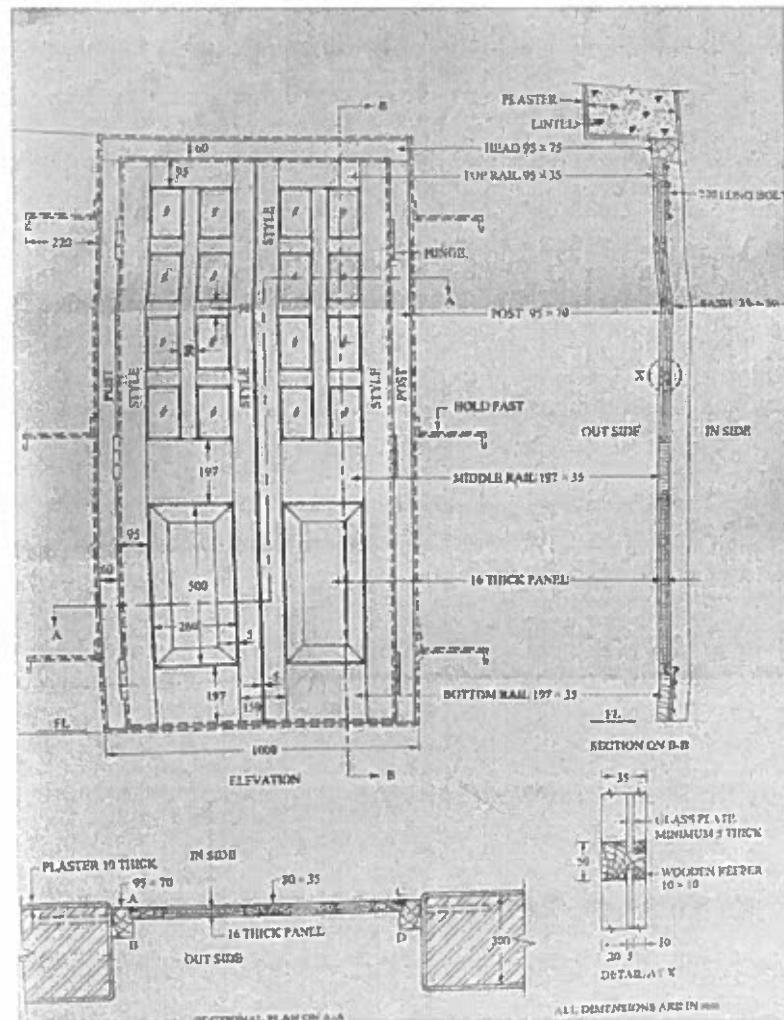
- A minimum plinth height of 750 mm creates effective drainage
- Head room greater than 3.5 m, and more width and height of doors and windows and use of superior building material considerably enhance its appearance beside reducing its cost of maintenance



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ANSWER KEY AND SCHEME OF EVALUATION
PART-B

6a) Sketch of partly paneled and glaze door



Lay out $\rightarrow$ 5M
Components indication
- 4M
Dimensions & Neatness
5M

Drawn in Drawing Sheet

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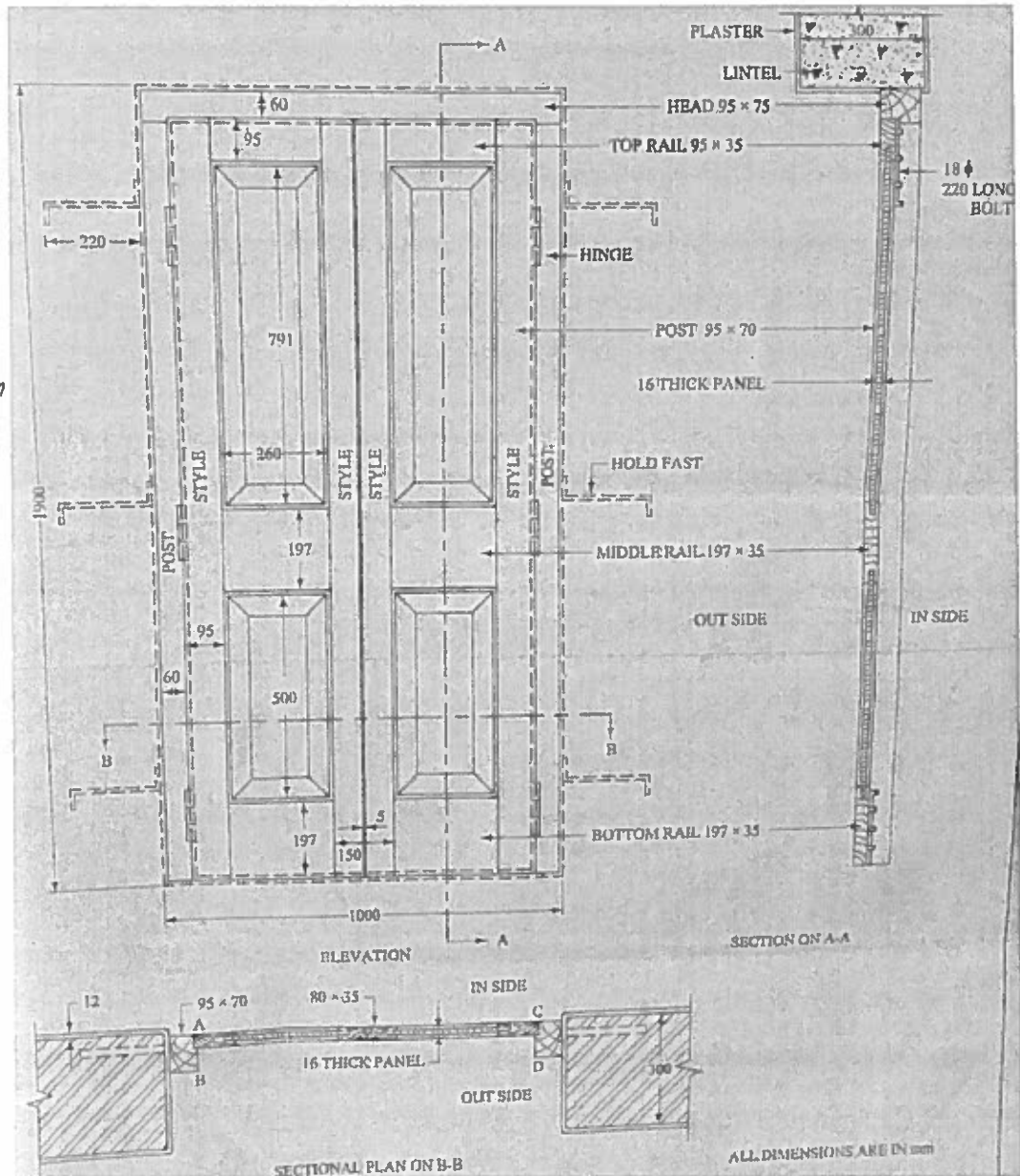
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ANSWER KEY AND SCHEME OF EVALUATION

6b) Fully panelled double shutter door:



Drawn in Drawing sheet



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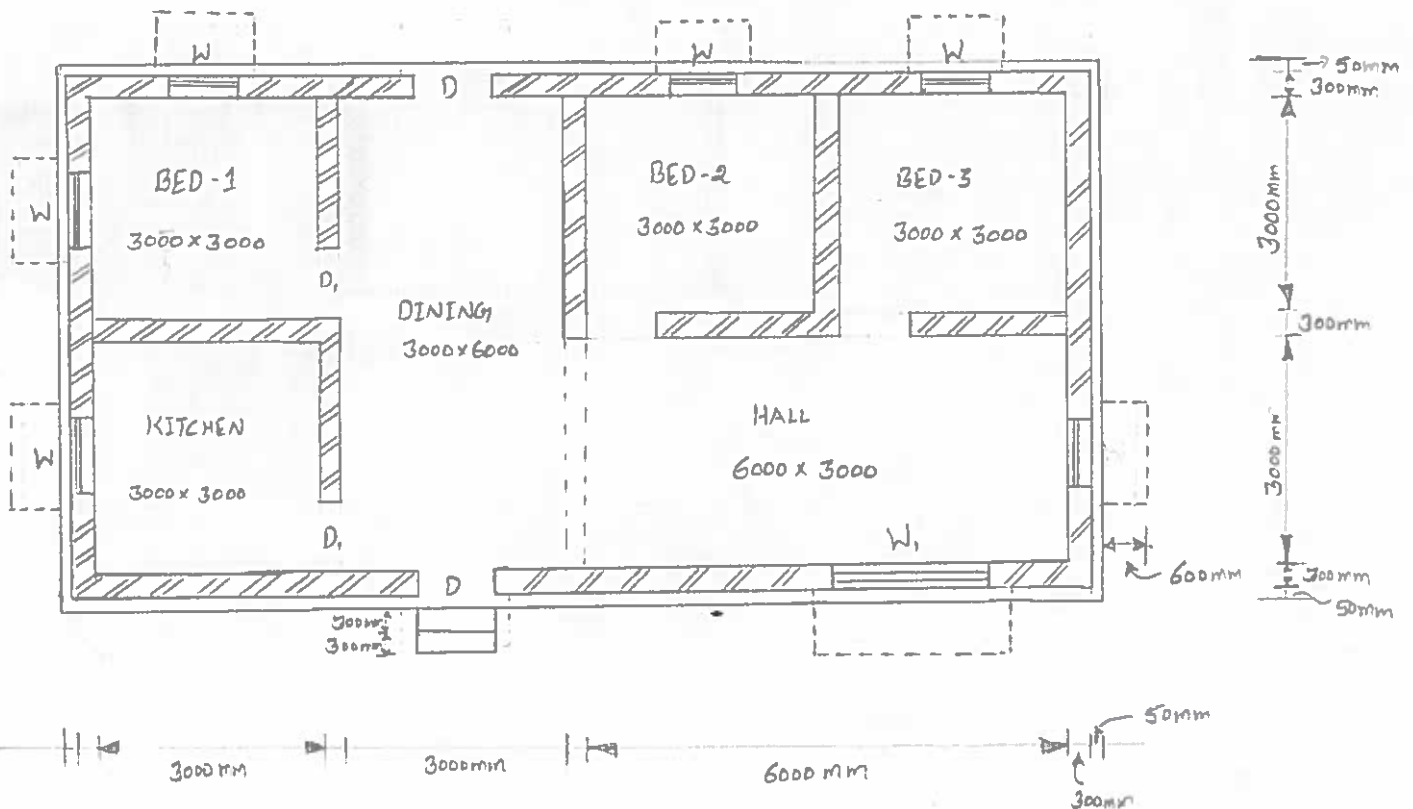
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ANSWER KEY AND SCHEME OF EVALUATION

- 7) Taking 1:100 scale.
All dimensions are in mm
Wall thickness taken as 300mm
Sunshades of projection 600mm

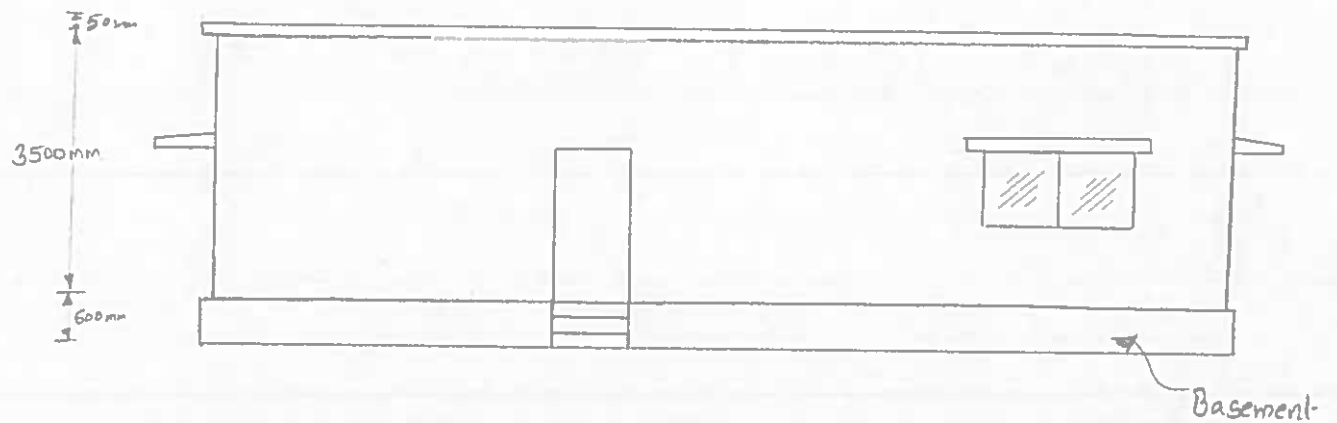
PLAN → 16M → Lay out - 5M
→ Material indication - 5M
→ Dimensions & Neatness → 6M



PLAN

Drawn on Drawing sheet

Elevation → 12M → Layout 6M
Components indication,
dimensions & Neatness 6M



FRONT ELEVATION

Drawn in Drawing Sheet.